PENERAPAN METODE RUNGE KUTTA PADA PERSAMAAN DIFFERENSIAL LINIER ORDE SATU

SKRIPSI

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MALANG
2020

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Diajukan Kepada
Fakultas Sains dan Teknologi
Universitas Islam Negeri Maulana Malik Ibrahim Malang
untuk Memenuhi Salah Satu Persyaratan dalam
Memperoleh Gelar Sarjana Matematika (S.Mat)

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PERNYATAAN KEASLIAN TULISAN

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Menyatakan dengan sebenarnya bahwa skripsi yang saya tulis ini benar-benar merupakan hasil karya saya sendiri, bukan pengambilan data, tulisan, atau pikiran orang lain yang saya akui sebagai hasil tulisan atau pikiran saya sendiri, kecuali dengan mencantumkan sumber cuplikan pada daftar rujukan. Apabila di kemudian hari terbukti atau dapat dibuktikan skripsi ini hasil jiplakan, maka saya bersedia menerima sanksi atas perbuatan tersebut.

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Yang membuat pernyataan

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MOTO

“Maka sesungguhnya bersama kesulitan ada kemudahan”

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Skripsi ini penulis persembahkan untuk:

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Malang, 14 Mei 2020

Penulis

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ABSTRAK

Unaizahroya, Isvina, 2020. **Penerapan Metode Runge Kutta Pada Persamaan Differensial Linier Orde Satu.** Skripsi. Jurusan Matematika, Fakultas Sains dan Teknologi, Universitas Islam Negeri Maulana Malik Ibrahim Malang. Pembimbing: (I) Ari Kusumastuti, M.Si, M.Pd. (II) Mohammad Nafie Jauhari, M.Si.

Kata kunci: Metode Runge Kutta Orde Lima, Penurunan, Persamaan Differensial Linier Orde Satu, Galat

Penelitian ini membahas tentang penurunan metode Runge Kutta orde lima dan penerapannya pada Persamaan Differensial Linier Orde Satu. Penurunan dilakukan dengan cara menurunkan $f(x, y)$ sebanyak empat kali menggunakan aturan rantai, Substitusikan hasil penurunan ke deret Taylor orde lima. Setelah itu, merubah k_2, k_3, k_4, k_5, ke dalam bentuk deret Taylor dua variabel dan disubstitusikan ke rumus Runge kutta orde lima. Kemudian menyamakan hasil dua langkah sebelumnya, sehingga diperoleh system persamaan yang nantinya digunakan untuk untuk menentukan nilai konstanta dari rumus runge kutta orde lima. Dalam penelitian ini Persamaan Differensial Linier Orde Satu disimulasikan menggunakan metode runge Kutta orde lima dengan hasil berupa iterasi, selanjutnya dari hasil iterasi dianalisis galat. Galat yang dihasilkan sangatlah besar dan menjahui nol. Sehingga disarankan untuk penelitian selanjutnya untuk memperbaiki penurunan metode runge kutta orde lima atau menurunkan runge kutta dengan orde yang lebih tinggi.

ABSTRACT

Unaizahroya, Isvina, 2020. **Application of Runge Kutta method on linear differential equations of first order**. Thesis. Department of Mathematics, Faculty of Science and Technology, Islamic State University of Maulana Malik Ibrahim Malang. Advisers: (I) Ari Kusumastuti, M.Si, M. Pd. (II) Mohammad Nafie Jauhari, M.Si.

Keyword: Fifth order Runge-Kutta method, Derivation, linear differential equations of first order, Error

This study discusses the derivation of the five-order Runge Kutta method and application of the linear differential equations of first order. Derivation is done by deriving $f(x, y)$ as much as four times using the chain rule, The result substituted for the five-order Taylor series. After that, change k_2, k_3, k_4, k_5, into the form of a two-variable Taylor series and substituted into the five-order Runge Kutta formula. Then equate the results of two steps, so that the system obtained the equation that later used to determine the value of constants of the formula five order Runge Kutta. In this research linear differential equations of first order simulated using the five-order Runge Kutta method with the result of iterating, then from the iteration results are analyzed for errors. The resulting error is very large and avoids zero. So it is suggested for further research to improve the reduction of the fifth order runge kutta method or to reduce the runge kutta with a higherorder.

