

**EQUIVALENCE OF PRIMAL AND DUAL SIMPLEX ALGORITHMS
FOR THE MAXIMUM FLOW PROBLEM**

THESIS

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THE STATE ISLAMIC UNIVERSITY MAULANA MALIK IBRAHIM
OF MALANG
2010**

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A Thesis Submitted to:
The Science And Technology Faculty
The State Islamic University Maulana Malik Ibrahim
Of Malang In Partial Fulfillment For The Requirements
For The Degree Of Bachelor Of Science (S.Si)

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MOTTO

إِعْمَلْ لِدُنْيَاكَ كَأَنَّكَ تَعِيشُ أَبَدًا وَاعْمَلْ لِآخِرَتِكَ كَأَنَّكَ تَمُوتُ غَدًا. (رواه ابن عساکر)

It means: "Working for the world as if you were to live forever, and work for your hereafter as if you were to die tomorrow"

“To move toward the point, do very well. Do not ever feel is ripe, because the sign will soon rot. Stay green felt, and you'll have plenty of room to learn and develop” (the forgotten man)



DEDICATION

This work is dedicated with love and grateful to beloved people in my life

for their great deal of support, encouragement and inspiration

My parents, Father and Mother, Abdul Aziz and Siti Ruqoiyah,

My older brother, Misbahul Munir, My sister in law, Puji Lestari (mba iput)

And for my cutest nephew, Darma Bintang Munir Putra

To people who dare to dream and make it true.



ACKNOWLEDGMENT

Praise be to Allah the Almighty and Merciful, who has given me great miracle in every struggle, so that I can finished in writing my thesis entitled **Equivalence of Primal and Dual Simplex Algorithms For the Maximum Flow Problem**. Peace be upon the prophet Muhammad SAW who has brought us into Islam to reach brighter life.

I would like to thank those who helped me in finishing this thesis. The first sincere gratitude goes to the Rector of UIN Maulana Malik Ibrahim Malang, Prof.Dr.H.Imam Suprayogo, the Dean of Science and Technology Faculty, Prof. Drs Sutiman Bambang Sumitro, SU, D.Sc the Dean of Mathematic Department, Abdussakir,M.Pd, for their valuable knowledge given to me. I am particularly grateful to my advisor, Hairur Rahman, M.Si and Dr. Ahmad Barizi, M.A for their encouragement, insightful suggestions, and critical comments, which became the best guide in writing this study.

I would also like to express my gratitude to all of lecturers whom I have over the years developed my knowledge and experience. In addition, I also say so many thanks for the director of al-Hikmah al-Fathimiyyah Islamic boarding house, Drs. H. Yahya Dja'far, MA and Dra. Hj. Syafiyah Fattah, MA, whose praying and guidance make me step forward easier.

I would like to express my sincere gratitude to my family who has provided me a great deal of support and encouragement during my study in UIN Maulana Malik Ibrahim Malang. Never ending love from me to my beloved parents for their love, cares, praying and support. And to my siblings who have given me cheerful and joyful days. Also for my cutest nephew, it is great to be your Tante.

Last but not least, I also thank all my friends in Mathematics Department 2006, especially for Siti Nurul Afiyah, Varhana, Nanang Kartiyadi, Vidya Desi Rahmawati and Erni Nur Indah Lestari my best friends for beautiful friendship, great experience and

motivation. Let's bring our dreams into reality. Success is ours. And for Mas Ridho, Mba Ninik, Mba Fifi, Mba Nilna and Mas Beny. Always smile in your life. And for my friends in AHAF, particularly for Lutfi, Emy, Fida, Hindun, Isna, Mida, Lely, Dewi, Hayu, Rahmatin and mba Nayla, who always support, understand, and give beautiful days. I am also very grateful to the one who taught me the audacity of hope and dream.

Finally, I truly realize that this thesis is still far from perfection. Thus, I will always appreciate for the constructive criticism and suggestion from the readers. Hopefully, this study can give a valuable contribution to the Mathematics field.

Malang, July 2, 2010

Mariyatul Azizah



ABSTRACT

Azizah, Mariyatul. 2010. **Equivalence Of Primal And Dual Simplex Algorithms For The Maximum Flow Problem.** *Thesis*, Mathematics Department, Science And Technology Faculty, The State Islamic University Maulana Malik Ibrahim Of Malang, Advisor (I) Hairur Rahman, M.Si. Advisor (II) Dr. Ahmad Barizi, M.A.

Key Words: Simplex Primal, Simplex Dual, Maximum Flow

Simplex algorithm is a mathematical procedure for solving linear programming problems over and over with a way to test the angle points that meet the constraints to find the extreme point of the corner points that will maximize or minimize the objective function.

Mathematical model of linear programming problems must first be modified in order to wake-matrix contains the identity of mathematics must be solved by using simplex algorithm. Building a slack variable is formed by bringing mathematics, surplus variables and variables in the form of artificial constraints that limit the need and requirement. In this case, the presence of an artificial variable as a variable which is zero at the optimal solution requires the use of a number M , ie a very large number of often called "Big M ", the coefficient of artificial variables in the objective function. If the objective function is maximized, then $-M$ is the coefficient of artificial variables. Conversely, if the objective function is minimized, then $+M$ is the coefficient.

Optimality condition: Entering Variable in maximizing (minimize) is a variable coefficient non basis the most negative (positive) in the equation destination z . Coefficient with the same value can be chosen arbitrarily. Optimum value is achieved if all the coefficients in the equation z non basic nonnegative (non positive). Feasibility conditions: either to maximize or minimize problem, the leaving variable is the current basic variables that have the smallest cut point (the minimum ratio in the denominator is strictly positive) to the variable entries. The same value can be chosen arbitrarily.

Pivot variable can be determined using Gauss-Jordan elimination. This method begins by identifying the columns under variable was included as an entry field (trespassing column). Lines associated with the variable of the equation called the pivot and elements in the intersection between the entrance and common pivot column called pivot elements. Gauss-Jordan method to make a change on the basis of the use of two types of calculations:

1. pivot equation:
new pivot equation= last pivot equation/ pivot element
2. all other equations including z
new equation= (last equation-entering column coefisien) x new pivot equation

Both types of calculations are basically looking for a new basic solution by substituting out the variables included in all equations, except in the pivot variable.

ملخص البحث

الباحث : مارية العزيزة, ٠٦٥١٠٠١٠ Equivalence Of Primal And Dual Simplex Algorithms For The Maximum Flow Problem. البحث الجامعي. في شعبة الرياضيات بكلية العلوم والتكنولوجيا الجامعة الإسلامية الحكومية مولانا مالك ابراهيم بمالانج ٢٠١٠ . تحت الإشراف : الأستاذ هيرر رحمان, الماجستير و الأستاذ الدكتور احمد بارزي, الماجستير.

Simplex Algorithm هو إجراء رياضي لحل هذه المشكلة المتكررة من البرمجة الخطية في Linear Programming نقطة الزاوية التي تقي القيود العثور على رأس المدقع من النقاط التي من شأنها تحقيق أقصى قدر من الزاوية أو التقليل من وظيفة الهدف

يجب أولاً أن النموذج الرياضي لمشكلة البرمجة Linear Programming يمكن تعديلها لتصبح بعد المحتوى على هوية مصفوفة الرياضية بحيث يتم حل التحيز باستخدام الخوارزمية البسيط. وتشكلت بناء عن طريق الجمع الحسابية متغير الركون ، وفائض المتغير والمتغيرات في شكل مصطنع من القيود التي تحد من شرط وضرورة. في هذه الحالة ، فإن وجود المتغير كمتغير الاصطناعية التي يتم صفر في الحل الأمثل يتطلب استخدام أرقام M ، أي عدد كبير جدا او في كثير من الأحيان كما دعا M كبيرة ، ومعامل صناعية المتغيرات في دالة الهدف. إذا كانت وظيفة تكبير الهدف ، ثم $-M$ في المناطق الجبلية هي معامل المتغيرات الاصطناعي. على العكس من ذلك ، إذا تم تصغير دالة الهدف ، ثم $+M$ هو معامل.

شرط الأمثلية : يتم تضمين المتغير في تعظيم (الحد) هو متغير معامل غير الأساسية والأكثر سلبية (إيجابية) في الوجهة المعادلة Z ويمكن اختيار معامل بنفس القيمة تعسفية. ويتم تحقيق القيمة المثلى إذا كانت كافة المعاملات في المعادلة ض غير الأساسية غير سلبية غير ايجابي. شروط الجدوى : إما تكبير أو تصغير المشكلة ، المتغير هو من المتغيرات الأساسية الحالية التي لديها نقطة أصغر قطع (نسبة الحد الأدنى في القاسم بدقة إيجابي) إلى إدخال متغير. ويمكن اختيار نفس القيمة تعسفية.

ويمكن تحديد المتغيرات المحورية باستخدام الغاوس والأردن القضاء هذا الأسلوب يبدأ من خلال تحديد الأعمدة في إطار متغير دخل كإدخال العمود دخول column يسمى خطوط المرتبطة متغير خارج معادلة محور دعا وعناصر في تقاطع بين العمود ومدخل معادلة محور العنصر المحوري . الغاوس والأردن طريقة لإجراء تغييرات على أساس استخدام نوعين من الحسابات: محور المعادلة:

- محور المعادلة الجديدة= طول محور المعادلة/ العنصر المحوري
- جميع المعادلات الأخرى ، بما في ذلك Z

معادلة جديدة=المعادلات طول -أعمدة دخول معامل X محور المعادلة الجديدة
كلا النوعين من العمليات الحسابية التي تبحث أساسا للتوصل إلى حل الأساسية الجديدة عن طريق الاستعاضة عن المتغيرات المدرجة في جميع المعادلات ، ما عدا في متغير محوري.

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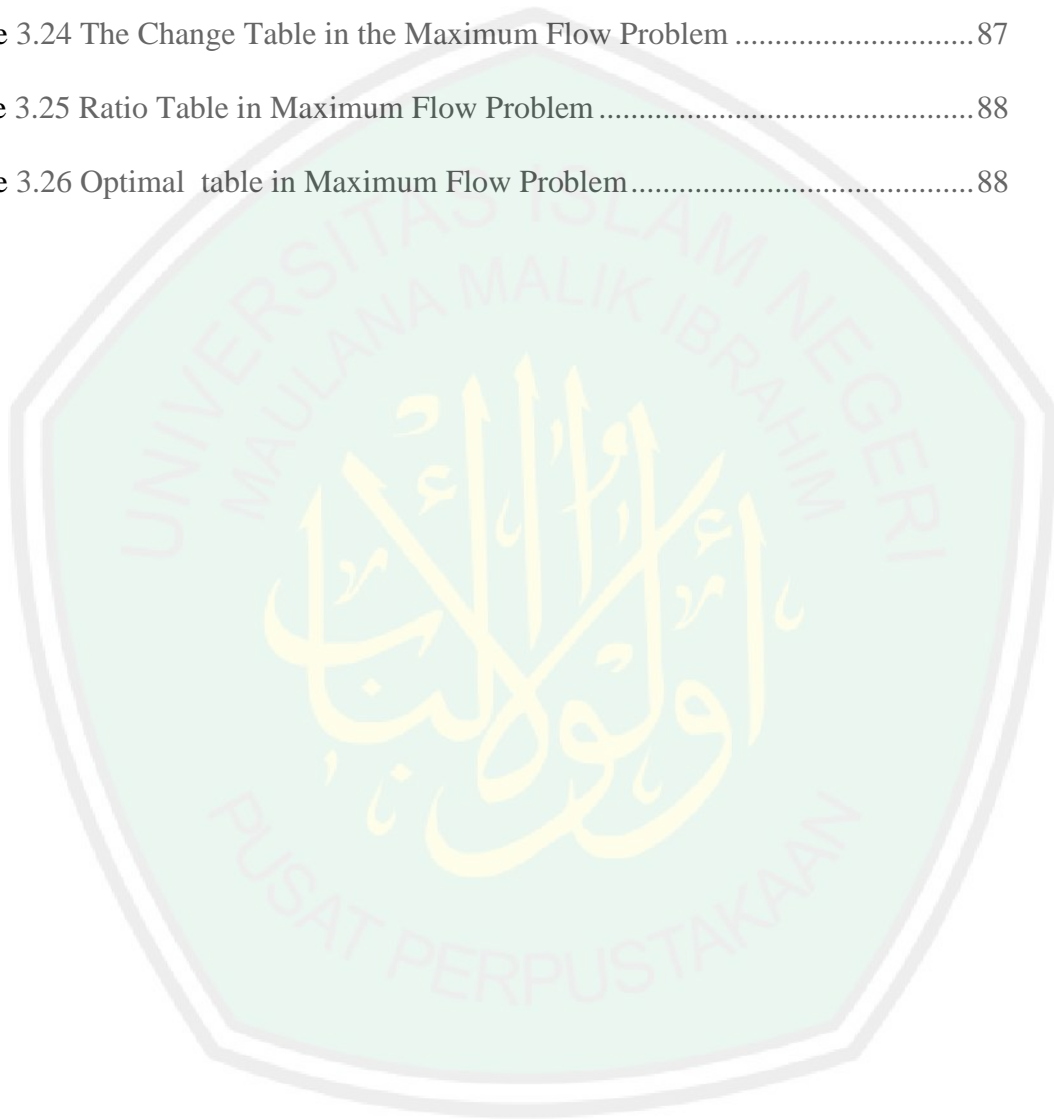
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CHAPTER I

INTRODUCTION

This chapter presents background of the study, statement of the problems, objectives of the study, limitation of the study, significant of the study, research methods, systematic discussion.

1.1 Background of the Study

ظَهَرَ الْفَسَادُ فِي الْبَرِّ وَالْبَحْرِ بِمَا كَسَبَتْ أَيْدِي النَّاسِ لِيُذِيقَهُمْ بَعْضَ الَّذِي عَمِلُوا لَعَلَّهُمْ

يَرْجِعُونَ ﴿٤١﴾

Meaning: After seeing the damage on land and at sea caused by the actions of human hands, so God told them to feel part of the (result of) their deeds, so they come back (to the right path). The Qur'an: Ar - Rum: 41.

Indonesia a country that continued to hit by tragedy in recent years. Should we as Muslims introspection, what's really happening in the world, such as landslides, floods, tsunamis, earthquakes and other disasters that Allah SWT give to this country.

Learning from the disaster that God gave to us as a servant. So we really introspection mistakes we have done the best as individuals but man contained in a sphere, including community organizations, companies and others. And then think how to change these conditions better than the original. Where God says:

إِنَّ اللَّهَ لَا يُغَيِّرُ مَا بِقَوْمٍ حَتَّى يُغَيِّرُوا مَا بِأَنْفُسِهِمْ ۗ وَإِذَا أَرَادَ اللَّهُ بِقَوْمٍ سُوءًا فَلَا مَرَدَّ لَهُ ۗ

وَمَا لَهُمْ مِّنْ دُونِهِ ۗ مِنْ وَالٍ ﴿١١﴾

"Verily Allah does not change the situation so that they change the situation which is in themselves. And if Allah wills evil for a nation, then no one can deny, and sometimes there is no protector for them besides Him". (The Qur'an, Ar-Ra'du: 11)

This verse actually gives such great wisdom of God will not change the state, nation and country, but they are in the (State) does not change and introspection what is causing the decline, chaos and multi-dimensional crisis that hit the country. (Suherlan, 2006)

The key of each individual is always loyal and obedient to Allah SWT. With faith and fear of the Lord will provide a good blessing from the heavens and the earth which is a priceless wealth, because so much that God gave to us.

وَمِنْهُمْ مَّنْ يَقُولُ رَبَّنَا آتِنَا فِي الدُّنْيَا حَسَنَةً وَفِي الْآخِرَةِ حَسَنَةً وَقِنَا عَذَابَ النَّارِ ﴿٢٠١﴾

Meaning: "And among them are those who say: Our Lord, grant us good in this world and good in the Hereafter us from the torment of hell". (The Qur'an, Al-Baqarah: 201)

The balance will be seen or shown in the maximum flow problem, or better known as the Maximum Flow Problem. The maximum flow problem which we encounter in many ways, such as the traffic that we can always meet every time, especially in big cities like Jakarta, Surabaya, then the maximum flow problem can be found also in the channel of water flowing from the center toward house - the house.

The maximum flow problem is a specialized Linear Programming problem (LP) and can be solved by specialized network simplex method. "Simplex Method" is the method used to identify the initial basic solution and then move systematically to the basic solution that has the potential to increase the value of objective function, which in the end the basic solution in accordance with the optimal value will be identified and the calculation process ends. In turn, the "simplex method" is a calculation procedure is repeated (again) in which each repetition (iteration) associated with the initial basic solution. (Hamdy, 1996: 61)

The primal simplex algorithm for the minimum cost flow problem, subsequently referred to as the network simplex algorithm, has been extensively studied in the literature. Researchers have also been interested in developing polynomial time network simplex algorithms for the minimum cost flow problem and its special cases. Polynomial time primal simplex algorithms have been developed for the shortest path problem, the assignment problem and the maximum flow problem. The only polynomial time simplex algorithm for the minimum cost flow problem is a dual simplex algorithm due to Orlin [1984] which

performs $O(n^{\log n})$ pivots for the uncapacitated minimum cost flow problem. Developing a polynomial time primal simplex algorithm for the minimum cost flow problem is still an open problem. (Revindra, 1996: 4)

In the table implementation of the primal simplex algorithm, the right-hand-side column is always nonnegative so the basic solution is feasible at every iteration. For purposes of this section, we will say that the basis for the tableau is primal feasible if all elements of the right-hand side are nonnegative. Alternatively, when some of the elements are negative, we say that the basis is primal infeasible. Up to this point we have always been concerned with primal feasible bases. For the primal simplex algorithm, some elements in row 0 will be negative until the final iteration when the optimality conditions are satisfied. In the event that all elements of row 0 are nonnegative, we say that the associated basis is dual feasible. Alternatively, if some of the elements of row 0 are negative, we have a dual infeasible basis.