ملخص

اوني زهرايا ، اسفنا. ٢٠٢٠. **تطبيق اسلوب *رونج كوتا* على المعادلات الخطية التفاضلية على الترتيب الأول.** البحث الجامعي. شعبة الرياضيات، كلية العلوم والتكنولوجيا، جامعة مولانا ملك إبراهيم الإسلامية الحكومية مالانج. المشرف: (١) أري كوسوماستوتي، الماجستير (٢) مُحَمَّدنافع جوهري، الماجستير.

الكلمات الرئيسية: الاسلوب رونج كوت على الترتيب الخامس ، الانخفاض، المعادلات الخطية التفاضلية على الترتيب الأول, خطأ

تناقش هذه الدراسة انخفاض في اسلوب رونج كوت على الترتيب الخامس و تطبيقه على المعادلات الخطية التفاضلية على الترتيب الأول, يتم الانخفاض عن طريق الخفض $f(x,y)$ بأربع مرات باستخدام قواعد السلسلة، استبدال النتائج المتناقصة لسلسلة تايلور على الترتيب الخامس. بعد ذلك, تغيير k_2, k_3, k_4, k_5 إلى شكل سلسلة من تايلور بمتغيرين واستبدالها بالصيغة رونج كوت على الترتيب الخامس. ثم تسوية نتائج من الخطوتين, السابقين وهكذا حتى اكتسبات معادلات النظام تستخدم لتحديد قيمة الثوابت بعده من رونج كوت على الترتيب الخامس. في هذه الدراسة الحاكى المعادلات الخطية التفاضلية الترتيب الأول بأسلوب رونج كوت على الترتيب الخامس يحتج إلى شكل تكرار, التالي من نتائج التكرار هو تحليل الخطأ، استنادا إلى المناقشة والتاخيص إلى أن رونج كوت على الترتيب الخامس يمكن استخدامه لإكمال المعادلات الخطية التفاضلية على الترتيب الأول

BAB I

PENDAHULUAN

1.1 Latar Belakang

Persamaan yang menyangkut satu atau lebih fungsi (peubah tak bebas) beserta turunannya terhadap satu atau lebih peubah bebas disebut persamaan diferensial (Pamuntjak dan Santosa, 1990). Persamaan diferensial dibedakan menjadi beberapa kategori, kategori pertama adalah persamaan diferensial biasa. Persamaan Diferensial Biasa merupakan persamaan diferensial yang hanya memiliki satu variabel bebas, Persamaan Diferensial Biasa dikategorikan menjadi Persamaan Diferensial Biasa orde satu, Persamaan Diferensial Biasa orde dua, Persamaan Diferensial Biasa orde tiga dan seterusnya. Kategori persamaan diferensial lainnya adalah Persamaan Diferensial Parsial yakni persamaan diferensial yang memiliki lebih dari satu variabel bebas.

Menurut Waluyo (2006) persamaan differensial yang berbentuk $F(t, y, \dot{y}, \ddot{y}, \dots, y^n) = 0$ dikatakan linier jika F adalah linier dalam variabel-variabelnya. Menurut Baiduri (2002) persamaan diferensial orde-n dikatakan linier jika memiliki ciri-ciri variabel terikat y dan derivatifnya hanya berderajat satu, tidak ada perkalian antara y dan derivatifnya serta antara derivatif. variabel terikat y bukan merupakan fungsi transenden.

Masalah persamaan differensial linier orde satu dapat diatasi dengan menggunakan metode numerik. Pada metode numerik terdapat beberapa variasi metode yang sering digunakan untuk menghitung solusi persamaan defferensial biasa salah satu diantaranya yaitu metode Euler, metode Heun, dan metode