As described, the primal simplex method works with primal feasible, but dual infeasible (non optimal) bases. At the final (optimal) solution, the basis is both primal and dual feasible. Throughout the process we maintain primal feasibility and drive toward dual feasibility. (Hamdy, 1996: 82)

In this section, a variant of the primal approach, known as the dual simplex method, is considered that works in just the opposite fashion. Until the final iteration, each basis examined is primal infeasible (some negative values on the right-hand side) and dual feasible (all elements in row 0 are nonnegative). At the final (optimal) iteration the solution will be both primal and dual feasible. Throughout the process we maintain dual feasibility and drive toward primal feasibility. For a given problem, both the primal and dual simplex algorithms will terminate at the same solution but arrive there from different directions.

The dual simplex algorithm is most suited for problems for which an initial dual feasible solution is easily available. It is particularly useful for reoptimizing a problem after a constraint has been added or some parameters have been changed so that the previously optimal basis is no longer feasible.

Although the simplex method is very efficient in practice, it may stall in intermediate stages due to degenerate pivoting. Recently, a new least-squares primal-dual (LSPD) algorithm which guarantees non-degenerate pivoting in each iteration has been proposed to solve LPs with good performance. In each primal-dual iteration, the LSPD algorithm solves a nonnegative least-squares (NNLS) sub problem to obtain an improving direction for its dual variables. Exchanging the role of the primal and dual formulations in LSPD, this paper investigates a new least-squares dual-primal (LSDP) algorithm which is also imperious to degenerate pivots. When solving for an s-t max-flow problem, we show the NNLS sub problem in our LSDP algorithm is equivalent to calculating the current on an electrical network with diodes, where the orientation and the capacity associated with an arc correspond to the orientation of the current and the resistance along that arc, respectively.

Allah SWT has given a mandate to man to be a caliph in the earth. Associated with the mandate of Almighty God to give authority to man to utilize all available resources on the earth within reasonable limits for the common weal. Allah SWT says:

وَابْتَغِ فِيمَا آتَاكَ اللَّهُ الدَّارَ الْآخِرَةَ وَلَا تَنْسَ نَصِيبَكَ مِنَ الدُّنْيَا وَأَحْسِن
 كَمَا أَحْسَنَ اللَّهُ إِلَيْكَ وَلَا تَبْغِ الْفَسَادَ فِي الْأَرْضِ إِنَّ اللَّهَ لَا يُحِبُّ الْمُفْسِدِينَ ﴿٧٧﴾

Meaning: "And seek what Allah has provided in the Hereafter, and do not forget the life of the world. And you do good as Allah has been good to you. And if you're looking for mischief on the earth". The Qur'an: al-Qasas: 77.

قُلْ هَلْ نُنَبِّئُكُمْ بِالْأَخْسَرِينَ أَعْمَالًا ﴿١٠٣﴾ الَّذِينَ ضَلَّ سَعْيُهُمْ فِي الْحَيَاةِ الدُّنْيَا وَهُمْ

تَحْسَبُونَ أَنَّهُمْ مُحْسِنُونَ صُنْعًا ﴿١٠٤﴾

Meaning: "Say: Shall we tell you about the people who lost most of the action? That people who have been in vain deeds in this life, while they thought that they were doing their best". The Qur'an: al-Kahfi: 103-104.

When we refer to implicit and explicit meaning of the verse shows that the philosophy of life interaction (economics) in Islam must be based on efforts to conduct economic activities by holding the commands and prohibitions of God, which is based awareness of man's relationship with Allah SWT .(Hatta,2006)

In an article entitled "Equivalence of Primal and Dual Simplex Algorithm for the Maximum Flow Problem" is the author hopes there are some benefits to be provided, both in terms of science or in anything else, and do not forget also the author also welcome any suggestions and criticisms for improvement in subsequent writing.

1.2 Statement of the Problems

1. How to know about Primal Simplex Algorithm?
2. How to know about Dual Simplex Algorithm?
3. How to know about Equivalence of Primal and Dual Simplex Algorithms for the Maximum Flow Problem?

1.3 Objectives of the Study

1. Knowing about Primal Simplex Algorithm.
2. Knowing about Dual Simplex Algorithm.
3. Knowing about Equivalence of Primal and Dual Simplex Algorithms for the Maximum Flow Problem.

1.4 Limitation of the Study

In this writing, writer only limiting how to finish Maximum Flow Problem in this research using Transportations Problems by using Primal and Dual Simplex Algorithms.

1.5 Significant of the Study

For Authors

1. Ability to apply Operations Research Subjects who had studied in college in everyday life.
2. As a learning tool and an exercise to examine a problem.
3. Add knowledge and insights.

For Readers

1. Enrich the insight in using mathematics.
2. Help readers who want to learn and broaden their knowledge about science, particularly in the application of mathematics.
3. As the supporting literature, especially for math students

1.6 Research Methods

1.6.1 Operational Definitions Of Variables

1.6.1.1 Independent Variables

The independent variable is a function of the variable constraints in the form of restrictions in the primal simplex method and dual simplex method.

1.6.1.2 Depend Variable

Bound variable is a function of the variables in this goal is represented by the maximum flow problem to be searched.

1.6.2 Research Design

This research is the study of literature , namely the search for references relating to the maximum flow problem and related references primal simplex algorithm and dual simplex. Equivalence of Primal and Dual Simplex Algorithms for the Maximum Flow Problem can be expressed in the objective function. In Primal Simplex method starting from the basic feasible solution (extreme point) and continue to repeat through the proper solution to the following basis to achieve optimum point and Simplex problems we usually call the main problem. The interesting thing to note, that the fundamental problem can be solved by other means, ie change the line into columns. Especially when the simplex problem is too many lines, while the number of columns a bit.

1.7 Systematic Discussion

Description in this thesis consists of 4 chapters that systematic are as follows:

CHAPTER I: Introduction of: Background of the Study, Statement Of The Problems, Objectives of the Study, Limitation of the Study, research methods , systematic discussion

CHAPTER II : Review of The Related Literature, consisting of: a description of the theories about Primal simplex algorithm, dual simplex algorithm and Max Flow Problem.

CHAPTER III: Discussion, consisting of: how to know using Primal and Dual Simplex Algorithm in Maximum Flow Problem.

CHAPTER IV: Conclusions and Suggestions

CHAPTER II

REVIEW OF THE RELATED LITERATURE

2.1 Linear Programming

Many people agree when developments are placed in a sequence of linear programming and the most important scientific advances, certainly since the invention of linear programming and the growth that is strong enough in various fields. Until now, linear programming is used as standard equipment can save thousands, even millions of dollars for many companies with different size scales. So also happened in other sectors indicated by the text and articles on the application of linear programming are numbered in the hundreds.

In the practice of everyday activities, some of the decisions relating to the business managers to utilize resources effectively can be: raw materials, labor, machinery, watches and money. Resources used to produce goods (goods of food, clothing or household) and services (transportation, cinema or salon). In the widely used linear programming and mathematical techniques to help managers make decisions about the allocation of (limited) resources.

Linear programming uses a mathematical model that describes the problem faced. Linear means that all the mathematical functions in the model should be a linear function, while the programming / planning can be defined as programming. So the linear programming can be defined as an activity program using a general model in solving resource allocation with limited resources optimally. Problems can arise when a person or company is faced with choices every level of activities undertaken, where each activity requires inputs of the same or nearly the same, but limited.

The concept of linear programming was first developed and introduced by George Dantzig in the form of LP method solution to the problem with many decision variables. Dantzig working in the field of research, mathematical techniques for solving problems of military logistics of the United States Air Force during World War II. His research is

supported by J Von Neumann, L. TC.Koopmans Hurwics and working in the same field. Original technique is the interdependence of program activities in a linear structure and then simplified into a linear programming. After the war ended, many experts who joined in developing the concept Dantzig Linear Programming. The first essay is a solution method called the simplex method, published in 1947. (Zulfikarijah, 2004: 15-17)

A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities. Here is a simple example.

Find numbers x_1 and x_2 that maximize the sum $x_1 + x_2$ subject to the constraints $x_1 \geq 0, x_2 \geq 0$ and

$$\begin{aligned}x_1 + 2x_2 &\leq 4 \\4x_1 + 2x_2 &\leq 12 \\-x_1 + x_2 &\leq 1\end{aligned}\tag{2.1}$$

In this problem there are two unknowns, and five constraints. All the constraints are inequalities and they are all linear in the sense that each involves an inequality in some linear function of the variables. The first two constraints, $x_1 \geq 0$ and $x_2 \geq 0$, are special. These are called non negativity constraints and are often found in linear programming problems. The other constraints are then called the main constraints. The function to be maximized (or minimized) is called the objective function. Here, the objective function is $x_1 + x_2$.

Since there are only two variables, we can solve this problem by graphing the set of points in the plane that satisfies all the constraints (called the constraint set) and then finding which point of this set maximizes the value of the objective function. Each inequality constraint is satisfied by a half-plane of points, and the constraint set is the intersection of all

the half-planes. In the present example, the constraint set is the five-sided figure shaded in Figure 1.

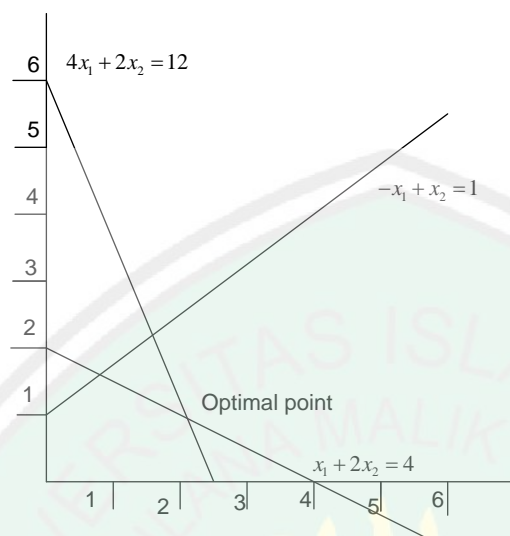


Figure 2.1: Graphing the set of Points

We seek the point (x_1, x_2) , that achieves the maximum of $x_1 + x_2$ as (x_1, x_2) ranges over this constraint set. The function $x_1 + x_2$ is constant on lines with slope -1 , for example the line $x_1 + x_2 = 1$, and as we move this line further from the origin up and to the right, the value of $x_1 + x_2$ increases. Therefore, we seek the line of slope -1 that is farthest from the origin and still touches the constraint set. This occurs at the intersection of the lines $x_1 + 2x_2 = 4$ and $4x_1 + 2x_2 = 12$, namely $(x_1, x_2) = (8/3, 2/3)$. The value of the objective function there is $(8/3) + (2/3) = 10/3$.

It is easy to see in general that the objective function, being linear, always takes on its maximum (or minimum) value at a corner point of the constraint set, provided the constraint set is bounded. Occasionally, the maximum occurs along an entire edge or face of the constraint set, but then the maximum occurs at a corner point as well.

Not all linear programming problems are so easily solved. There may be many variables and many constraints. Some variables may be constrained to be nonnegative and others unconstrained. Some of the main constraints may be equalities and others inequalities.

However, two classes of problems, called here the standard maximum problem and the standard minimum problem, play a special role. In these problems, all variables are constrained to be nonnegative, and all main constraints are inequalities.

We are given an m -vector, $b = (b_1, \dots, b_m)^T$, an n -vector, $c = (c_1, \dots, c_n)^T$, and an $m \times n$ matrix,

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

of real numbers.

The Standard Maximum Problem: Find an n -vector, $x = (x_1, \dots, x_n)^T$, to maximize

$$c^T x = c_1 x_1 + \dots + c_n x_n \tag{2.2}$$

subject to the constraints

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m \end{aligned} \tag{2.3} \quad (\text{or } Ax \leq b)$$

And

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0 \tag{or } x \geq 0$$

The Standard Minimum Problem: Find an m -vector, $y = (y_1, \dots, y_m)$, to minimize

$$y^T b = y_1 b_1 + \dots + y_m b_m \tag{2.4}$$

Subject to the constraint

$$\begin{aligned} y_1 a_{11} + y_2 a_{21} + \dots + y_m a_{m1} &\geq c_1 \\ y_1 a_{12} + y_2 a_{22} + \dots + y_m a_{m2} &\geq c_2 \\ &\vdots \\ y_1 a_{1n} + y_2 a_{2n} + \dots + y_m a_{mn} &\geq c_n \end{aligned} \tag{2.5}$$

$$\text{or } y^T A \geq c^T$$

And

$$y_1 > 0, y_2 > 0, \dots, y_m > 0 \quad (\text{or } y > 0)$$

Note that the main constraints are written as \leq for the standard maximum problem and \geq for the standard minimum problem. The introductory example is a standard maximum problem. (Ferguson, 1991:1-4)

Linear programming is one of the most successful disciplines within the field of operation research. In its standard form, the linear programming problem calls for finding nonnegative x_1, \dots, x_n so as to maximize a linear function $\sum_{j=1}^n c_j x_j$ subject to a system of linear equations:

$$\begin{aligned} a_{11}x_1 + \dots + a_{1n}x_n &= b_1 \\ &\vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n &= b_m \end{aligned} \quad (2.6)$$

This problem can be stated in vector notation as

$$\text{Maximize } c^T x$$

$$\text{Subject to } Ax = b$$

$$x \geq 0$$

Where $A \in R^{m \times n}$ is assumed to have linearly independent rows, and $b \in R^m$ and $c, x \in R^n$. In fact, any problem of maximizing or minimizing a linear function subject to linear equations and inequalities can be easily reduced to the standard form.

The dual problem of the linear programming problem in standard form is

$$\text{minimize } b^T y$$

$$\text{subject to } A^T y \geq c$$

The former problem is then referred to as the primal. The duality theorem asserts that

(i) for any x that satisfies the constraints of primal and for any y that satisfies the conditions of the dual, $c^T x \leq b^T y$, and (ii) if there exist such x and y , then the maximum of the primal

equals the minimum of the dual. The duality theorem plays a central role in the theory of linear programming.

The linear programming model has been applied in large number of areas including military applications, transportations and distributions, scheduling, production and inventory management, telecommunication, agriculture and more. Many problems simply lend themselves to a linear programming solution but in many cases some ingenuity is required for the modeling. Linear programming also has interesting theoretical applications in combinatorial optimization and complexity theory.

The classical tool for solving the linear programming problem in practice is the class of simplex algorithms proposed and developed by George Dantzing. The method is based on generating a sequence of bases. A basis is a nonsingular submatrix of \mathbf{A} of order $m \times m$. The fundamental characteristic of the method is that at some point a basis is reached which provides a solution to the problem. A suitable basis can certify either that the problem has no solution at all or that it is unbounded, otherwise a basis will be reached which defines optimal solutions for both the primal and the dual problems. Due to the problems of degeneracy, special care is needed to guarantee that the method will not cycle. Bland proved that the “least index” rules guarantees that.

Recently, methods of nonlinear programming methods have also become practical tools for certain classes of linear programming problems.

The computational complexity of linear programming had puzzled researchers even before the field of computational complexity started to develop. The question of finding bounds on the diameter and height of polytopes is closely related to the complexity of the simplex method.

Practitioners have noticed long ago that the simplex algorithms performed surprisingly well. In particular, the number of iterations seemed linear in the number of rows

m and sublinear in the number of columns n in the standard form. Klee and Minty, however, found examples where the number of iterations performed by certain variants of the method was exponential.

The area of computational complexity was developed mainly during the 1970's and 1980's. Within this field, the question of the complexity of linear programming was given a new meaning. Complexity theorists were interested in the relation between the running time measured in bit operations and the length L of the representation of the problem with integer data in binary encoding. Khachiyan was the first to show that the linear programming problem was in the class P, that is, it could be solved in time polynomial in the length of the binary encoding of the input. Khachiyan's result was based on the ellipsoid algorithm which had been first proposed by Shor for general convex programming. In the ellipsoid method the problem is reduced to finding a solution to a system of strict inequalities $Ax > b$ whose set of solutions is bounded. It generates a sequence of ellipsoids each of which is guaranteed to contain all the solutions. If a center of any ellipsoid in this sequence is a solution, then it is discovered. Otherwise the process stops when the volume of the current ellipsoid is too small to contain all the solutions if there exist any, so the conclusion in this case is that there are no solutions.

Theorists were quite disappointed when it became clear that the ellipsoid algorithm was not useful for solving linear programming problems in practice. The contrast between the ellipsoid and the simplex methods gave an excellent example that theory could not always be relied upon for predicting applicability. Variants of the simplex method, which were proven to be exponential in the worst case, were very efficient in practice, while the polynomial ellipsoid method was very inefficient. With this observation, researchers became interested in analyzing the behavior of the simplex method from a more practical point of view and at the same time were also searching for other methods of solving the problem.

Breakthroughs in the analysis of the simplex method were made by Borgwardt and Smale. The former was the first to show that under a certain probabilistic model of distribution of inputs, a certain variant of the method runs in polynomial time. Subsequently, under a different model, a bound of $O(m^2)$ was proven by Todd, Adler and Megiddo, and Adler, Karp and Shamir.

A new polynomial-time algorithm for linear programming was proposed by Karmarkar in 1984. His algorithm works on the problem in the form:

$$\begin{aligned} & \text{minimize } c^T x \\ & \text{subject to } Ax = 0 \\ & e^T x = 1 \\ & x \geq 0 \end{aligned}$$

Where $e = (1, \dots, 1)^T \in R^n$, assuming (without loss of generality) that the minimum equals 0 and $Ae=0$. Karmarkar's algorithm generates a sequence of points in the interior of the domain defined by constraint which converges to an optimal point. The algorithm is based on repeated centering by a projective scaling transformation. Given the current iterate x^k , the transformation maps any vector $x \in R^n$ such that $e^T x = 1$ to a vector $x' \in R^n$, where $x'_j = (x_j/x_j^k)/(1 + \sum_i x_i/x_i^k)$ ($j = 1, \dots, n$). Karmarkar's projective scaling algorithm provided an improved upper bound on the complexity of linear programming under the bit operations model. It runs in $O(nL)$ iterations and was reported to be very practical, but most of the computational experience was done with a simplified version of it, called *primal affine scaling*. Here the scaling transformation is simply $x'_j = x_j/x_j^k$. The primal affine scaling algorithm is not believed to run in solving system of m inequalities and n variables, with at most two variables per inequality (where as the general case can be reduced to at most three variables per inequality), which runs in a number arithmetic operations bounded by a

polynomial in m and n . Such an algorithm is said to run in strongly polynomial time. Improved algorithms for this problem were recently proposed by Cohen and Megiddo. (Megiddo, 1991:1-5)

LP Model Formulation

Usual decision problem faced by analysts is the optimum allocation of scarce resources. Resources can be capital, labor, raw materials, machine capacity, time, space or technology. Our task is to achieve the best results with limited resources. Desired results can be expressed as the maximization of some measures such as profit, sales and welfare, or minimization such as cost, time and distance.

Once a problem is identified, the purpose is applied, the next step is to construct a mathematical model that includes three stages:

1. Determine the unknown variables (decision variables) and expressed in mathematical symbols
2. Form of the objective function is expressed as a linear relationship (not multiplication) decision variables
3. Determine all the problems and constraints expressed in equations and inequalities is also a linear relationship of decision variables that reflect the problem of limited resources.

This model is not a science but an art and it will be understood primarily because of the practice.

Examples of Combination Products Issues

A company wants to determine how much each of three different products to be produced with the availability of limited resources for maximum benefit. Inventories of raw materials and labor and profit contribution of each product as follows:

Table 2.1: Combination Products Issues Table

Product	resource needs		Profit
	Labour (hours/unit)	Source (kg/unit)	
A	5	4	3
B	2	6	5
C	4	3	2

Available 240 hours of work and materials as much as 400 Kg. The problem is to determine the amount of each product for maximum profits. His formulation of the LP model are:

2.1.1 Decision Variables

Three variables in this problem is the product of A, B and C that will be generated.

This number can be expressed as:

X_1 = number of products A

X_2 = number of products B

X_3 = number of products C

2.1.2 Objective Function

The objective problem is a combination of products to maximize total profits. Obvious advantage is the amount of profits earned from each product. Gains from product A is a multiplication of the number of product A with earnings per unit (USD 3, -). Benefits of products B and C determined in the same way. So the total profit of Z, can be written:

$$Z = 3X_1 + 5X_2 + 2X_3 \quad (2.7)$$

2.1.3 Constraint System

In this issue the problem is the amount of labor and raw materials is limited. Each product requires that both labor and raw materials. Product A, the labor required to produce each unit is 5 hours, so that the workers needed for the product is $5X_1$ hours. in similar

fashion products requires $2X_2$ hours of labor. while the number of labor hours available is 240 hours. So it can be written:

$$5X_1 + 2X_2 + 4X_3 \leq 240 \quad (2.8)$$

Raw material constraints formulated in the same way, namely for product A requires as much as 4 kg of raw materials per unit, product B takes 6 kg per unit and C takes 3 kg of product per unit. Because that is available is as much as 400 kg of raw material, it can be written:

$$4X_1 + 6X_2 + 3X_3 \leq 400 \quad (2.9)$$

We also limit each variable on only positive values, because it is not possible to produce a number of negative products. This obstacle is known as a non-negative constraints and mathematically can be written:

$$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0 \text{ or } X_1, X_2, X_3 \geq 0$$

Question that arises is why the constraints are written with the sign of inequality (\leq) instead of equation ($=$). The equation implies that the entire capacity of resource use, while allowing use of capacities of inequality is used wholly or partly of capacity. In some cases a solution by enabling the capacity of the unused resources will provide a better solution, which means bigger profits, from the use of all resources. Thus, the imbalance shows flexibility.

The above formulation of the problem can be written in full LP

$$\text{Maximize } Z = 3X_1 + 5X_2 + 2X_3 \quad (2.10)$$

with constraint

$$\begin{aligned} 5X_1 + 2X_2 + 4X_3 &\leq 240 \\ 4X_1 + 6X_2 + 3X_3 &\leq 400 \\ X_1, X_2, X_3 &\geq 0 \end{aligned} \quad (2.11)$$

2.1.4 General Form of LP Models

From the above examples that have been written in depth look at the pattern typical to formulate a general LP problem. In each issue, the decision variable is set, the objective function and constraint system, which together form a mathematical model of the real world.

General form of the LP model are:

$$\text{Maximize (minimize) } Z = \sum_{j=1}^n c_j x_j \quad (2.12)$$

with constraint: $a_{ij}x_j (\leq, =, \geq) b_i$, for all $i(i=1,2,\dots,m)$ for $x_j \geq 0$

explanation:

x_j : many activities j, with $j = 1, 2, \dots, n$, which means there is a decision n variable

Z : the objective function value

c_j : contribution per unit of activity j, for a maximization problem also does not indicate or revenue per unit, while in the case was shown to minimize the cost per unit.

b_i : amount of resources $i(i=1,2,\dots,m)$ means that there are types m of resources

x_{ij} : amount of resources i consumed resources j

To remember is the sign of inequality will not be the same for every obstacle!

2.1.5 LP Model Assumptions

LP model contains certain implicit assumptions that must be met in order for the definition as a LP problem becomes invalid. That assumption requires that the functional relationship of the problem is linear and additive, can be divided and deterministic.

2.1.6 Linearity and Additivity

That the objective function and all constraints must be linear. In other words, if a problem involving two decision variables, in two-dimensional diagram he will form a straight line. Similarly, a constraint involving three variables will result in flat terrain and obstacles involving n variables will produce a hyperplane (a flat geometry) in the space dimension n.

Linear word implies that the proportional relationship which means that the rate of change or slope of the functional relationship that is constant and therefore change the variable value will result in changes in the relative value of the objective function in the same amount.

LP also requires that the total number of criteria and use variable resources should be additive. For example, Z is the total profit criterion variable, equal to the amount of profits obtained from their respective activities, c_j, x_j . Also, all resources used for all activities, must be equal to the amount of resources used for each activity.

Additives can be interpreted as a lack of adjustment to the calculation criteria for the interaction variables. For example, the problem of combination products is mentioned that the profit per unit of product A USD 3, -, product B USD 5, -, and product C Rp 2, -. If each product sold separately. But it could be, if sold simultaneously on the same area can cause a decrease in profits, so that adjustments need to enter into the interaction variable criteria, such as:

$$Z = 3X_1 + 5X_2 + 2X_3 - X_1X_2X_3 \quad (2.13)$$

2.1.7 Divisibility

This assumption means that the value obtained by the solution x, not necessarily integers. This means the value of x can be corrupted values. Therefore, decision variables are continuous variables, as opposed to discrete or integer variables.

2.1.8 Deterministic

All parameters of the model (it does not work, a_{ij} and b_i) are known assumed to be constant. LP does not directly consider the decision problem in a static framework in which all parameters are known with certainty. In fact, rare model parameters is deterministic, because they reflect the future circumstances and conditions present, and future are rarely known for certain.

There are several ways to overcome the uncertainty parameters in the LP model. Sensitivity analysis is a technique developed to test the value of the solution, how the sensitivity to parameter changes.

2.1.9 Applications of Linear Programming

Linear Programming has a large number of applications. Any textbook on operations research is filled with examples of linear programming, and it is now a standard tool taught to student in most business schools. The election scenario is one typical example. Two more examples of linear programming are the following:

1. An airline wishes to schedule its flight crews. The Federal Aviation Administration imposes many constraint, such as limiting the number of consecutive hours that each crew member can work and insisting that a particular crew work only on one model of aircraft during each month. The airline wants to schedule crews on all of its flights using as few crew members as possible.
2. An oil company wants to decide where to drill for oil. Siting a drill at a particular location has an associated cost and based on geological surveys, an expected pay off of some number of barrels of oil. The company has a limited budget for locating new drills and want to maximize the amount of oil it expects to find, given this budget.

Linear Programs also are useful for modeling and solving graph and combinatorial problems.

2.1.10 Algorithms For Linear Programming

This algorithm, when implemented carefully, often solves general linear programs quickly in practice. With some carefully contrived inputs, however the simplex algorithm can require exponential time. The first polynomial-time algorithm for linear programming was the ellipsoid algorithm, which runs slowly in practice. A second class of polynomial-time

algorithms are known as interior- point method. In contrast to the simplex algorithm, which moves along the exterior of the feasible region and maintains a feasible solution that is a vertex of the simplex at each iteration, these algorithms move through the interior of the feasible region. The intermediate solution, while feasible, are not necessarily vertices of the simplex, but the final solution is a vertex. The first such algorithm was discovered by Karmarkar. For large inputs, the performance of interior- point algorithms can be competitive with and sometimes faster than the simplex algorithm. (Cormen, 2004: 776-777)

2.2 The Simplex Algorithm

The Simplex Algorithm was an algebraic approach designed to solve linear programming problems without having to resort to drawing the graphs of the inequalities. It is the best way to solve more complex linear programming problem if you have more than two variables.

LP Standard Form

An LP model could include the prohibition of all kinds ($\leq, \geq, =$). In addition, variables can be nonnegative or not limited to the mark. To develop a method of solving the general LP problem must be placed within the same format, which is called the standard format. This properties as follows:

1. all constraints are equality
2. all variables are nonnegative
3. objective function can be either maximize or minimasi

The second trait that requires all variables must be non-negative is very important in the development of simplex method (primal and dual).

2.2.1 Limitation

1. A variable of type (\leq, \geq) can be transformed into an equation by adding the slack variables (subtracting surplus variables from the left side of the boundary). For example, in

$$3x_1 + x_2 \geq 3 \quad \text{or} \quad -3x_1 - x_2 \leq -3 \quad (2.14)$$

we add a slack $x_3 \geq 0$ to the left side to obtain the equation

$$-3x_1 - x_2 + x_3 = -3, x_3 \geq 0 \quad (2.15)$$

Then consider the limit

$$4x_1 + 3x_2 \geq 6 \quad (2.16)$$

Because the left side is now no smaller than the right side, we subtract a surplus variable $x_4 \geq 0$ from the left side to obtain the equation

$$4x_1 + 3x_2 - x_4 = 6, x_4 \geq 0 \quad (2.17)$$

2. The right side of the equation can always be made nonnegative by multiplying both sides by -1. for example, ~~$2x_1 + 3x_2 - 7x_3 = -5$~~ is mathematically equivalent to

~~$$-2x_1 - 3x_2 + 7x_3 = 5$$~~

3. the inequality is reversed when both sides are multiplied by -1 such as $2 < 4$, $-2 > -4$

4. So inequality $2x_1 - x_2 \leq -5$ can be replaced with $-2x_1 + x_2 \leq 5$.

2.2.2 Variable

Variable unrestricted y_i can be expressed in the form of two nonnegative variables using the substitution

$$y_i = y_i' - y_i'' \quad y_i', y_i'' \geq 0 \quad (2.18)$$

substitution should be applied at all boundaries and in the objective function.

LP problem is usually solved in the form of y_i' and y_i'' , that it y_i determined by back substitution. The interesting properties y_i' and y_i'' is that in LP (simplex) are optimum only one of the two variables can have a positive value, but never both. So when $y_i' > 0, y_i'' = 0$, and vice versa in the case where y_i (unrestricted) representing both slack and surplus we can see y_i' as a slack variable and y_i'' as a surplus variable because only one of them can have a

positive value in one moment. This observation is widely used in programming the target and in fact is the basis for the idea of conversion.

2.2.3 Objective Function

Although the standard LP model that maximizes or minimize manifolds, conversion from one form to another is sometimes useful. Maximizing the function equals the negative of minimize the same function, and vice versa. For example

$$\max \quad z = 3x_1 + 2x_2 \quad (2.19)$$

is mathematically equivalent to

$$\min \quad (-z) = -3x_1 - 2x_2 \quad (2.20)$$

Equality means that for the same group of constraints, the optimal value x_1 and x_2 is the same in both cases. The only difference is that the value of objective function, although the same numerical, will appear with signs differently. (Hamdy,1996: 62-64)

Write the following models in standard form

$$\min \quad z = 3x_1 + 2x_2 \quad (2.21)$$

with restrictions

$$\begin{aligned} -3x_1 - x_2 &\leq -3 \\ -4x_1 - 3x_2 &\leq -6 \\ x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned} \quad (2.22)$$

The following changes must be made as below:

1. Add the slack variable $x_3 \geq 0$ to the left side of the first restriction
2. Add the slack variable $x_4 \geq 0$ to the left side of the second restriction
3. Add the slack variable x_5 to the left side of the third restriction

So the restrictions become

$$\begin{aligned} -3x_1 - x_2 + x_3 &= -3 \\ -4x_1 - 3x_2 + x_4 &= -6 \\ x_1 + x_2 + x_5 &= 3 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

Example of Simplex Algorithm

min $z = 3x_1 + 2x_2$
with restriction

$$\begin{aligned} -3x_1 - x_2 &\leq -3 \\ -4x_1 - 3x_2 &\leq -6 \\ x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

With add slack variables, so the equations becomes

min $z = 3x_1 + 2x_2 + 0x_3 + 0x_4 + 0x_5$

with restriction

$$\begin{aligned} -3x_1 - x_2 + x_3 &= -3 \\ -4x_1 - 3x_2 + x_4 &= -6 \\ x_1 + x_2 + x_5 &= 3 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

The equation of objective function can be write

$$z - 3x_1 - 2x_2 - 0x_3 - 0x_4 - 0x_5 = 0 \tag{2.23}$$

Table 2.2: Table of Simplex Algorithm

Z_j	Z	x_1	x_2	x_3	x_4	x_5	Solutions
C_j	1	-3	-2	0	0	0	0
x_3	0	-3	-1	1	0	0	-3
x_4	0	-4	-3	0	1	0	-6
x_5	0	1	1	0	0	1	3

x_1 become entering variable

x_3 become leaving variable

-3 is pivot variable and should become the main dividing the value of -3, so that the entire column of the x_3 should be divided by -3.

Every line that exists between the primary, must be zero, so the column should follow the equation $z = 3x_1 + z$

Column x_4 should follow the equation $x_4 = 4x_1 + x_4$

Column x_5 should follow the equation $x_5 = -x_1 + x_5$

Table 2.3: Table change in the Simplex Algorithm

Z_j	Z	x_1	x_2	x_3	x_4	x_5	Solutions
c_j	1	0	-1	-1	0	0	3
x_1	0	1	1/3	-1/3	0	0	1
x_4	0	0	-5/3	-4/3	1	0	-2
x_5	0	0	2/3	1/3	0	1	2

x_2 become entering variable

x_4 become leaving variable

-5/3 is pivot variable and should become the main by dividing the value of -5 / 3, so the entire column x_4 should be divided with the -5/3.

Every line that exists between the primary, must be zero, so the column should follow the equation $z, z = x_2 + z$

Column x_1 should follow the equation $x_1 = -1/3x_2 + x_1$

Column x_5 should follow the equation $x_5 = -2/3x_2 + x_5$

Table 2.4: Optimum Table in the Simplex Algorithm

Z_j	Z	x_1	x_2	x_3	x_4	x_5	Solutions
C_j	1	0	0	-1/5	-3/5	0	21/5
x_1	0	1	0	-3/5	1/5	0	3/5
x_2	0	0	1	4/5	-3/5	0	6/5
x_5	0	0	0	-1/5	2/5	1	6/5

Table above are feasible by $x_1 = 3/5$, $x_2 = 6/5$, and $z = 21/5$. it can be proved by inserting the value x_1 and x_2 to the original objective function is $z = 3x_1 + 2x_2$.