Runge-Kutta. Metode Runge-Kutta adalah alternatif lain dari metode deret taylor dan tidak membutuhkan perhitungan turunan. Metode ini berusaha mendapatkan derajat ketelitian yang lebih tinggi (Munir, 2008). Metode Runge-Kutta sendiri mempunyai banyak tipe, mulai dari Rungge Kutta orde satu, orde dua, orde tiga, orde empat, orde lima, dan Runge Kutta Felberg. Dalam penelitian ini dipilih metode Runge Kutta orde lima. Metode Runge Kutta orde lima merupakan salah satu metode numerik yang dapat digunakan untuk menyelesaikan persamaan differensial. Metode Runge Kutta dipandang sebagai metode yang baik untuk menyelesaikan persoalan numerik karena mempunyai tingkat ketelitian yang cenderung lebih tinggi, semakin tinggi orde Runge Kutta maka semakin tinggi pula tingkat ketelitiannya. Sebelum menggunakan rumus metode Runge Kutta orde lima, kita perlu mengetahui penurunan metode Runge Kutta itu sendiri untuk mengetahui kebenaran dari metode Runge Kutta tersebut. Dalam metode numerik terlebih metode Runge Kutta hasil yang didapat berupa proses iterasi untuk menghampiri nilai sebenarnya, dan selisih yang merupakan galat dari metode numerik. Kesalahan (error/galat) adalah besarnya perbedaan atau selisih antara nilai taksiran (hampiran/aproksimasi) dengan nilai sesungguhnya (eksak), kesalahan ini biasa timbul karena proses pengukuran atau penggunaan aproksimasi.

Allah Swt berfirman dalam al-Quran surat Maryam ayat 94:

"Dia (Allah)benar-benar telah menentukan jumlah mereka dan menghitung mereka dengan hitungan teliti".

Dalam tafsir kementrian agama RI, ayat tersebut Allah menjelaskan bahwa semua amal dan takwa mereka itu telah tercatat dalam kitab yang amat teliti dan terperinci tidak seorang pun terluput dalam catatan itu, semua amal perbuatan

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Dalam penelitian ini akan dibahas persamaan differensial linier orde satu, penurunan Runge Kutta orde lima, solusi numerik menggunakan Runge Kutta orde lima, analisis galat.

Penelitian ini merujuk pada penelitian Sihombing dan Dahlia pada tahun 2018 yang mengkaji persamaan differensial linier orde satu menggunakan Runge kutta butcher dan felhberg. Kemudian pada tahun 2013 Prian Peisisa Putri menganalisis model predator prey yang merupakan salah dari sistem persamaan differensial linier orde satu dengan penyelesaian numerik di tahun yang sama Darmiyanti mengkaji tentang persamaan lotka voltera yang juga contoh dari salah dari persamaan differensial linier orde satu secara numerik. Maka pada penelitiam ini dapat dikembangkan dengan memilih metode Runge Kutta orde lima sebagai metode numerik yang digunakan untuk menyelesaiakan persamaan differensial linier orde satu.

Berdasarkan uraian di atas, penulis tertarik untuk mengkaji persamaan differensial linier orde satu yang diselesaikan menggunakan metode Rungge Kutta. Oleh sebab itu, maka penulis mengambil judul skripsi "Penerapan Metode Runge Kutta pada Persamaan Differensial Linier Orde Satu".

1.2 Rumusan Masalah

Berdasarkan latar belakang tersebut, maka rumusan masalah pada penelitian ini adalah:

1. Bagaimana penurunan metode Runge-Kutta orde lima?
2. Bagaimana solusi numerik dari persamaan differensial linier orde satu dengan menggunakan metode Runge-Kutta orde lima?
3. Bagaimana analisis galat pada persamaan differensial linier orde satu?

1.3 Tujuan Penelitian

Berdasarkan rumusan masalah tersebut, maka tujuan dari penelitian ini adalah:

1. Mengetahui penurunan Runge-Kutta orde lima.
2. Mengetahui solusi numerik dari persamaan differensial linier orde satu dengan menggunakan Runge-Kutta orde lima.
3. Mengetahui hasil analisis pada persamaan differensial linier orde satu.

1.4 Manfaat Penelitian

Hasil penelitian ini diharapkan dapat memberikan manfaat sebagai berikut:

1. Mendapatkan hasil penurunan Runge-Kutta orde lima.
2. Mendapatkan solusi numerik dari persamaan differensial linier orde satu dengan menggunakan Runge-Kutta orde lima.
3. Mendapatkan hasil analisis galat dari persamaan differensial linier orde satu.