2.3 The Relationship Between Primal Simplex and Dual Simplex Objective Value

Scored on a pair of primal and dual problem must satisfy the following relationship:

1. For each pair of primal and dual solutions are feasible

$$\left(\begin{array}{l} \text{purpose value of} \\ \text{maximizing problems} \end{array} \right) \leq \left(\begin{array}{l} \text{purpose value of} \\ \text{minimization problems} \end{array} \right)$$

2. At the optimum solution for both problems

$$\left(\begin{array}{l} \text{purpose value of} \\ \text{maximizing problems} \end{array} \right) = \left(\begin{array}{l} \text{purpose value of} \\ \text{minimization problems} \end{array} \right)$$

Note carefully that the results do not say anything about where the problem where the primal and dual problems. What is important in this case is the meaning of optimization (maximization and minimization).

To prove the validity of the two results, suppose that $(\mathbf{X}_I, \mathbf{X}_{II})$ and \mathbf{Y} is the primal and dual feasible solutions, which meet the definition of primal-dual is given in the form of a matrix. Then, by multiplying the previous primal constraint by \mathbf{Y} , we obtain

$$YAX_I + YX_{II} = Yb \equiv w \quad (2.24)$$

Then, by multiplying the dual constraint afterwards with X_I and X_{II} , we obtain

$$\begin{aligned} YAX_I &\geq C_I X_I \\ YX_{II} &\geq C_{II} X_{II} \end{aligned} \quad (2.25)$$

(Observe that $X_I \geq 0$ and $X_{II} \geq 0$. so the inequality remains unchanged). then adding the second restriction will produce

$$YAX_I + YX_{II} \geq C_I X_I + C_{II} X_{II} \equiv z \quad (2.26)$$

Because the left side of the identity of the above w and z are the same, we conclude that

$$z \leq w$$

which proves the result given above first.

Now to show that that $z = w$ on the optimal solution, observe that z is related to the maximization, while w is related to the minimization. This means that the highest value of z for all values (X_I, X_{II}) w decent and pursue the lowest among all feasible Y , because $z \leq w$ for all feasible solutions (including the optimal solution), the second issue will reach optimal only if $\max z = \min w$.

Dual Optimal Solution

Dual optimal solution can be determined directly from the table are the optimal primal.

standard primal

$$\text{Max } z = C_I X_I + C_{II} X_{II} \quad (2.27)$$

with restriction

$$\begin{aligned} AX_I + IX_{II} &= b \\ X_I, X_{II} &\geq 0 \end{aligned} \quad (2.28)$$

Dual

$$\text{Min } w = Yb \quad (2.29)$$

with restriction

$$\begin{aligned} YA &\geq C_I \\ Y &\geq C_{II} \end{aligned} \quad (2.30)$$

Y vector that is not restricted

Suppose **B** is the main basis of optimal and assumes that **C_B** is associated with the coefficient of objective function, then

$$Y = C_B B^{-1} \quad (2.31)$$

is the optimal dual solution. To show that this result is correct, we need to examine both the following conditions.

1. $Y = C_B B^{-1}$ is a feasible dual solution
2. Max z in the primal same min w in dual

Dual solution $Y = C_B B^{-1}$ is feasible if the solution meets the dual constraints $YA \geq C_I$ and $Y \geq C_{II}$. the optimality of primal problem we have $z_j - c_j \geq 0$ for all j

$$C_B B^{-1} A - C_I \geq 0 \text{ and } C_B B^{-1} - C_{II} \geq 0 \quad (2.32)$$

with regard $Y = C_B B^{-1}$, we immediately see that both the dual constraints are satisfied. (Hamdy, 1996, 151-154)

The second requirement is verified by showing that $z = w$ for $Y = C_B B^{-1}$. this is directly applicable because

$$\begin{aligned} w &= Yb = C_B B^{-1} b \\ z &= C_B X_B = C_B B^{-1} b \end{aligned} \quad (2.33)$$

No one who does not want the lives of the survivors, happy and prosperous during the inhabitants of the earth. Many people who provide the benchmark of safety, happiness and prosperity only on the material, which will provide a certain socioeconomic status in society. People like living in a world judged that only one time, therefore, must be enjoyed maximally, by doing the things he likes. Intellect and mind fixed on the effort to produce

something that can give out pleasure, as always deal with good food / bad, well dressed, driving luxury, living in a large building with many amenities, which are served by many attendants to complete all business and other purposes and so on. Moreover, even if it might save a lot of wealth in many places, so many of them already do not know how much truth and how to use.

Human life on this earth is always between the two extreme circumstances, such as very good and a bad pair or a noble and despicable or very rich and very poor or the faithful and infidels, and others. Word of the balance when viewed from the two extreme conditions, will cause an error, because it understands that each lot or powerful or as complete. Balance in life does not mean as much or little to attitude is good and the bad or the same quality of faith and believe in thinking, and behavior. Enterprises looking for a balance to live like that, someone will face losses in the world and the Hereafter.

The balance referred to is the ability to think, act and behave the best in facing the need to pursue the safety, happiness and prosperity in this world and the hereafter. Both conditions must be different in both categories for the best. In a more simple sentences to say the balance between material and spiritual life. These people have enough wealth to own property, or the position / positions in government or society to respect, which is used in the path of Allah SWT. People who succeed at the top of the success of living material and spiritual life of believers. Material support for the life of faith is always doing good deeds. Instead his faith to guide and control how and effort in the pursuit of material success and use it in a way that God blessed.(Kaelani, 1982)

In fact, not a few people who found success in material life but failed and became a loser in the spiritual life. In contrast, not a few successful people who are spiritually, but failed in material life, to become people who are not losers, but life does not have to face the

light. For those who believed that if the conditions experienced after making various efforts, not a failure, but the examination of Allah SWT.

Human achievement in realizing a balance between the achievement with the achievement of psychic material in the form of faith, is highly dependent on each individual business. Allah Almighty says in An-Najm 39

وَأَنْ لَّيْسَ لِلْإِنْسَانِ إِلَّا مَا سَعَى ﴿٣٩﴾

Meaning: "And behold, a man no other benefit than what they have done."

Likewise in the surah Al-Baqarah verse 134 Allah says

تِلْكَ أُمَّةٌ قَدْ خَلَتْ لَهَا مَا كَسَبَتْ وَلَكُمْ مَا كَسَبْتُمْ وَلَا تُسْأَلُونَ عَمَّا كَانُوا يَعْمَلُونَ ﴿١٣٤﴾

Meaning: "Those are a people who died for him what you've earned and what you've got, and you will not be asked about what coverage they have done."

Of the two words mentioned above clearly a qualified individual, and living in balance with the material and spiritual attainment of the maximum, depending on their respective businesses. Later in responsible before Allah SWT is also a personal expense of each individual. Therefore, in verses Surah Al-Baqarah: 286 Allah said

لَا يُكَلِّفُ اللَّهُ نَفْسًا إِلَّا وُسْعَهَا لَهَا مَا كَسَبَتْ وَعَلَيْهَا مَا اكْتَسَبَتْ ﴿٢٨٦﴾

Meaning: "Allah burdens not a person but according to his ability he's got the reward (of virtue), it has earned. And he got a punishment (the crime) was doing."

Word of Almighty Allah which shows that human ability is not the same. Therefore, the results of his efforts is not the same. Containing such circumstances, which means that there are people in communities rich and poor, or between them, it is a condition in which normal or human. Similarly, in society there are people who believe that faith who believe in a high or modest, and even many people who kufr and kafir, in a fair and humane conditions. Among them the most fortunate are rich and faithful high. Man was lucky enough that the socioeconomic conditions of mediocrity, but his faith high. The latter was also fortunate for the poor, however high his faith. Conversely a person become very lost when his ability to

become rich, but low faith or unbelievers. Humans become losers if the socioeconomic conditions of mediocrity, his faith or someone who is Kufr thin. In addition, losses are not rivals, when according to his ability to be poor and thin as well as faith or Kufr.

That connection Rasulullah SAW said:

الْمُؤْمِنُ الْقَوِيُّ خَيْرٌ وَأَحَبُّ إِلَى اللَّهِ مِنَ الْمُؤْمِنِ الضَّعِيفِ وَفِي كُلِّ خَيْرٍ

Meaning: "a strong believer is better and more beloved of God than the believer who is weak or for good."

Strong understanding by the word of God that touches all aspects of life. Physically healthy and strong tool for doing good, such as prayer, fasting, hajj, to help others, struggling to establish the religion of Allah Almighty, devotion to work and others without interference ill more frequently. strong believer also means that the property itself to perform such charitable Zakat and alms to the poor, creating a small mosque or the mosque, making way for the public interest, preserving and helping orphans in order to improve the world's fate in life and in the Hereafter, etc. strong believer also means having the mind of language or noble and lofty morality, such as respect for parents, not selfish, like consensus, maintain the honor based on the values taught by Allah SWT to live, love hanging out and respect others and other-Other . Means also strongly believe that people believe that high in doing all the commands and prohibitions of Allah SWT on leave, such as diligent, humble and diligent and disciplined in the pilgrimage, and always avoid alcohol, gambling, prostitution or other sinful acts.

At the next change in the balance as the quality of human life are also displayed in the form of independence, which has been trained since childhood. The lack of independence among others, dependence on others, able to compete and cooperate well with others, have goals and know how to make it happen, both for life in the world and the Hereafter, capable and courageous decisions, to solve problems, save money, careful thought, thorough and

honest, trustworthy and willing for other people believe, not easily discouraged, and positive and forward, etc. (Hadari, 1992: 271-277).

2.4 Maximum Flow Problem

Just as we can model a road map as a directed graph in order to find the shortest path from one point to another, we can also interpret a directed graph as a “flow network” and use it to answer questions about material flows. Imagine a material coursing through a system from a source, where the material is produced, to a sink, where it is consumed. The source produces the material at same steady rate, and the sink consumes the material at the same rate. The “flow” of the material at any point in the system is intuitively the rate at which the material moves. Flow networks can be used to model liquids flowing through pipes, parts through assembly lines, current through electrical networks, information through communication networks, and so forth.

Each directed edge in a flow network can be thought of is a conduit for the material. Each conduit has a stated capacity, given a maximum rate at which the material can flow through the conduit, such as 200 gallons of liquid per hour through a pipe or 20 amperes of electrical current through a wire. Vertices are conduit junctions, and other than the source and sink, material flows through the vertices without collecting in them. In other words, the rate at which material enters a vertex must equal the rate at which it leaves the vertex. We call this property “flow conservation”, and it is equivalent to Kirchhoff’s Current Law when the material is electrical current.

In the maximum-flow problem, we wish to compute the greatest rate at which material can be shipped from the source to the sink without violating any capacity constraints. It is one of the simplest problems concerning flow networks, this problem can be solved by efficient algorithms. Moreover, the basic techniques used in maximum-flow algorithms can be adapted to solve other network-flow problems.

2.4.1 Flow Networks

In this section, we give a graph-theoretic definition of flow networks, discuss their properties, and define the maximum-flow problem precisely. We also introduce some helpful notation.

Flow Networks and Flows

A flow network $G=(V,E)$ is a directed graph in which each edge $(u,v) \in E$ has a nonnegative capacity $c(u,v) \geq 0$. If $(u,v) \notin E$, we assume that $c(u,v) = 0$. We distinguish two vertices in a flow network: a source s and a sink t . For convenience, we assume that every vertex lies on some path from the source to the sink. That is, for every vertex $v \in V$, there is a path $s \rightarrow v \rightarrow t$. The graph is therefore connected, and $|E| \geq |V| - 1$. Figure 2 shows an example of a flow network.

We are now ready to define flows more formally. Let $G=(V,E)$ be a flow network with a capacity function c . Let s be the source of the network, and let t be the sink. A flow in G is a real-valued function $f : V \times V \rightarrow \mathbb{R}$ that satisfies the following three properties:

Capacity constraint: for all $u, v \in V$, we require $f(u, v) \leq c(u, v)$

Skew symmetry: for all $u, v \in V$, we require $f(u, v) = -f(v, u)$

Flow conservation: for all $u \in V - \{s, t\}$, we require

$$\sum_{v \in V} f(u, v) = 0 \quad (2.34)$$

The quantity $f(u, v)$, which can be positive, zero, or negative, is called the flow from vertex u to vertex v . The value of a flow f is defined as

$$|f| = \sum_{v \in V} f(s, v) \quad (2.35)$$

That is, the total flow out of the source. In the maximum-flow problem, we are given a flow network G with source s and sink t , and we wish to find a flow of maximum value.

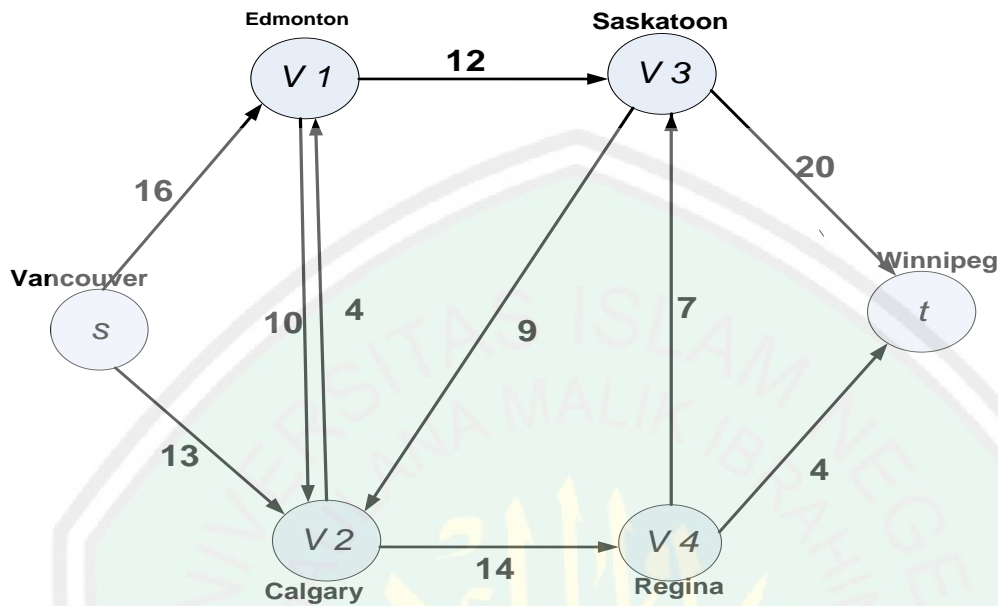


Figure 2.2: A flow network $G = (V, E)$ for the Lucky Puck Company's trucking problem. The Vancouver factory is the source s , and the Winnipeg warehouse is the sink t . Pucks are shipped through intermediate cities, but only $c(u, v)$ crates per day can go from city u to city v . Each edge is labeled with its capacity

Before seeing an example of a network-flow problem, let us briefly explore the three flow properties. The capacity constraint simply says that the flow from one vertex to another must not exceed the given capacity. Skew symmetry is a notational convenience that says that the flow from a vertex u to a vertex v is the negative of the flow in the reverse direction. The flow-conservation property says that the total flow out of a vertex other than the source or sink is 0. By skew symmetry, we can rewrite the flow-conservation property as

$$\sum_{u \in V} f(u, v) = 0 \tag{2.36}$$

For all $v \in V - \{s, t\}$. That is, the total flow into a vertex is 0.

When neither (u, v) nor (v, u) is in E , there can be no flow between u and v , and

$$f(u, v) = f(v, u) = 0.$$

Our last observation concerning the flow properties deals with flows that are positive.

The total positive flow entering a vertex v is defined by

$$\sum_{\substack{u \in V \\ f(u,v) > 0}} f(u,v) \quad (2.37)$$

The total positive flow leaving a vertex is defined symmetrically. We define the total net flow at a vertex to be the total positive flow leaving a vertex minus the total positive flow entering a vertex. One interpretation of the flow-conservation property is that the total positive flow entering a vertex other than the source or sink must equal the total positive flow leaving that vertex. This property, that the total net flow at a vertex must equal 0, is often informally referred to as “flow in equals flow out”.

2.4.2 An Example of Flow

A flow network can model the trucking problem shown in Figure 2. The Lucky Puck Company has a factory (source s) in Vancouver that manufactures hockey pucks, and it has a warehouse (sink t) in Winnipeg that stocks them. Lucky Puck leases space on trucks from another firm to ship the pucks from the factory to the warehouse. Because the trucks travel over specified routes (edges) between cities (vertices) and have a limited capacity, Lucky Puck can ship at most $c(u, v)$ crates per day between each pair of cities u and v in Figure 2. Lucky Puck has no control over these routes and capacities and so cannot alter the flow network shown in Figure 2. Their goal is to determine the largest number p of crates per day that can be shipped and then to produce this amount, since there is no point in producing more pucks than they can ship to their warehouse. Lucky Puck is not concerned with how long it takes for a given puck to get from the factory to the warehouse, they care only that p crates per day leave the factory and p crates per day arrive at the warehouse.

On the surface, it seems appropriate to model the “flow” of shipments with a flow in this network because the number of crates shipped per day from one city to another is subject

to a capacity constraint. Additionally, flow conservation must be obeyed, for in steady state, the rate at which pucks enter an intermediate city must equal the rate at which they leave. Otherwise, crates would accumulate at intermediate cities.