1.5 Batasan Masalah

Adapun batasan masalah pada penelitian ini adalah:

1. Persamaan differensial linier orde satu

$$\frac{dy}{dx} = x + y, \quad y(1) = 0$$

2. Menggunakan Metode Runge-Kutta orde lima

$$y_{i+1} = y_i + (k_1 + k_2 + k_3 + k_4 + k_5)h.$$

1.6 Metode Penelitian

Metode yang digunakan pada penelitian ini menggunakan jenis penelitian kepustakaan (*library research*). Berikut langkah-langkah penelitian:

1. Penurunan metode Runge kutta orde lima
   a. Menurunkan $f(x, y)$ sebanyak empat kali menggunakan aturan rantai.
   b. Substitusikan hasil penurunan ke deret Taylor orde lima.
   c. Ubah k_2, k_3, k_4 dan k_5 ke dalam bentuk deret Taylor dua variabel.
   d. Substitusikan hasilnya ke rumus Runge Kutta orde lima.
   e. Samakan hasil langkah (b) dan (d) sehingga diperoleh sistem persamaan.
   f. Mencari semua nilai konstanta.
   g. Mensubstitusikan nilai konstanta ke rumus Runge Kutta orde lima.
2. Menentukan solusi numerik persamaan differensial linier orde satu dengan menggunakan Runge-Kutta orde lima.
   a. Menghitung solusi numerik dari persamaan differensial linier orde satu.
   b. Menginterpretasikan hasil simulasi numerik.
3. Analisis Galat
   a. Menghitung nilai galat.
   b. Menginterpretasi hasil analisis galat.

1.7 Sistematika Penulis

Penulisan skripsi ini menggunakan sistematika penulisan yang tersusun dari empat bab. Masing-masing bab terdiri dari beberapa subbab dengan sistematika sebagai berikut:

Bab I Pendahuluan

Pendahuluan terdiri dari latar belakang, rumusan masalah, tujuan penelitian, manfaat penelitian, batasan masalah, metode penelitian, dan sistematika penulisan.

Bab II Kajian Pustaka

Kajian pustaka menguraikan mengenai teori-teori yang mendukung untuk memecahkan masalah yang berkaitan dengan persamaan differensial linier orde satu, metode Runge-Kutta orde lima.

Bab III Pembahasan

Pembahasan menguraikan semua langkah yang disebutkan pada metode penelitian.

Bab IV Penutup

Penutup berisi kesimpulan yang diperoleh dari pembahasan sebelumnya dan saran untuk penelitian selanjutnya.

BAB II

KAJIAN PUSTAKA

2.1 Persamaan differensial

Persamaan yang menyangkut satu atau lebih fungsi (peubah tak bebas) beserta turunannya terhadap satu atau lebih peubah bebas disebut persamaan diferensial (Pamuntjak dan Santosa, 1990). Menurut Triatmodjo (2002) persamaan diferensial adalah suatu persamaan yang mengandung turunan fungsi. Derajat (order) dari persamaan diferensial ditentukan oleh derajat tertinggi dari turunannya.

Berdasarkan jumlah variabel bebas, persamaan diferensial dibagi menjadi dua, yaitu persamaan diferensial biasa dan persamaan diferensial parsial. Jika variabel terikat dalam suatu persamaan diferensial merupakan suatu fungsi satu variabel bebas maka turunannya disebut turunan biasa dan persamaan tersebut dinamakan persamaan diferensial biasa. Jika variabel terikat suatu fungsi dua variabel atau lebih maka turunannya disebut turunan parsial dan persamaannya dinamakan persamaan diferensial parsial (Triatmodjo, 2002).

Persamaan diferensial biasa adalah persamaan yang memuat turunan terhadap fungsi yang memuat satu variabel bebas. Jika *x* adalah fungsi dari *t*, maka contoh persamaan diferensial biasa adalah

$$\frac{dx}{dt} = t^2 \cos x$$

dimana persamaan tersebut memiliki order satu. Order dari persamaan diferensial adalah turunan tertinggi pada fungsi tak diketahui (peubah tak

bebas) yang muncul dalam persamaan diferensial (Campbell & Haberman, 2008).

Berdasarkan sifat kelinieran dari peubah tak bebasnya, persamaan diferensial biasa dapat dibedakan menjadi persamaan diferensial biasa linier dan persamaan diferensial biasa nonlinier.