There is one subtle difference between shipments and flows, however. Lucky Puck may ship pucks from Edmonton to Calgary, and they may also ship pucks from Calgary to Edmonton. Suppose that they ship 8 crates per day from Edmonton (v_1 in Figure 2) to Calgary (v_2) and 3 crates per day from Calgary to Edmonton. It may seem natural to represent these shipments directly by flows, but we cannot. The skew-symmetry constraint requires that $f(v_1, v_2) = -f(v_2, v_1)$, but this is clearly not the case if we consider $f(v_1, v_2) = 8$ and $f(v_2, v_1) = 3$.

Lucky Puck may realize that it is pointless to ship 8 crates per day from Edmonton to Calgary and 3 crates from Calgary to Edmonton, when they could achieve the same net effect by shipping 5 crates from Edmonton to Calgary and 0 crates from Calgary to Edmonton (and presumably use fewer resources in this process). We represent this latter scenario with a flow: we have $f(v_1, v_2) = 5$ and $f(v_2, v_1) = -5$. In effect, 3 of the 8 crates per day from v_1 to v_2 are canceled by 3 crates per day from v_2 to v_1 .

In general, cancellation allows us to represent the shipments between two cities by a flow that is positive along at most one of the two edges between the corresponding vertices. That is, any situation in which pucks are shipped in both directions between two cities can be transformed using cancellation into an equivalent situation in which pucks are shipped in one direction only: the direction of positive flow.

Given a flow f that arose from, say, physical shipments, we cannot reconstruct the exact shipments. If we know that $f(v_1, v_2) = 5$, this flow may be because 5 units were shipped from u to v and 3 units were shipped from v to u . Typically, we shall not care how the actual physical shipments are set up, for any pair of vertices, we care only about the net amount that

travels between them. If we do care about the underlying shipments, then we should be using a different model, one that retains information about shipments in both directions.

2.4.3 Networks With Multiple Sources and Sinks

A maximum-flow problem may have several sources and sinks, rather than just one of each. The Lucky Puck Company, for example, might actually have a set of m factories $\{s_1, s_2, \dots, s_m\}$ and a set of n warehouse $\{t_1, t_2, \dots, t_n\}$ as shown in Figure 3(a). Fortunately, this problem is no harder than ordinary maximum flow.

We can reduce the problem of determining a maximum flow in a network with multiple sources and multiple sinks to an ordinary maximum-flow problem. Figure 3(b) shows how to network from (a) can be converted to an ordinary flow network with only a single source and a single sink. We add a supersource s and add a directed edge (s, s_i) with capacity $c(s, s_i) = \infty$ for each $i = 1, 2, \dots, m$. We also create a new supersink t and add a directed edge (t_i, t) with capacity $c(t_i, t) = \infty$ for each $i = 1, 2, \dots, n$. Intuitively, any flow in the network in (a) corresponds to a flow in the network in (b), and vice versa. The single source s simply provides as much flow as desired for the multiple sources s_i , and the single sink t likewise consume as much flow as desired for the multiple sinks t_i .

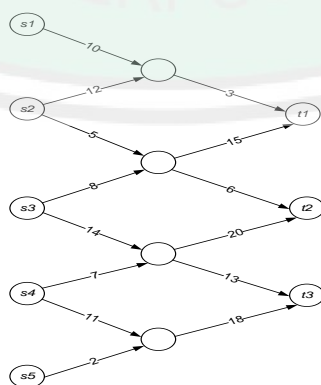


Figure 2.3 : Converting a multiple-source, multiple-sink maximum-flow problem into a problem with a single source and a single sink. A flow network with five sources $S = (s_1, s_2, s_3, s_4, s_5)$ and three sinks $T = (t_1, t_2, t_3)$.

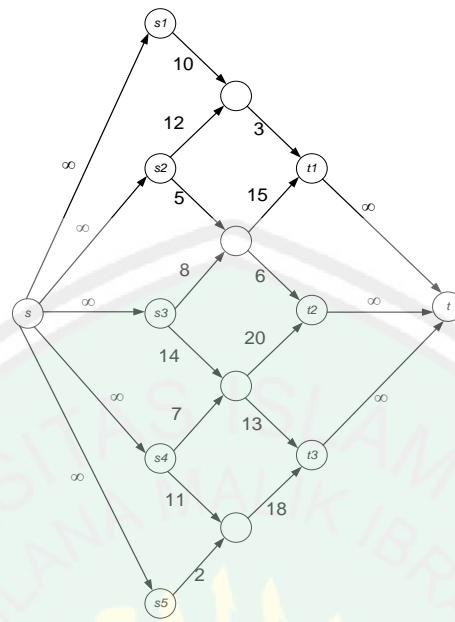


Figure 2.4 : An equivalent single-source, single-sink flow network. We add a supersource s and an edge with infinite capacity from s to each of the multiple sources. We also add a supersink t and an edge with infinite capacity from each of the multiple sinks to t .

2.4.4 Working with Flows

We shall be dealing with several functions (like f) that take as arguments two vertices in a flow network. In this chapter, we shall use an implicit summation notation in which either argument, or both, may be a set of vertices, with the interpretation that the value denoted is the sum of all possible ways of replacing the arguments with their members. For example, if X and Y are sets of vertices, then

$$f(X, Y) = \sum_{x \in X} \sum_{y \in Y} f(x, y) \tag{2.38}$$

Thus, the flow-conservation constraint can be expressed as the condition that $f(u, V) = 0$ for all $u \in V - \{s, t\}$. Also, for convenience, we shall typically omit set braces when they would otherwise be used in the implicit summation notation. For example, in the equation $f(s, V - s) = f(s, V)$, the term $V - s$ means the set $V - \{s\}$. (Cormen, 2004: 808-811)

CHAPTER III DISCUSSION

3.1 The Primal Simplex Algorithm

Primal simplex method starting from the basic feasible solution (extreme point) and continue to repeat through the proper solution to the following basis to achieve optimum point. For example, consider the model in Reddy Mikks standard form given below:

$$\max z = 3x_E + 2x_I + 0s_1 + 0s_2 + 0s_3 + 0s_4 \quad (3.1)$$

with restrictions

$$\begin{aligned} x_E + 2x_I + s_1 &= 6 \\ 2x_E + x_I + s_2 &= 8 \\ -x_E + x_I + s_3 &= 1 \\ x_I + s_4 &= 2 \\ x_E, x_I, s_1, s_2, s_3, s_4 &\geq 0 \end{aligned} \quad (3.2)$$

This model has $m=4$ equalities and $n=6$ variables. So the number of non-basic variables must be equal to zero $6 - 4 = 2$. If we choose $x_I=0$ dan $x_E=0$ as a non-basic variables, directly and without calculation, we obtain the basic feasible solution $s_1=6$, $s_2=8$, $s_3=1$, and $s_4=2$. This basic solution is an initial solution or initial repetition of this simplex method. The appropriate destination determined by the value of objective function expresses the following form (which we will call the equations z):

$$z = 3x_E + 2x_I = \epsilon \quad (3.3)$$

because x_E and x_I is zero, the corresponding z values will be automatically recognized by the right side of the equation above ($=0$).

Simple algebraic determination to the basic solution in the model initial Reddy Mikks is caused by :

1. Each equation has a slack variable
2. The right side of the boundary is nonnegative

The first properties to ensure that the amount of slack variables is equal to the number of equations. So all the remaining variables can be used as a variable non basic (zero). Because it is based on the nature of the right side of the equation 2 nonnegative, the resulting basic solution automatically is feasible, as required by the primal simplex method.

The next logical point that followed the identification of initial solution is to check the movement for a new basic solution. From the viewpoint of optimization, we are interested to move to other basic solution only if we can improvement in the objective function value. Remember the first of the new basic solution is only obtained by making at least one of the variables non basic (zero) now becomes the basic variable. In the simplex method, we make this change by including variables non basic Intuitively, a variable can only non-basic incorporated into the solution if it improves the value of objective function. In the form Reddy Mikks models, x_E and x_I is non basic ($= 0$). By considering again equation z objective

$$z - 3x_E - 2x_I = 0 \quad (3.4)$$

We see that one unit increase in z increases x_E with 3 and one unit increase in z increases x_I by 2. Because we are maximizing, one of the two variables can increase the value of the goal. However, calculations for designing a clear rule, simplex method using the heuristic, ie, in the case of maximizing, non-basic selected variable is a variable that has the most negative coefficient in the equation destination z . Using this heuristic, it is expected (but not guaranteed) that the simplex method will produce a "jump" the largest in the value of the goal, moving from one iteration to the next iteration, thus achieving the minimum optimal iteration. Application of this condition will result in a model x_E Reddy Mikks as entry variable.

New basic solution and that obtained by entering the variable must include the appropriate entry in basic variables. This means that one of the current basic variables must be removed from the solution. We can choose the variables out of the confines of direct

equation simply by counting the intersection of all the nonnegative axis limits x_E . Smallest intersection will identify the variables out. Reddy Mikks In the model, only the limits of 1 and 2 intersect with the axis of x_E in a positive direction, then the intersection of each equal to $6 / 1 = 6$ and $8 / 2 = 4$. Because the crossing is smaller ($= 4$) associated with the second restriction, the basic variables must leave s_2 solution.

How we can automate the process of selecting variables out without the benefit of the solution space graph. All we need to do is to compute all the intersection with the axis limits x_E as the ratio of the right side with boundaries that match the coefficient of x_E ,

$$\text{intersection constraints 1: } \frac{6}{1} = 6$$

$$\text{intersection constraints 2: } \frac{8}{2} = 4$$

$$\text{intersection constraints 3: } \frac{1}{-1} = -1$$

$$\text{intersection constraints 4: } \frac{2}{0} = \notin R$$

The procedure given above to select the variables in and out of variable called a condition of optimality and feasibility conditions. We note that feasibility conditions (minimum point of intersection) can be applied either to maximize or minimize problems. In contrast, the optimal conditions for the different issues in minimize include variables related to the most non basic positive coefficient (compared with the most negative in the case of maximizing).

The following is a formal summary of these two conditions simplex:

Optimality condition: Variable is included in maximizing (minimize) is a variable coefficient non basic the most negative (positive) in the equation destination z . Coefficient with the same value can be chosen arbitrarily. Optimum value is achieved if all the coefficients in the equation z non basic nonnegative (non positive).

Feasibility conditions: either to maximize or minimize problem, the variable out is the current basic variables that have the smallest cut point (the minimum ratio in the denominator is strictly positive) to the variable entries. The same value can be chosen arbitrarily.

We are now ready to present the formal steps primal simplex iteration method:

Step 1: Using the standard form (with the right side of all non-negative), determine the initial basic solution is feasible.

Step 2: Choose a variable in the variables used non basic optimality condition.

Step 3: Choose variables from the basic flow variables using the feasibility condition

Step 4: Determine the value of the new basic variable with variable entry as basis variables and variables come out as non-basic variables.

With some thought, primal simplex method is based on a plausible argument. In particular, at the point of the basic solution (extreme point) specially, we are looking for a new basic solution only if the increase in the value of one variable non basic can enhance the value of the current objective (optimality condition). If we find such non basic variables, one of the basic variables must now leave the solution to meet the requirement that the amount of the basic variables must be precisely equal to m . The selection of variables out achieved by using the feasibility condition. The process of exchange with the basic variables is exactly the same variables non basic movement between extreme points along the edge solutions. Actually this is the essence of the primal simplex method. However, keep in mind that when you begin to explain how the method called Gauss-Jordan method is used to make the switch variables in and out variables.

3.2 Artificial Initial Solution For Primal Simplex Method

We do this by adding artificial variables or where the additional variables needed to play the role of slack variables. However, because the artificial variables such as no physical meaning in the original model (thus given the name "artificial"), provisions should be made

to create a zero at the optimal iteration. In other words, we use artificial variables to start the solution, and then left them after their mission is fulfilled.

We achieve this by using the feedback information, which would make these variables are not interesting from the viewpoint of optimization. One logical way to achieve this goal is to give punishment to the artificial variables in the objective function. Two closely related methods based on the use of punishment available for this purpose: (1) the M method or methods penalty and (2) stage two methods.

3.2.1 M Method (Penalty Method)

We describe this method by using the following numeric example:

$$\min z = 3x_1 + 2x_2 \quad (3.5)$$

With restrictions

$$\begin{aligned} 3x_1 + x_2 &\geq 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned} \quad (3.6)$$

standard form of this model becomes

$$\begin{aligned} \min z &= 3x_1 + 2x_2 \\ \text{with restrictions} \\ 3x_1 + x_2 - x_3 &= 3 \\ 4x_1 + 3x_2 - x_4 &= 6 \\ x_1 + x_2 + x_5 &= 3 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned} \quad (3.7)$$

The first and second equation has no variables that play a role slack variables. So, we add two artificial variables R_1 and R_2 in the second equation is as follows:

$$\begin{aligned} 3x_1 + x_2 - x_3 + R_1 &= 3 \\ 4x_1 + 3x_2 - x_4 + R_2 &= 6 \\ x_1 + x_2 + x_5 &= 3 \end{aligned} \quad (3.8)$$

We may impose penalties on the R_1 and R_2 in the objective function by providing a positive coefficient is very large in the objective function. Let $M > 0$ is a constant that is very large, so the LP with artificial variable becomes

$$\min z = 3x_1 + 2x_2 + MR_1 + MR_2 \quad (3.9)$$

With restriction

$$\begin{aligned} 3x_1 + x_2 - x_3 + R_1 &= 3 \\ 4x_1 + 3x_2 - x_4 + R_2 &= 6 \\ x_1 + x_2 + x_5 &= 3 \\ x_1, x_2, x_3, x_4, x_5, R_1, R_2 &\geq 0 \\ R_1 &= 3 - 3x_1 - x_2 + x_3 \\ R_2 &= 6 - 4x_1 - 3x_2 + x_4 \end{aligned} \quad (3.10)$$

So the equation of objective function is

$$\begin{aligned} \min z &= 3x_1 + 2x_2 + M(3 - 3x_1 - x_2 + x_3) + M(6 - 4x_1 - 3x_2 + x_4) \\ &= 3x_1 + 2x_2 + 3M - 3Mx_1 - Mx_2 + Mx_3 + 6M - 4Mx_1 - 3Mx_2 + Mx_4 \\ &= (3 - 3M - 4M)x_1 + (2 - M - 3M)x_2 + Mx_3 + Mx_4 + 9M \\ &= (3 - 7M)x_1 + (2 - 4M)x_2 + Mx_3 + Mx_4 + 9M \\ z - (3 - 7M)x_1 - (2 - 4M)x_2 - Mx_3 - Mx_4 &= 9M \\ z + (-3 + 7M)x_1 + (-2 + 4M)x_2 - Mx_3 - Mx_4 &= 9M \end{aligned}$$

with restrictions

$$\begin{aligned} 3x_1 + x_2 - x_3 + R_1 &= 3 \\ 4x_1 + 3x_2 - x_4 + R_2 &= 6 \\ x_1 + x_2 + x_5 &= 3 \\ x_1, x_2, x_3, x_4, x_5, R_1, R_2 &\geq 0 \end{aligned}$$

Table 3.1: Table's Of M Method

z_j	z	x_1	x_2	x_3	x_4	x_5	R_1	R_2	Solution
c_j	1	(7M-3)	(4M-2)	-M	-M	0	0	0	9M
R_1	0	3	1	-1	0	0	1	0	3
R_2	0	4	3	0	-1	0	0	1	6
x_5	0	1	1	0	0	1	0	0	3

Since the minimization problem, so the variable entry should have the most positive coefficient in the equation z . (Hamdy, 1996: 76)

x_1 become entering variable

R_1 become leaving variable

3 is pivot variable and should become the main by dividing the value of 3, so the entire column R_1 should be divided with the 3.

Every line that exists between the primary, must be zero, so the column should follow the equation z , $z = -(7M - 3)x_1 + z$

Column R_2 should follow the equation $R_2 = -4x_1 + R_2$

Column x_5 should follow the equation $x_5 = -x_1 + x_5$

Table 3.2 : Table Change in the M Method

z_j	z	x_1	x_2	x_3	x_4	x_5	R_1	R_2	Solution
c_j	1	0	$\left(\frac{5M}{3}-1\right)$	$\left(\frac{4M}{3}-1\right)$	-M	0	$\left(\frac{-7M}{3}+1\right)$	0	2M+3
x_1	0	1	1/3	-1/3	0	0	1/3	0	1
R_2	0	0	5/3	4/3	-1	0	-4/3	1	2
x_5	0	0	2/3	1/3	0	1	-1/3	0	2

x_2 become entering variable

R_2 become leaving variable

5/3 is pivot variable and should become the main by dividing the value of 5/3, so the entire column R_2 should be divided with the 5/3.