Menurut Waluya (2006) persamaan diferensial biasa yang berbentuk

$$F(t, y, \dot{y}, \ddot{y}, \dots, y^n) = 0 \quad (2.1)$$

Dikatakan liner jika F adalah linier dalam variabel-variabel $y, \dot{y}, \ddot{y}, \dots, y^n$

Persamaan diferensial biasa linier memiliki bentuk umum

$$a_n(t)\, y^n + a_{n-1}(t) y^{n-1} + \cdots + a_1(t)\dot{y} + a_0(t) y = f(t) \quad (2.2)$$

Menurut Baiduri (2002), persamaan (2.2) merupakan persamaan diferensial orde-n dikatakan linier jika memiliki ciri-ciri sebagai berikut:

a. Variabel terikat y dan derivatifnya hanya berderajat satu.
b. Tidak ada perkalian antara y dan derivatifnya serta antara derivatif.
c. Variabel terikat y bukan merupakan fungsi transenden.

Persamaan diferensial yang bukan persamaan linier disebut persamaan diferensial non linier. Dikatakan persamaan diferensial non linier jika persamaan (2.1) memenuhi salah satu dari sifat berikut:

1. F tidak berbentuk polinom dalam $y, \dot{y}, \ddot{y}, \dots, y^n$
2. F tidak berbentuk polinom berpangkat lebih dari 2 dalam $y, \dot{y}, \ddot{y}, \dots, y^n$

2.2 Persamaan Differensial Linier Orde Satu

Pada umumnya PDB linier order satu dapat dinyatakan dengan

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (2.3)$$

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \qquad (2.4)$$

Untuk persamaan (2.3) dapat ditulis dalam bentuk

$$\big(P(x)y - Q(x)\big)dx + dy = 0$$

Sehingga

$M(x,y) = P(x)y - Q(x)$ dan $N(x,y) = 1$

Sekarang $\frac{\partial M(x,y)}{\partial y} = P(x)$ dan $\frac{\partial N(x,y)}{\partial x} = 0$

dengan demikian persamaan ini bukan merupakan PDB eksak sehingga perlu ditentukan faktor integrasinya, kita pilih faktor integrasi yang hanya tergantung pada x yaitu $\mu(x)$ sedemikian

$$\big(\mu(x)P(x)y - \mu(x)Q(x)\big)dx + \mu(x)dy = 0$$

merupakan PDB eksak, yang berakibat bahwa

$$\frac{\partial M\big(\mu(x)P(x)y - \mu(x)Q(x)\big)}{\partial y} = \frac{\partial \mu(x,y)}{\partial x}$$

bentuk ini dapat diselesaikan menjadi

$$P(x)dx = \frac{1}{\mu(x)}\partial\mu(x)$$

$$ln|\mu| = \int P(x)\,dx$$

$$\therefore \mu = e^{\int P(x)dx} \quad \mu > 0$$

Kalikan μ terhadap persamaan (2.3) sehingga didapat

$$e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y - Q(x)e^{\int P(x)dx}$$

yang mana hal ini sama dengan

$$\frac{d}{dx}\big(e^{\int P(x)dx}y\big) - Q(x)e^{\int P(x)dx}$$

atau

$$\big(e^{\int P(x)dx}y\big) - \int e^{\int P(x)dx}Q(x)y + c$$

atau

$\therefore\ y = e^{-\int P(x)dx} \int e^{\int P(x)dx} Q(x)dx + c$, Persamaan ini disebut Persamaan Bernaoulli.

Selanjutnya untuk persamaan (2.4) dapat kita tulis dalam

$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$

misal $v = y^{1-n}$ maka $\frac{dy}{dx} = \frac{1}{(1-n)}\ y^n \frac{dv}{dx}$ sehingga persamaan diatas menjadi

$\frac{dv}{dx} + (1-n)P(x)v = Q(x)(1-n)$

misal $P_p(x) = (1-n)P(x)$ dan $Q_q(x) = (1-n)Q(x)$ maka persamaan diatas dapat direduksi ke dalam bentuk

$\therefore \frac{dv}{dx} + P_p(x)v = Q_q(x)$

Contoh:

Tentukan solusi PDB berikut ini

$(x^2+1)\frac{dy}{dx} + 4xy = x, y(2) = 1$

Solusi

$(x^2+1)\frac{dy}{dx} + 4xy = x, y(2) = 1$

Persamaan tersebut menjadi

$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y = \frac{x}{(x^2+1)}$

Maka $P(x) = \frac{4x}{(x^2+1)}$ dan $Q(x) = \frac{x}{(x^2+1)}$ sehingga dengan menggunakan

$y = e^{-\int P(x)dx} \int e^{\int P(x)dx} Q(x)dx + c$

y dapat ditentukan sebagai

$y = \frac{x^4}{4\,(x^2+1)^2} + \frac{x^2}{2\,(x^2+1)^2} + \frac{c}{(x^2+1)^2}$

Untuk $y(2) = 1$ maka subtitusikan ke persamaan ini didapat $c = 19$, akhirnya solusi khususnya adalah

$$y = \frac{x^4}{4\,(x^2+1)^2} + \frac{x^2}{2\,(x^2+1)^2} + \frac{19}{(x^2+1)^2}$$

(Dafik, 1999)

2.3 Metode Runge Kutta

Metode Runge Kutta merupakan metode yang memberikan ketelitian hasil yang lebih besar dan tidak memerlukan turunan dari fungsi (Triatmojo, 2002). Untuk mendapatkan hasil yang lebih teliti perlu diperhatikan suku yang lebih banyak dari deret Taylor atau dengan menggunakan interval Δx yang lebih kecil. Kedua cara tersebut tidak menguntungkan. Perhitungan suku yang lebih banyak memerlukan turunan yang lebih tinggi dari fungsi $y(x)$, sedangkan penggunaan Δx yang kecil menyebabkan waktu hitungan lebih panjang. Bentuk umum dari metode Rungge-Kutta adalah: (Triatmojo, 2002)

$$y_{i+1} = y_i + \phi\,(x_i, y_i, \Delta x)\Delta x \tag{2.5}$$

Dengan $\phi(x_i, y_i, \Delta x)$ adalah fungsi pertambahan yang merupakan kemiringan rerata pada interval. Fungsi pertambahan dapat ditulis dalam bentuk umum:

$$\phi = a_1 k_1 + a_2 k_2 + \cdots \ldots \ldots + a_n k_n \tag{2.6}$$

Dengan a adalah konstan dan k adalah:

$$k_1 = f(x_i, y_i) \tag{2.7}$$

$$k_2 = f(x_i + p_1\Delta x, y_i + q_{11} k_1 \Delta x) \tag{2.8}$$

$$k_3 = f(x_i + p_1\Delta x, y_i + q_{21} k_1 \Delta x + q_{22} k_2 \Delta x) \tag{2.9}$$

$$\vdots$$

$$k_n = f\big(x_i + p_{n-1}\Delta x, y_i + q_{n-1}k_1\Delta x + q_{n-1,2}k_2\Delta x + \cdots \ldots \ldots + q_{n-1,n-1}k_{n-1}\Delta x\big) \quad (2.10)$$

Persamaan tersebut menunjukkan bahwa nilai k mempunyai hubungan berurutan. Nilai k_1 muncul dalam persamaan untuk menghitung k_3, dan seterusnya. Hubungan yang berurutan ini membuat Rungge-Kutta adalah efisiensi dalam hitungan.

Metode Rungge-Kutta mempunyai tiga sifat yang utama

1. Metodenya satu langkah: untuk mencapai y_{m+1} hanya diperlukan keterangan yang tersedia pada titik sebelumnya yaitu x_m, y_m
2. Mendekati ketelitian metoda deret Taylor sampai suku dalam h^p, dimana nilai p berbeda untuk metoda yang berbeda, dan p ini disebut *derajat* dari metoda.
3. Tidak memerlukan penghitungan turunan $f(x,y)$ tetapi hanya memerlukan fungsi itu sendiri.