Every line that exists between the primary, must be zero, so the column should follow the

equation z , $z = -\left(\frac{5M}{3} - 1\right)x_2 + z$

Column x_1 should follow the equation $x_1 = -\frac{1}{3}x_2 + x_1$

Column x_5 should follow the equation $x_5 = -\frac{2}{3}x_2 + x_5$

Table 3.3: Optimal Table M Method

z_j	z	x_1	x_2	x_3	x_4	x_5	R_1	R_2	Solution
c_j	1	0	0	-1/5	-3/5	0	$\left(-M - \frac{9}{5}\right)$	$\left(-M + \frac{3}{5}\right)$	21/5
x_1	0	1	0	-3/5	1/5	0	3/5	-1/5	3/5
x_2	0	0	1	4/5	-3/5	0	-4/5	3/5	6/5
x_5	0	0	0	-1/5	2/5	1	1/5	-2/5	6/5

table above are feasible by $x_1 = 3/5$, $x_2 = 6/5$, and $z = 21/5$. it can be proved by inserting the value x_1 and x_2 to the original objective function is $z = 3x_1 + 2x_2$.

3.2.2 Two-Stage Technique

One drawback of the technique of M is likely that the results of the calculation of the errors can give too much value to the constant M. Phase Two methods designed to overcome these difficulties. Although the artificial variables are added in the same way as used in the technique of M, using a constant M eliminated by solving the problem in two stages (called method of "two stage"). The second stage is described as follows:

Stage 1. Add artificial variables needed to get the initial solutions. Create a new objective function to minimation number of artificial variables with the original problem variables limitation by this artificial modified If the minimum value of the new objective function is zero (which means that all the artificial variables zero), this problem has a space, fragment right. Proceed to step 2. If not, if the positive minimum value, the problem has no feasible solution. Stop

Step 2. Use the optimum basic solution of step 1 as an initial solution to the original problem.

Stage 1. Because we need artificial variables R_1 and R_2 on first and second equation, the problem of phase 1 can be read

$$\min r = R_1 + R_2 \quad (3.11)$$

with restrictions

$$3x_1 + x_2 - x_3 + R_1 = 3$$

$$4x_1 + 3x_2 - x_4 + R_2 = 6$$

$$x_1 + x_2 + x_5 = 3$$

$$x_1, x_2, x_3, x_4, x_5, R_1, R_2 \geq 0$$

$$R_1 = 3 - 3x_1 - x_2 + x_3$$

$$R_2 = 6 - 4x_1 - 3x_2 + x_4$$

Because R_1 and R_2 , including the initial solution, they should the substitution the following objective function:

$$r = R_1 + R_2 \quad (3.12)$$

$$= (3 - 3x_1 - x_2 + x_3) + (6 - 4x_1 - 3x_2 + x_4)$$

$$= -7x_1 - 4x_2 + x_3 + x_4 + 9$$

$$r + 7x_1 + 4x_2 - x_3 - x_4 = 9$$

Table 3.4: Table Two-Stage Method

<i>basis</i> \ <i>c_j</i>	<i>r</i>	<i>x₁</i>	<i>x₂</i>	<i>x₃</i>	<i>x₄</i>	<i>x₅</i>	<i>R₁</i>	<i>R₂</i>	Solution
<i>c_j</i>	1	7	4	-1	-1	0	0	0	9
<i>R₁</i>	0	3	1	-1	0	0	1	0	3
<i>R₂</i>	0	4	3	0	-1	0	0	1	6
<i>x₅</i>	0	1	1	0	0	1	0	0	3

x_1 become entering variable

R_1 become leaving variable

3 is pivot variable and should become the main by dividing the value of 3, so the entire column R_1 should be divided with the 3.

Every line that exists between the primary, must be zero, so the column should follow the equation r , $r = -7x_1 + r$

Column R_2 should follow the equation $R_2 = -4x_1 + R_2$

Column x_5 should follow the equation $x_5 = -x_1 + x_5$

Table 3.5: Table Changes In a Two-Stage Method

<i>basis</i> \ <i>c_j</i>	<i>r</i>	<i>x₁</i>	<i>x₂</i>	<i>x₃</i>	<i>x₄</i>	<i>x₅</i>	<i>R₁</i>	<i>R₂</i>	Solution
<i>c_j</i>	1	0	5/3	4/3	-1	0	-7/3	0	2
<i>x₁</i>	0	1	1/3	-1/3	0	0	1/3	0	1
<i>R₂</i>	0	0	5/3	4/3	-1	0	-4/3	1	2
<i>x₅</i>	0	0	2/3	1/3	0	1	-1/3	0	2

x_2 become entering variable

R_2 become leaving variable

5/3 is pivot variable and should become the main by dividing the value of 5/3, so the entire column R_2 should be divided with the 5/3.

Every line that exists between the primary, must be zero, so the column should follow the

equation r , $r = -\frac{5}{3}x_2 + r$

Column x_1 should follow the equation $x_1 = -\frac{1}{3}x_2 + x_1$

Column x_5 should follow the equation $x_5 = -\frac{2}{3}x_2 + x_5$

Table 3.6: Tables Of Optimum Two-Stage Method

<i>basis</i>	<i>r</i>	x_1	x_2	x_3	x_4	x_5	R_1	R_2	Solution
c_j	1	0	0	0	0	0	-1	-1	0
x_1	0	1	0	-3/5	1/5	0	3/5	-1/5	3/5
x_2	0	0	1	4/5	-3/5	0	-4/5	3/5	6/5
x_5	0	0	0	-1/5	2/5	1	1/5	-2/5	6/5

Because the minimum $r = 0$, this problem has a feasible solution, therefore we continue to step 2.

Step 2. Artificial variables must now be functioning and should be eliminated in all subsequent calculations. This means that the equation in the table at the optimum phase 1 can be written as

$$\begin{aligned}
 x_1 - \frac{3}{5}x_3 + \frac{1}{5}x_4 &= \frac{3}{5} \\
 x_2 + \frac{4}{5}x_3 - \frac{3}{5}x_4 &= \frac{6}{5} \\
 -\frac{1}{5}x_3 + \frac{2}{5}x_4 + x_5 &= \frac{6}{5}
 \end{aligned}
 \tag{3.13}$$

This equation is exactly the same as the equation in the standard form of the original problem (before the artificial variable is added). So the original problem can be written as

$$\min z = 3x_1 + 2x_2$$

with restrictions

$$\begin{aligned} x_1 - \frac{3}{5}x_3 + \frac{1}{5}x_4 &= \frac{3}{5} \\ x_2 + \frac{4}{5}x_3 - \frac{3}{5}x_4 &= \frac{6}{5} \\ -\frac{1}{5}x_3 + \frac{2}{5}x_4 + x_5 &= \frac{6}{5} \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{3}{5} + \frac{3}{5}x_3 - \frac{1}{5}x_4 \\ x_2 &= \frac{6}{5} - \frac{4}{5}x_3 + \frac{3}{5}x_4 \end{aligned} \tag{3.14}$$

To overcome this problem, we need to change the basic variables x_1 and x_2 in the objective function, which is achieved by using the following boundary equations:

$$\begin{aligned} z &= 3x_1 + 2x_2 \\ &= 3\left(\frac{3}{5} + \frac{3}{5}x_3 - \frac{1}{5}x_4\right) + 2\left(\frac{6}{5} - \frac{4}{5}x_3 + \frac{3}{5}x_4\right) \\ &= \frac{9}{5} + \frac{9}{5}x_3 - \frac{3}{5}x_4 + \frac{12}{5} - \frac{8}{5}x_3 + \frac{6}{5}x_4 \\ &= \frac{1}{5}x_3 + \frac{3}{5}x_4 + \frac{21}{5} \end{aligned}$$

Table 3.7: Table Early For The Second Phase

z_j	Z	x_1	x_2	x_3	x_4	x_5	Solution
c_j	1	0	0	-1/5	-3/5	0	21/5
x_1	0	1	0	-3/5	1/5	0	3/5
x_2	0	0	1	4/5	-3/5	0	6/5
x_5	0	0	0	-1/5	2/5	1	6/5

Table is not optimal, since x_3 and x_4 must include a solution. If we do calculations simplex, we obtain the optimal solution in one iteration.

Elimination of artificial variables at the end of phase 1 was carried out only when all these variables non basic. However, there is the possibility that the artificial variables or more permanently, but at the zero level at the end of stage 1. In such case, the variable must be part of the initial solution stage 2. Thus, the calculation of phase 2 should be changed to prevent the artificial variables have positive values during the iteration in step 2.

Regulations to ensure that zero artificial variable will never be positive in phase 2 is simple. Notice that the column entry, the coefficient restrictions relating to the line of artificial variables can be positive, zero, or negative. If positive, it will automatically be a pivot element (as it will coincide with the minimum ratio of zero) and the artificial variables can definitely be leaving the basic solution for a variable non-basic in the next iteration. Then, if the coefficient is zero, although the pivot element will be somewhere else in the column entry, the nature of line operations to ensure that the artificial lines remain unchanged, which makes artificial basic variable is zero at the desired level. The only case that remains is a negative coefficient. In this case the pivot element must be located elsewhere in the column entry, and if the minimum ratio of a positive result occurs, then the artificial variables will have a positive value on the next iteration. To prevent this from happening, all that did was force must be artificial variable to keep leaving the solution, simply by choosing a negative coefficient as a pivot element for iteration. Although we violate the minimum ratio rule, we will not violate the feasibility problem (which is the core of the regulatory minimum ratio) because the artificial variables have a zero value. Thus, when line operations, the right side of the table remains unchanged and is therefore feasible.

To summarize, the new rules for the selection phase 2 requires artificial variables to solve each time leaving the base as a variable coefficient restrictions in the field of entries has not zero (positive or negative).

3.3 Dual Simplex Method

In this section, we use artificial variables to solve LP problem that has no initial basic solution is feasible and it is a slack variable. There are a bunch of LP problems that do not have the initial basic solution is feasible and is variable loose, but can be resolved without the use of artificial variables. The procedure for solving this problem is called the dual simplex method. In this method, the solution is not feasible and begin optimal (compared to the primal simplex method starting feasible but non optimal).

Example :

$$\min z = 3x_1 + 2x_2$$

with restrictions

$$3x_1 + x_2 \geq 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

If we change the constraint into the form of equations by adding a surplus or slack variables, the problem can be written

$$\min z = 3x_1 + 2x_2$$

with restrictions

$$-3x_1 - x_2 + x_3 = -3$$

$$-4x_1 - 3x_2 + x_4 = -6$$

$$x_1 + x_2 + x_5 = 3$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

(3.15)

Form of the above can be seen as a form of double standard simplex method. This conversion is done in such a way that all variables within the limits of the surplus will have a coefficient of 1 only by multiplying the equation by -1. In this case, the right side boundary to be negative. We can immediately see that the initial basic solution

$$x_3 = -1 \quad x_4 = -6 \quad x_5 = 3$$

not feasible. This is also the optimal solution (in fact, better than the optimum) because the value associated with z is zero ($x_1 = x_2 = 0$) that can not be even smaller because $x_1 \geq 0$ and $x_2 \geq 0$, because the coefficients are both positive (ie, 3 and 2). This initial condition is a common condition for the application of dual simplex method.

You will see that the dual simplex iterations continue to do with the point angle (or extreme), as in the primal simplex method. Thus, successive iterations can be obtained by applying the row operations used Gauss-Jordan. The main difference between this procedure occurs in how the variables in and out of the selected variables.

Table 3.8 : Table Simplex Algorithm For the Dual Start

Basis	x_1	x_2	x_3	x_4	x_5	Solution
Z	-3	-2	0	0	0	0
x_3	-3	-1	1	0	0	-3
x_4	-4	-3	0	1	0	-6
x_5	1	1	0	0	1	3

In the table above, note that the goal line meets optimality condition (minimization). This solution is not feasible because of x_3 and x_4 has a negative value. This is the condition (optimal and not worthy) required for initial iteration in the dual simplex method.

Since the purpose of the dual simplex is rid of this incongruity, then what should be done is to force out the negative basic variables of the solution. Although both x_3 (= - 3) or x_4 (= - 6) will meet this goal, the general rule that requires removal of at least worthy of variables (most negative) among all candidates in the hope that it will bring us to a proper solution in a way fastest. So, we chose a variable x_4 out.

Now a variable to be chosen from among current non basic variable with the stipulation that optimality should be maintained. This is achieved by taking the ratio of the

coefficient on the left side of the equation z consistent with variable coefficients in the equation out. To maintain the optimum, with the denominator of the ratio of positive or zero deleted. Input variables associated with the ratio variable has the smallest value.

Table 3.9: Table Ratio is Calculated as Follows

Variables	x_1	x_2	x_3	x_4	x_5
equality z	-3	-2	0	0	0
equality x_4	-4	-3	0	1	0
Ratio	3/4	2/3	-	-	-

The ratio shows that the solution will enter the x_2 . Thus, the following table is obtained by using the usual row line operations, thus leading to the following table:

Table 3.10: The Change Table Dual Simplex Method

Basis	x_1	x_2	x_3	x_4	x_5	Solution
z	-1/3	0	0	-2/3	0	4
x_3	-5/3	0	1	-1/3	0	-1
x_2	4/3	1	0	-1/3	0	2
x_5	-1/3	0	0	1/3	1	1

Then x_3 leave the solution and x_1 in the bottom, which produces the following table:

Table 3.11: Optimal Table Dual Simplex Method

Basis	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	-1/5	-3/5	0	21/5
x_1	1	0	-3/5	1/5	0	3/5
x_2	0	1	4/5	-3/5	0	6/5
x_5	0	0	-1/5	2/5	1	6/5

This table is now feasible and optimal. The appropriate solution is $x_1 = 3/5$, $x_2 = 6/5$ and $z = 21/5$.

Dual simplex method applied in the same way to maximize the problem-solving with the stipulation that once again the beginning of the optimal but not feasible. The only difference is, occurs in a condition to select the variables included, as described below.

Feasibility condition: the variable out is the basic variable that has the most negative values (if same, arbitrarily set). If all basic variables non-negative, the process end.

Optimality conditions: variable in non basic variables associated with the smallest ratio if minimize or smallest absolute value of the ratio if it minimizes (if same, arbitrarily set). The ratio is determined by dividing the coefficient of the left side of the equation z with a negative coefficient in the equation associated with a negative coefficient associated with the variable out. If the denominator is zero or positive, there is no feasible solution.

Dual simplex method, in addition to its use for a group of special LP problem, it is useful to conduct a post optimization analysis in the form of sensitivity analysis and parametric programming. (Hamdy, 1996: 82-85)

3.4 Primal-Dual in Example

Simplex problems we usually call the main problem. The interesting thing to note, that the fundamental problem can be solved by other means, ie change the line into columns. Especially when the simplex problem is too many lines, while the number of columns a bit. So how to change rows into columns called-Dual Simplex. hoped, with the dual solution can save time. To this there are rules as follows:

1. Choosing how to make progress with a number of formulations simplex matrix rows become smaller (more original line).
2. If you find a way of primal and dual matrices obtained the same number of rows and select number of columns less. (Prawirosentono, page: 34-35)

Primal

$$\max z = 10x_1 + 24x_2 + 8x_3$$

With constrain

$$2x_1 + 4x_2 + 2x_3 \leq 20 \rightarrow 2x_1 + 4x_2 + 2x_3 + x_4 = 20$$

$$4x_1 + 2x_2 + 6x_3 = 16 \rightarrow 4x_1 + 2x_2 + 6x_3 + 0x_4 = 16$$

So objective function becomes

$$\max z = 10x_1 + 24x_2 + 8x_3 + 0x_4$$

$$z - 10x_1 - 24x_2 - 8x_3 - 0x_4$$

Table 3.12 : Primal Table

z_j	z	x_1	x_2	x_3	x_4	Solution
c_j	1	-10	-24	-8	0	0
x_4	0	2	4	2	1	20
x_4	0	4	2	6	0	16

x_2 is entering variable

4 is pivot variable

$$z = 24x_2 + z$$

$$x_4 = -2x_2 + x_4$$

Table 3.13: Optimal Primal Table

z_j	z	x_1	x_2	x_3	x_4	Solution
c_j	1	2	0	4	6	120
x_2	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	5
x_4	0	3	0	5	$-\frac{1}{2}$	6

Because the table above is the maximization of the table, the table above have been in

vain because artificial variable non-negative, with the values obtained $x_2=5$ and $x_4=6$.

This can be proved by inserting these values into the objective function

$$\begin{aligned} \max z &= 10x_1 + 24x_2 + 8x_3 \\ &= 10(0) + 24(5) + 8(0) \\ &= 120 \end{aligned}$$

Dual

$$\min z = 20y_1 + 16y_2$$

with constrains

$$\begin{aligned} 2y_1 + 4y_2 &\geq 10 &\rightarrow -2y_1 - 4y_2 &\leq -10 &\rightarrow -2y_1 - 4y_2 + y_3 &= -10 \\ 4y_1 + 2y_2 &\geq 24 &\rightarrow -4y_1 - 2y_2 &\leq -24 &\rightarrow -4y_1 - 2y_2 + y_4 &= -24 \\ 2y_1 + 6y_2 &\geq 8 &\rightarrow -2y_1 - 6y_2 &\leq -8 &\rightarrow -2y_1 - 6y_2 + y_5 &= -8 \\ y_1 + 0y_2 &\geq 0 &\rightarrow -y_1 - 0y_2 &\leq 0 &\rightarrow -y_1 - 0y_2 + y_6 &= 0 \end{aligned}$$

So objective function becomes

$$\begin{aligned} \min z &= 20y_1 + 16y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 \\ z - 20y_1 - 16y_2 - 0y_3 - 0y_4 - 0y_5 - 0y_6 &= 0 \end{aligned}$$

Table 3.14: Dual Table

z_j	z	y_1	y_2	y_3	y_4	y_5	y_6	Solution
c_j	1	-20	-16	0	0	0	0	0
y_3	0	-2	-4	1	0	0	0	-10
y_4	0	-4	-2	0	1	0	0	-24
y_5	0	-2	-6	0	0	1	0	-8
y_6	0	-1	0	0	0	0	1	0

Leaving Variable y_4

Table 3.15: Table ratio is calculated as follows

	y_1	y_2	y_3	y_4	y_5	y_6
Z	-20	-16	0	0	0	0
y_4	-4	-2	0	1	0	0
Ratio	5	8	-	-	-	-

this ratio shows y_1 that the solution will enter

-4 is pivot variable

$$z = 20y_1 + z$$

$$y_3 = 2y_1 + y_3$$

$$y_5 = 2y_1 + y_5$$

$$y_6 = y_1 + y_6$$

Table 3.16: The Optimum Dual Table

z_j	z	y_1	y_2	y_3	y_4	y_5	y_6	Solution
c_j	1	0	-6	0	-5	0	0	120
y_3	0	0	-3	1	$-\frac{1}{2}$	0	0	2
y_1	0	1	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	6
y_5	0	0	-5	0	$-\frac{1}{2}$	1	0	4
y_6	0	0	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	1	6

Because it is a minimization process, then the table is feasible if the column of non-positive values of objective function and positive solutions, so the table above decent. In the tables on the values obtained $y_1=6$, $y_3=2$, $y_5=4$ and $y_6=6$.

This can be proved by inserting these values into the objective function

$$\begin{aligned} \min z &= 20y_1 + 16y_2 \\ &= 20(6) + 16(0) \\ \min z &= 120 \end{aligned}$$

From the results of Primal and Dual obtain the same value, it proves equivalence between primal simplex and dual simplex.

The transportation problem and cycle cancelling methods are classical in optimization. The usual attributions are to the 1940's and later. However, as early as 1930, A.N. Tolstoi [1930] published, in a book on transportation planning issued by the National Commissariat of Transportation of the Soviet Union, an article called Methods of finding the minimal total kilometrage in cargo-transportation planning in space, in which he studied the

transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost cycle in its residual graph. He might have been the first to observe that the cycle condition is necessary for optimality. Moreover, he assumed, but did not explicitly state or prove, the fact that checking the cycle condition is also sufficient for optimality.

Tolstoi illuminated his approach by applications to the transportation of salt, cement, and other cargo between sources and destinations along the railway network of the Soviet Union. In particular, for that time large-scale, instance of the transportation problem was solved to optimality.

We briefly review the article. Tolstoi first considered the transportation problem for the case where there are only two sources. He observed that in that case one can order the destinations by the difference between the distances to the two sources. Then one source can provide the destinations starting from the beginning of the list, until the supply of that source has been used up. The other source supplies the remaining demands. Tolstoi observed that the list is independent of the supplies and demands, and hence it.

Next, Tolstoi studied the transportation problem in the case when all sources and destinations are along one circular railway line, in which case the optimum solution is readily obtained by considering the difference of two sums of costs. He called this phenomenon *circle dependency*.

Finally, Tolstoi combined the two ideas into a heuristic to solve a concrete transportation problem coming from cargo transportation along the Soviet railway network. The problem has 10 sources and 68 destinations, and 155 links between sources and destinations (all other distances are taken to be infinite), as given in the following table. (Alexander, 1996: 150)

But in the discussion this time is only used three sources and four goals with the inventory and supply 27. So we get

Table 3.17: Transportation Table

Source	Objective								Supply
	Kinel		Omsk		Berendeevo		Bobrinskaya		
Arkhangelsk	2	0	3	0	x_{13}	0	x_{14}	0	5
Kishert	x_{21}	1208	2	1159	0	0	10	0	12
Iledzks	x_{31}	454	x_{32}	1746	10	0	x_{34}	0	10
Demand	2		5		10		10		27

STEP 1

Feasible 1

$$\begin{aligned}
 z &= (2 \times 0) + (3 \times 0) + (2 \times 1159) + (0 \times 0) + (10 \times 0) + (10 \times 0) \\
 &= 0 + 0 + 2318 + 0 + 0 + 0 \\
 &= 2318
 \end{aligned}$$

STEP 2

Basic variable loop : $x_{13}, x_{14}, x_{21}, x_{31}, x_{32}, x_{34}$

loop $x_{13} : x_{13} \rightarrow x_{11} \rightarrow x_{31} \rightarrow x_{33} \rightarrow x_{13}$

- $$\begin{aligned}
 z_{13} - c_{13} &= 0 - 454 + 0 - 0 \\
 &= -454
 \end{aligned}$$

loop $x_{14} : x_{14} \rightarrow x_{12} \rightarrow x_{22} \rightarrow x_{24} \rightarrow x_{14}$

- $$z_{14} - c_{14} = 0 - 1159 + 0 - 0$$

$$= -1159$$

loop $x_{21} : x_{21} \rightarrow x_{22} \rightarrow x_{12} \rightarrow x_{11} \rightarrow x_{21}$

- $$z_{21} - c_{21} = 1159 - 0 + 0 - 1208$$

$$= -49$$

loop $x_{31} : x_{31} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{22} \rightarrow x_{12} \rightarrow x_{11} \rightarrow x_{31}$

- $$z_{31} - c_{31} = 0 - 0 + 1159 - 0 + 0 - 454$$

$$= 705$$

loop $x_{32} : x_{32} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{22} \rightarrow x_{32}$

- $$z_{32} - c_{32} = 0 - 0 + 1159 - 1746$$

$$= -587$$

loop $x_{34} : x_{34} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{24} \rightarrow x_{34}$

- $$z_{34} - c_{34} = 0 - 0 + 0 - 0$$

$$= 0$$

Entering Variable : $\max (z_{ij} - c_{ij})$

$$\max (-1159, -1159, -49, 705, -587, 0)$$

Entering Variable x_{31}

STEP 3

Determining Leaving Variable

loop $x_{31} : x_{31} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{22} \rightarrow x_{12} \rightarrow x_{11} \rightarrow x_{31}$

- $$z_{31} - c_{31} = 0 - 0 + 1159 - 0 + 0 - 454$$

Leaving Variable: $\min \{c_{ij} (+)\}$

$$\min \{+1159\}$$

❖ Leaving Variable : x_{22}

Table 3.18: Changes In The Transportation Table

Source	Objective								Supply
	Kinel		Omsk		Berendeevo		Bobrinskaya		
Arkhangelsk	0	0	5	0	x_{13}	0	x_{14}	0	5
Kishert	x_{21}	1208	x_{22}	1159	2	0	10	0	12
Iledzks	2	454	x_{32}	1746	8	0	x_{34}	0	10
Demand	2		5		10		10		27

STEP 1

$$\begin{aligned}
 z &= (0 \times 0) + (5 \times 0) + (2 \times 0) + (10 \times 0) + (2 \times 454) + (8 \times 0) \\
 &= 0 + 0 + 0 + 0 + 908 + 0 \\
 &= 908
 \end{aligned}$$

STEP 2

Basic variable loop : $x_{13}, x_{14}, x_{21}, x_{22}, x_{32}, x_{34}$

$$\text{loop } x_{13} : x_{13} \rightarrow x_{11} \rightarrow x_{31} \rightarrow x_{33} \rightarrow x_{13}$$

- $$\begin{aligned}
 z_{13} - c_{13} &= 0 - 454 + 0 - 0 \\
 &= -454
 \end{aligned}$$

loop $x_{14} : x_{14} \rightarrow x_{11} \rightarrow x_{31} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{24} \rightarrow x_{14}$

- $$z_{14} - c_{14} = 0 - 454 + 0 - 0 + 0 - 0$$

$$= -454$$

loop $x_{21} : x_{21} \rightarrow x_{23} \rightarrow x_{33} \rightarrow x_{31} \rightarrow x_{21}$

- $$z_{21} - c_{21} = 0 - 0 + 454 - 1208$$

$$= -754$$

loop $x_{22} : x_{22} \rightarrow x_{23} \rightarrow x_{33} \rightarrow x_{31} \rightarrow x_{11} \rightarrow x_{12} \rightarrow x_{22}$

- $$z_{22} - c_{22} = 0 - 0 + 454 - 0 + 0 - 1159$$

$$= -705$$

loop $x_{32} : x_{32} \rightarrow x_{31} \rightarrow x_{11} \rightarrow x_{12} \rightarrow x_{32}$

- $$z_{32} - c_{32} = 454 - 0 + 0 - 1746$$

$$= -1292$$

loop $x_{34} : x_{34} \rightarrow x_{33} \rightarrow x_{23} \rightarrow x_{24} \rightarrow x_{34}$

- $$z_{34} - c_{34} = 0 - 0 + 0 - 0$$

$$= 0$$

$$z_{feasible} = 0x_{11} + 5x_{12} + 2x_{23} + 10x_{24} + 2x_{31} + 8x_{33}$$

3.5 PRIMAL SIMPLEX METHOD IN MAXIMUM FLOW

Objective function

$$\begin{aligned}\max z &= 0x_{11} + 0x_{12} + 0x_{13} + 0x_{14} + 1208x_{21} + 1159x_{22} + 0x_{23} + 0x_{24} + \\ & 454x_{31} + 1746x_{32} + 0x_{33} + 0x_{34} \\ z &= 1208x_{21} + 1159x_{22} + 454x_{31} + 1746x_{32}\end{aligned}$$

resource constraints

$$0x_{11} + 0x_{12} + 0x_{13} + 0x_{14} \leq 5$$

$$1208x_{21} + 1159x_{22} + 0x_{23} + 0x_{24} \leq 12$$

$$454x_{31} + 1746x_{32} + 0x_{33} + 0x_{34} \leq 10$$

Note:

$x_{11} = a$	$x_{21} = e$	$x_{31} = i$
$x_{12} = b$	$x_{22} = f$	$x_{32} = j$
$x_{13} = c$	$x_{23} = g$	$x_{33} = k$
$x_{14} = d$	$x_{24} = h$	$x_{34} = l$

limitation purposes

$$0x_{11} + 1208x_{21} + 454x_{31} \leq 2$$

$$0x_{12} + 1159x_{22} + 1746x_{32} \leq 5$$

$$0x_{13} + 0x_{23} + 0x_{33} \leq 10$$

$$0x_{14} + 0x_{24} + 0x_{34} \leq 10$$

So that the objective function becomes

$$\begin{aligned}\max z &= 0a + 0b + 0c + 0d + 1208e + 1159f + 0g + 0h + 454i + 1746j + 0k + 0l \\ z &= 1208e + 1159f + 454i + 1746j\end{aligned}$$

resource constraints

$$0a + 0b + 0c + 0d \leq 5$$

$$1208e + 1159f + 0g + 0h \leq 12$$

$$454i + 1746j + 0k + 0l \leq 10$$

limitation purposes

$$0a + 1208e + 454i \leq 2$$

$$0b + 1159f + 1746j \leq 5$$

$$0c + 0g + 0k \leq 10$$

$$0d + 0h + 0l \leq 10$$

Objective Function

$$\max z = 0a + 0b + 0c + 0d + 1208e + 1159f + 0g + 0h + 454i + 1746j + 0k + 0l + 0m + 0n + 0o + 0p + 0q + 0r + 0s$$

resource constraints

$$0a + 0b + 0c + 0d + 1m = 5$$

$$1208e + 1159f + 0g + 0h + 1n = 12$$

$$454i + 1746j + 0k + 0l + 1o = 10$$

limitation purposes

$$0a + 1208e + 454i + 1p = 2$$

$$0b + 1159f + 1746j + 1q = 5$$

$$0c + 0g + 0k + 1r = 10$$

$$0d + 0h + 0l + 1s = 10$$

So objective function become:

$$\text{objective function} = \max z - 0a - 0b - 0c - 0d - 1208e - 1159f - 0g - 0h - 454i - 1746j - 0k - 0l - 0m - 0n - 0o - 0p - 0q - 0r - 0s$$

3.19: Primal Tabel in Maximum Flow Problem

z_j	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	Solution
c_j	1	0	0	0	0	-1208	-1159	0	0	-454	-1746	0	0	0	0	0	0	0	0	0	0
m	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	5
n	0	0	0	0	0	1208	1159	0	0	0	0	0	0	0	1	0	0	0	0	0	12
o	0	0	0	0	0	0	0	0	0	454	1746	0	0	0	0	1	0	0	0	0	10
p	0	0	0	0	0	1208	0	0	0	454	0	0	0	0	0	0	0	0	0	0	2
q	0	0	0	0	0	0	1159	0	0	0	1746	0	0	0	0	0	0	1	0	0	5
r	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	10
s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10

Entering Variable (j)

Leaving Variable (q)

Variable Pivot : 1746

$$z = 1746j + z$$

$$o = -1746j + o$$

Table 3.20: The Change Primal Table in Maximum Flow Problem

z_j	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	Solution
c_j	1	0	0	0	0	-	0	0	0	-454	0	0	0	0	0	0	0	1	0	0	5
m	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	5
n	0	0	0	0	0	1208	1159	0	0	0	0	0	0	0	1	0	0	0	0	0	12
o	0	0	0	0	0	0	-	0	0	454	0	0	0	0	0	1	0	-1	0	0	5
p	0	0	0	0	0	1208	0	0	0	454	0	0	0	0	0	0	1	0	0	0	2
j	0	0	0	0	0	0	$\frac{1159}{1746}$	0	0	0	1	0	0	0	0	0	0	$\frac{1}{1746}$	0	0	$\frac{5}{1746}$
r	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	10
s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10

Entering Variable (e)

Leaving Variable (p)

Variable Pivot : 1208

$$z = 1208e + z$$

$$n = -1208e + n$$

Table 3.21: Optimal Table in Maximum Flow Problem

z_j	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	Solution
c_j	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	7
m	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	5
n	0	0	0	0	0	0	1159	0	0	-454	0	0	0	0	1	0	1	0	0	0	10
o	0	0	0	0	0	0	-1159	0	0	454	0	0	0	0	0	1	0	-1	0	0	5
e	0	0	0	0	0	1	0	0	0	$\frac{454}{1208}$	0	0	0	0	0	0	$\frac{1}{1208}$	0	0	0	$\frac{2}{1208}$
j	0	0	0	0	0	0	$\frac{1159}{1746}$	0	0	0	1	0	0	0	0	0	0	$\frac{1}{1746}$	0	0	$\frac{5}{1746}$
r	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	10
s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	10

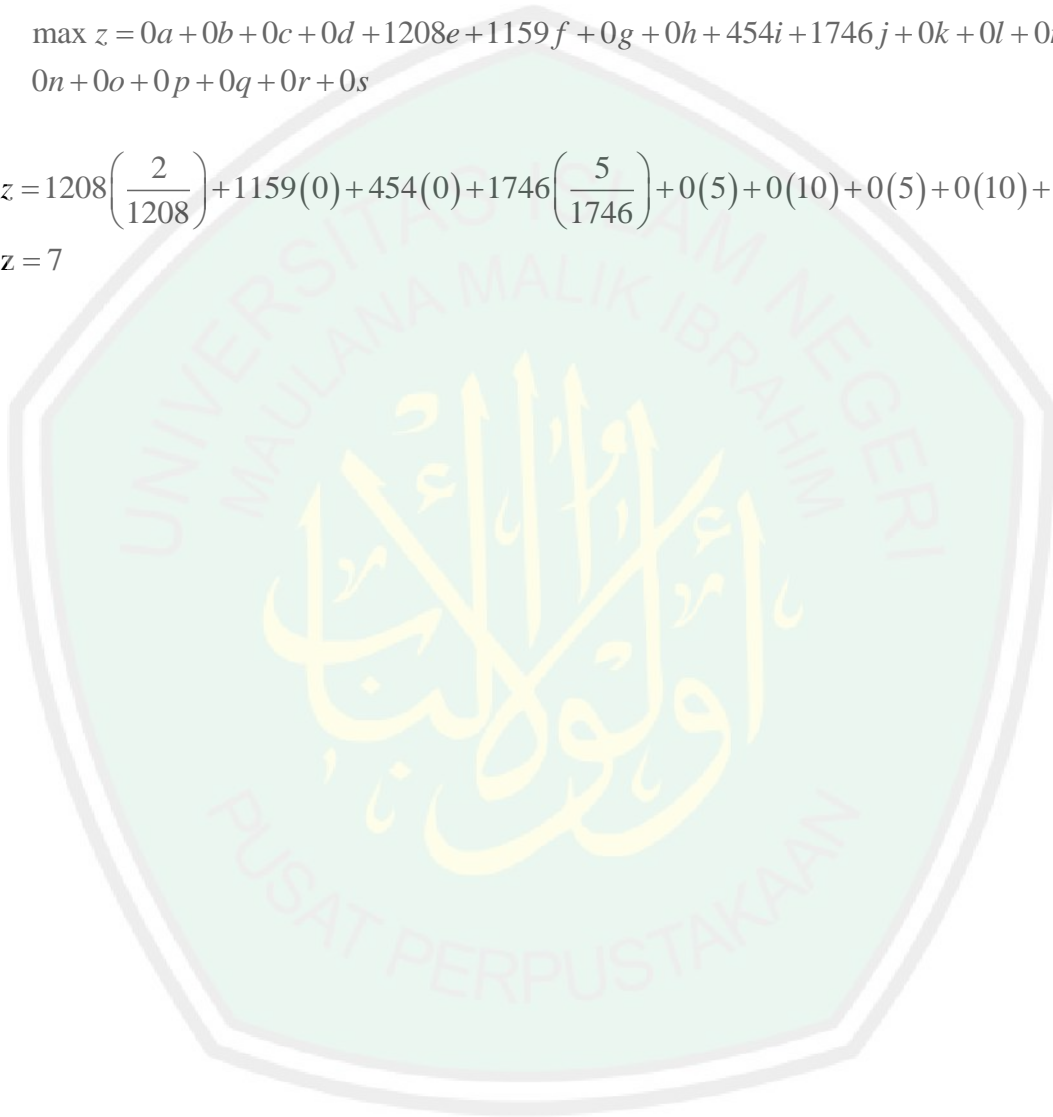
Because the table above is the maximization of the table, the table above have been in vain because all values of artificial variable non-negative, with the values obtained $m=5$, $n=10$, $o=5$, $e=2/1208$, $j=5/1746$, $r=10$ and $s=10$