Ada beberapa tipe Rungge-Kutta yang tergantung pada nilai n yang digunakan. Untuk $n = 1$, disebut Rungge-kutta orde satu, sehingga persamaan (2.6) menjadi:

$$\phi = a_1 k_1 = a_1 f\,(x_i, y_i) \quad (2.11)$$

Untuk $a_1 = 1$ maka persamaan (2.10) menjadi:

$$y_{i+1} = y_i + f\,(x_i, y_i)\Delta x \quad (2.12)$$

Untuk $n = 2$ disebut Rungge-Kutta orde dua dengan bentuk: (Triatmojo, 2002)

$$y_{i+1} = y_i + (a_1 k_1 + a_2 k_2)\Delta x \quad (2.13)$$

dengan:

$$k_1 = f(x_i, y_i) \quad (2.14)$$

$$k_2 = f(x_i + p_1\Delta x, y_i + q_{11}k_1\Delta x) \tag{2.15}$$

Nilai a_1, a_2, p_1 dan q_{11} dievaluasi dengan menyamakan persamaan (2.13) dengan deret Taylor orde 2, yang mempunyai bentuk:

$$y_{i+1} = y_i + (x_i + y_i)\Delta x + f'(x_i, y_i)\frac{\Delta x}{2} \tag{2.16}$$

dengan $f'(x_i, y_i)$ dapat ditentukan dari hukum berantai (*chain rule*) berikut:

$$f'(x_i, y_i) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} \tag{2.17}$$

Subtitusi persamaan (2.17) ke dalam persamaan (2.16) menghasilkan:

$$y_{i+1} = y_i + (x_i + y_i)\Delta x + \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx}\right)\frac{\Delta x}{2} \tag{2.18}$$

Dalam metode Rungge-Kutta ini dicari nilai a_1, a_2, p_1 dan q_{11} sedimikian sehingga persamaan (2.10) ekuivalen dengan persamaan (2.18). Untuk itu digunakan deret Taylor untuk mengembang persamaan (2.15). Deret Taylor untuk fungsi dengan dua variabel mempunyai bentuk:

$$g(x + r, y + s) = g(x, y) + r\frac{\partial g}{\partial x} + s\frac{\partial g}{\partial y} + \cdots. \tag{2.19}$$

Dengan cara tersebut persamaan (2.18) dapat ditulis dalam bentuk

$$\begin{aligned} & f(x_i + p_1\Delta x, y_i + q_{11}k_1\Delta x) \\ & \quad = f(x_i, y_i) + p_1\Delta x\frac{\partial f}{\partial x} + q_{11}k_1\Delta x\frac{\partial f}{\partial y} \\ & \quad + 0(\Delta x^2) \end{aligned} \tag{2.20}$$

Kemudian bentuk di atas dan persamaan (2.13) disubtitusikan ke dalam persamaan (2.12) sehinggan menjadi:

$$y_{i+1} = y_i + a_1\Delta x\, f(x_i, y_i) + a_2\Delta x\, f(x_i, y_i) + a_2 p_1\Delta x^2\frac{\partial f}{\partial x}$$

$$+a_2 q_{11}\Delta x^2 f(x_i, y_i)\frac{\partial f}{\partial x} + 0(\Delta x^3)$$

atau

$$y_{i+1} = y_i + [a_1 f(x_i, y_i) + a_2 f(x_i, y_i)]\Delta x + [a_2 p_1 \frac{\partial f}{\partial x} + a_2 q_{11} f(x_i, y_i) \frac{\partial f}{\partial x}]\Delta x^2 + 0(\Delta x^3) \quad (2.21)$$

Dengan membandingkan persamaan (2.20) dan persamaan (2.21), dapat disimpulkan bahwa kedua persamaan ekuivalen apabila:

$$a_1 + a_2 = 1 \quad (2.22)$$

$$a_2 p_1 = \frac{1}{2} \quad (2.23)$$

$$a_2 q_{11} = \frac{1}{2} \quad (2.24)$$

Sistem persamaan diatas yang terdiri dari tiga persamaan mengandung empat bilangan yang tidak diketahui, sehingga tidak bisa diselesaikan. Untuk itu salah satu bilangan yang tidak diketahui ditetapkan, dan kemudian dicari ketiga bilangan yang lain. Dianggap bahwa a_2 ditetapkan, sehingga persamaan (2.22) sampai (2.24) dapat diselesaikan dan menghasilkan:

$$a_1 = 1 - a_2 \quad (2.25)$$

$$p_1 = \frac{1}{2a_2} \quad (2.26)$$

Karena nilai a_2 dapat dipilih sembarang, maka akan terdapat banyak metode Rungge-Kutta orde 2 .