This can be proved by inserting these values into the objective function

$$\max z = 0a + 0b + 0c + 0d + 1208e + 1159f + 0g + 0h + 454i + 1746j + 0k + 0l + 0m + 0n + 0o + 0p + 0q + 0r + 0s$$

$$\max z = 1208\left(\frac{2}{1208}\right) + 1159(0) + 454(0) + 1746\left(\frac{5}{1746}\right) + 0(5) + 0(10) + 0(5) + 0(10) + 0(10)$$

$$\max z = 7$$



3.6 DUAL SIMPLEX METHOD IN MAXIMUM FLOW

Objective function

$$\min z = 5m + 12n + 10o + 2p + 5q + 10r + 10s$$

constraints

$$0m + 1208n + 0o + 1208p + 0q + 0r + 0s \geq 1208$$

$$0m + 1159n + 0o + 0p + 1159q + 0r + 0s \geq 1159$$

$$0m + 0n + 454o + 454p + 0q + 0r + 0s \geq 454$$

$$0m + 0n + 1746o + 0p + 1746q + 0r + 0s \geq 1746$$

Constraints become

$$-0m - 1208n - 0o - 1208p - 0q - 0r - 0s \leq -1208$$

$$-0m - 1159n - 0o - 0p - 1159q - 0r - 0s \leq -1159$$

$$-0m - 0n - 454o - 454p - 0q - 0r - 0s \leq -454$$

$$-0m - 0n - 1746o - 0p - 1746q - 0r - 0s \leq -1746$$

Because the constraint equations on the function \leq , it must be coupled with slack variable so that the constraint function becomes:

$$-0m - 1208n - 0o - 1208p - 0q - 0r - 0s + t = -1208$$

$$-0m - 1159n - 0o - 0p - 1159q - 0r - 0s + u = -1159$$

$$-0m - 0n - 454o - 454p - 0q - 0r - 0s + v = -454$$

$$-0m - 0n - 1746o - 0p - 1746q - 0r - 0s + w = -1746$$

So, Objective function becomes:

$$\min z = 5m + 12n + 10o + 2p + 5q + 10r + 10s + t + u + v + w$$

$$z - 5m - 12n - 10o - 2p - 5q - 10r - 10s - t - u - v - w$$

Problems mentioned above, it can be concluded that the method of Primal Simplex and Dual Simplex not only in the usual objective function that states equality, but also in the Maximum Flow Primal simplex and Dual simplex can be expressed equality.

Table 3.22: Dual Table in Maximum Flow Problem

$c_j \backslash z_j$	z	m	n	o	p	q	r	s	t	u	v	w	Solution
	1	-5	-12	-10	-2	-5	-10	-10	0	0	0	0	0
t	0	0	-1208	0	-1208	0	0	0	1	0	0	0	-1208
u	0	0	-1159	0	0	-1159	0	0	0	1	0	0	-1159
v	0	0	0	-454	-454	0	0	0	0	0	1	0	-454
w	0	0	0	-1746	0	-1746	0	0	0	0	0	1	-1746

Leaving Variable : w

Determining Entering Variable

Table 3.23: Ratio Table in Maximum Flow Problem

	m	n	o	p	q	r	s	t	u	v	w
z	-5	-12	-10	-2	-5	-10	-10	0	0	0	0
w	0	0	-1746	0	-1746	0	0	0	0	0	1
<i>ratio</i>	-	-	10/1746	-	5/1746	-	-	-	-	-	-

this ratio shows q that the solution will enter

-1746 pivot variable

$$z = 5q + z$$

$$u = 1159q + u$$

Table 3.24: The Change Table in Maximum Flow Problem

z_j	z	m	n	o	p	q	r	s	t	u	v	w	Solution
c_j	1	-5	-12	-5	-2	0	-10	-10	0	0	0	$\frac{-5}{1746}$	5
t	0	0	-1208	0	-1208	0	0	0	1	0	0	0	-1208
u	0	0	-1159	1159	0	0	0	0	0	1	0	$\frac{-1159}{1746}$	0
v	0	0	0	-454	-454	0	0	0	0	0	1	0	-454
q	0	0	0	1	0	1	0	0	0	0	0	$\frac{-1}{1746}$	1

Leaving Variable : t

Determining Entering Variable

Table 3.25: Ratio Table in Maximum Flow Problem

	m	n	o	p	q	r	s	t	u	v	w
z	-5	-12	-5	-2	0	-10	-10	0	0	0	$\frac{-5}{1746}$
t	0	-1208	0	-1208	0	0	0	1	0	0	0
<i>ratio</i>	-	12/1208	-	2/1208	-	-	-	-	-	-	-

this ratio shows p that the solution will enter

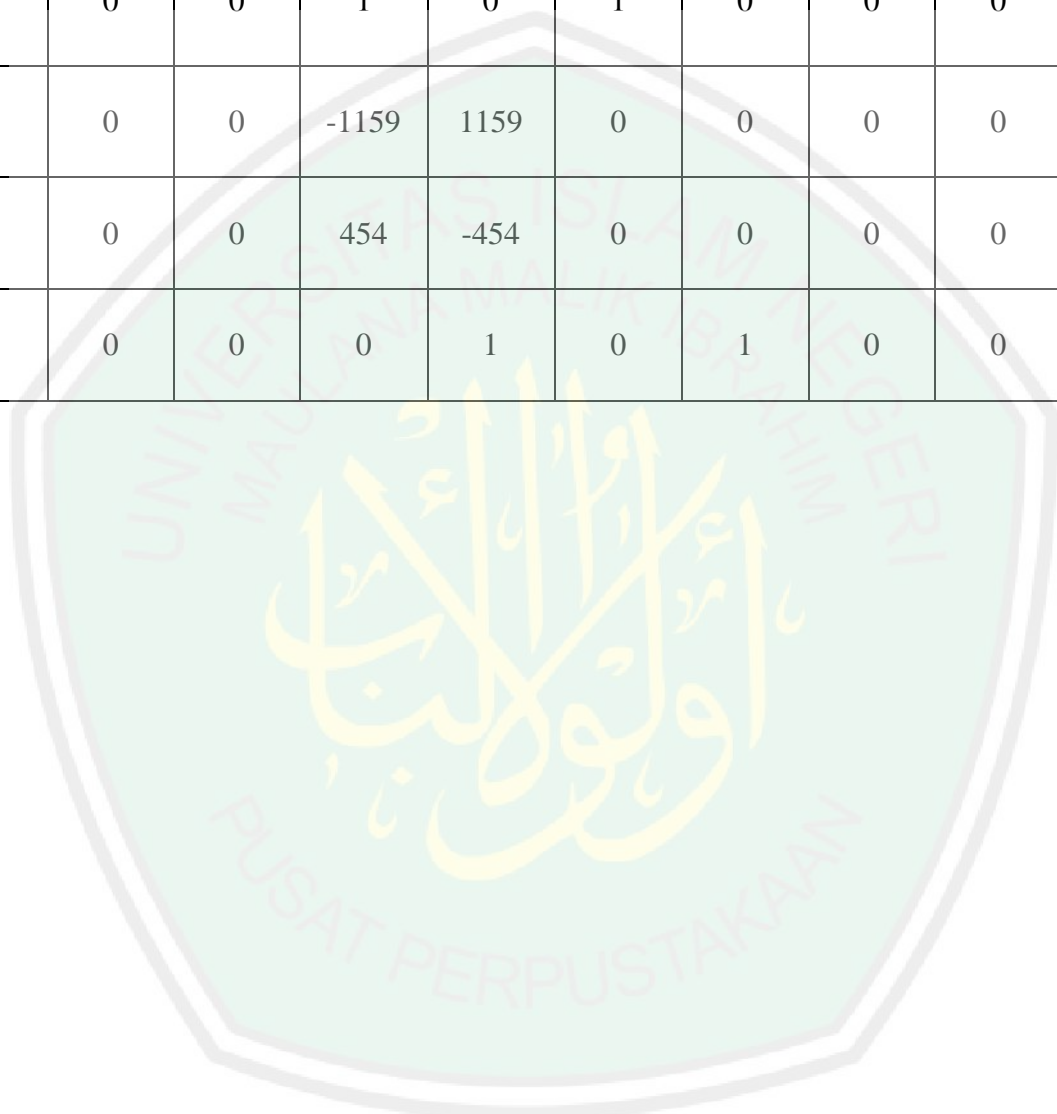
-1208 is pivot variable

$$z = 2p + z$$

$$v = 454p + v$$

Table 3.26: Optimal Table in Maximum Flow Problem

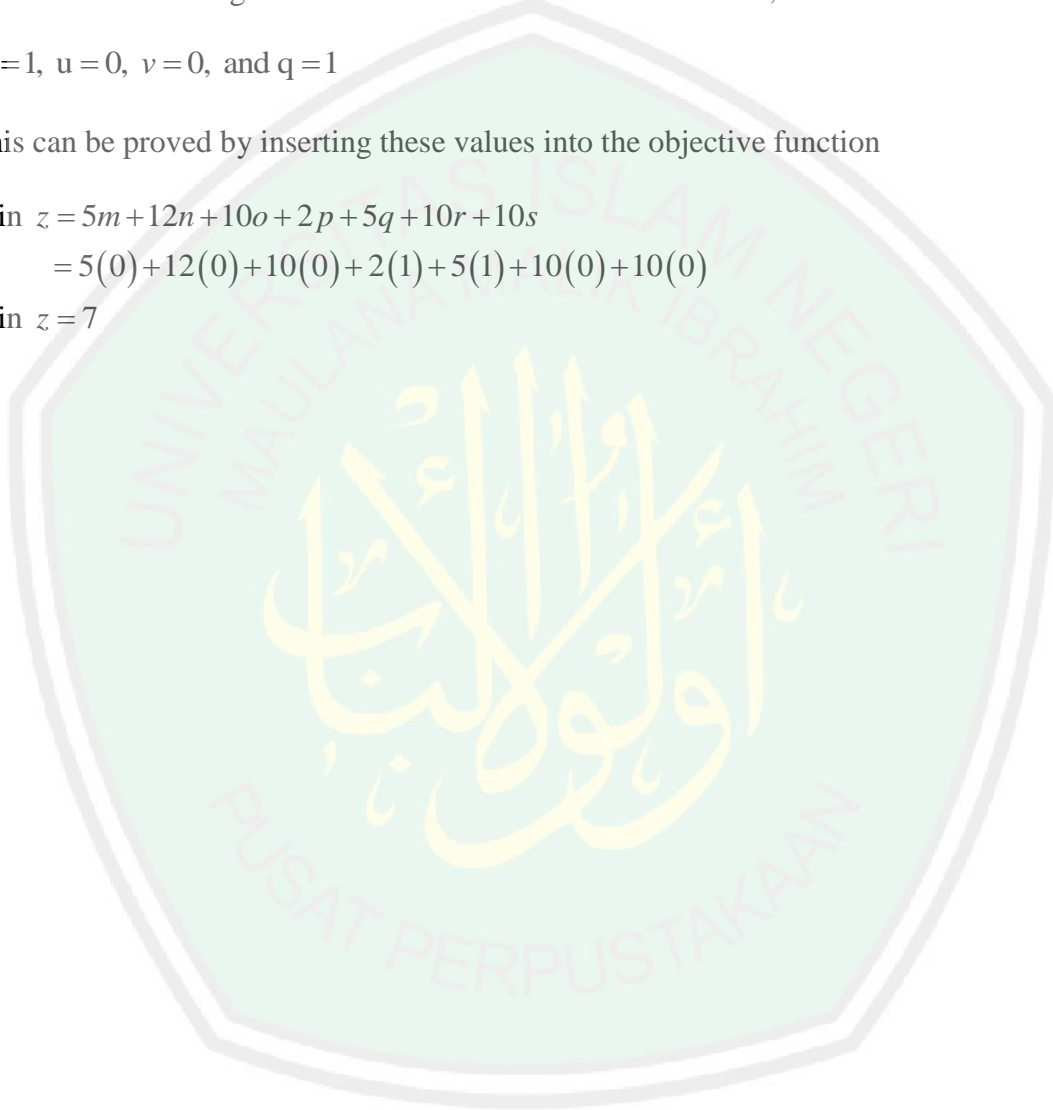
$z_j \backslash c_j$	z	m	n	o	p	q	r	s	t	u	v	w	Solution
	1	-5	-10	-5	0	0	-10	-10	$\frac{-2}{1208}$	0	0	$\frac{-5}{1746}$	7
p	0	0	1	0	1	0	0	0	$\frac{-1}{1208}$	0	0	0	1
u	0	0	-1159	1159	0	0	0	0	0	1	0	$\frac{-1159}{1746}$	0
v	0	0	454	-454	0	0	0	0	$\frac{-454}{1208}$	0	1	0	0
q	0	0	0	1	0	1	0	0	0	0	0	$\frac{-1}{1746}$	1



The table above is feasible, because the minimization process is said to have a decent table if non-positive objective function value and the value at the right side of the table are non-negative value. In the table above, the value obtained $p=1$, $u=0$, $v=0$, and $q=1$

This can be proved by inserting these values into the objective function

$$\begin{aligned}\min z &= 5m + 12n + 10o + 2p + 5q + 10r + 10s \\ &= 5(0) + 12(0) + 10(0) + 2(1) + 5(1) + 10(0) + 10(0) \\ \min z &= 7\end{aligned}$$



CHAPTER IV

CONCLUSION AND SUGGESTION

4.1 Conclusions

1. Primal simplex method starting from the basic feasible solution (extreme point) and continue to repeat through the proper solution to the following basis to achieve optimum point.

We are now ready to present the formal steps primal simplex iteration method:

Step 1: Using the standard form (with the right side of all non-negative), determine the initial basic solution is feasible.

Step 2: Choose a variable in the variables used non basic optimality condition.

Step 3: Choose variables from the basic flow variables using the feasibility condition

Step 4: Determine the value of the new basic variable with variable entry as basic variables and variables come out as non-basic variables.

2. Simplex problems we usually call the main problem. The interesting thing to note, that the fundamental problem can be solved by other means, ie change the line into columns. Especially when the simplex problem is too many lines, while the number of columns a bit. So how to change rows into columns called-Dual Simplex. hoped, with the dual solution can save time. To this there are rules as follows:

3. Choosing how to make progress with a number of formulations simplex matrix rows become smaller (more original line).

4. If you find a way of primal and dual matrices obtained the same number of rows and select number of columns less.

3. Equivalence of Primal and Dual Simplex Algorithms for the Maximum Flow Problem can be expressed in the objective function. The research in Chapter III did received the value in Primal Simplex is $\max z=7$, and in Dual Simplex is $\min z=7$.

4.2 Suggestion

Simplex method in particular Simplex Primal and Simplex Dual Simplex can be used to solve the problem of maximizing or minimizing, in this case the author uses the Maximum Flow problem is about the transportation problem. Author aware of the deficiencies in describing the existing problems, so author hope there will be a constructive input for further research.

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No	Tanggal	Materi Konsultasi	Tanda Tangan	
1	23 September 2009	Proposal	1.	
2	28 Desember 2009	Bab I		2.
3	25 Januari 2010	Revisi Bab I	3.	
4	06 Februari 2010	Bab II		4.
5	20 Februari 2010	Revisi Bab II	5.	
6	13 Maret 2010	Bab III		6.
7	27 Maret 2010	Bab III	7.	
8	05 April 2010	Revisi Bab III		8.
9	24 April 2010	Revisi Bab III	9.	
10	21 Juni 2010	Kajian Agama Bab I & Bab II		10.
11	26 Juni 2010	ACC Kajian Agama Bab I & Bab II	11.	

12	28 Juni 2010	Bab IV		12.
13	29 Juni 2010	ACC Keseluruhan	13.	

Malang, 03 Juli 2010

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