Selain Rungge Kuta orde dua, juga ada Runge Kutta orde tiga dengan $n = 3$. Metode Rungge Kutta orde tiga mempunyai bentuk umum:

$y_{i+1} = y_i + (\frac{1}{6}k_1 + \frac{2}{3}k_2 + \frac{1}{6}k_3)$ (Butcher, 2008)

Metode runge kutta orde empat dengan $n = 4$ adalah metode yang paling popular, karena metode ini sering digunakan Dalam komputasi. Berikut rumus metode Runge Kutta ode empat:

$y_{i+1} = y_i + (\frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4)$ (Dukkipati, 2010)

2.4 Metode Runge Kutta orde 5

Bentuk umum Runge kutta orde 5 yaitu

$$y_{i+1} = y_i + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4 + a_5k_5)h$$

dimana:

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + p_1h, y_i + q_{11}k_1h)$

$k_3 = f(x_i + p_2h, y_i + q_{21}k_1h + q_{22}k_2h)$

$k_4 = f(x_i + p_3h, y_i + q_{31}k_1h + q_{32}k_2h + q_{33}k_3h)$

$k_5 = f(x_i + p_4h, y_i + q_{41}k_1h + q_{42}k_2h + q_{43}k_3h + q_{44}k_4h)$

2.5 Integrasi Al-Quran

Allah berfirman dalam Q.S al-Baqoroh ayat 164 yang artinya:

"Sesungguhnya pada penciptaan langit dan bumi, dan pergantian malam dan siang, kapal yang berlayar di laut dengan (muatan) yang bermanfaat bagi manusia, apa yang diturunkan Allah dari langit berupa air lalu dari air itu dihidupkan bumi setelah mati (kering), dan dia tebarkan di dalamnya bermacam-macam binatang, dan perkisaran angin dan awan yang dikendalikan antara langit dan bumi, (semua itu) sungguh merupakan tanda-tanda (kebesaran Allah)bagi orang-orang yang mengerti"

Ibnu katsir dalam tafsirnya menerangkan bahwa dalam ayat ini Allah menunjukkan kejadian langit, tinggi halusnya, luasnya disamping bintang, bulan dan mataharinya, kemudian bumi dan semua yang ada di atasnya, bukit dan gunung-Nya, sungai dan lautannya, kota dan dusunnya, hutan dan kebun ladangnya, silih bergantinya malam dan siang, lambat dan cepatnya atau panjang dan pendeknya juga bahtera yang berjalan di laut membawa segala kepentingan hidup manusia, dan turunnya hujan dari langit untuk menumbuhkan berbagai

macam tumbuh-tumbuhan, dan tersebarnya berbagai binatang serangga dan ternak, dan datangnya angin menghalau awan sebagai bukti kekuasaan kebesaran Allah bagi kaum yang berakal

Abu Bakar jabir al-Jazairi dalam tafsir al-Aisar menjelaskan makna perkata dari ayat tersebut adalah:

(*ikhtilafullaili wannahari*): muncul yang satu dan tenggelam yang lain untuk kemaslahatan umat manusia, sehingga tidak menjadikan siang selamanya atau malam selamanya.

(*wabatsta fiha min kulli dabbatin*): dia Sebarkan dan tebarkan di atas bumi segala jenis hewan.

(*tashrifurriyah*): dan pengisaran angin, dan segala arahnya, terkadang dari barat, timur, utara, selatan terkadang dapat membantu terjadinya penyerbukan tanaman, namun terkadang juga tidak.

Ayat tersebut mengandung enam ayat kauniyah (bukti fenomena alam) yang menunjukkan adanya Allah, kekuasaan, ilmu dan rahmat-Nya, semua itu mengharuskan ibadah kepada Allah semata, yaitu:

1. Penciptaan langit dan bumi adalah penciptaan yang agung, tidak ada yang mampu melakukannya kecuali tuhan yang maha kuasa.
2. Pergantian malam dan siang secara beraturan, yang satu masanya panjang dan yang satu masanya pendek.
3. Berlayarlah perahu dan kapal di atas permukaan air laut dengan muatan dan bobot yang berton-ton baik dari makanan ataupun kebutuhan hidup manusia.
4. Allah menurunkan hujan dari langit untuk kehidupan bumi dan menghidupkan tumbuh-tumbuhan dan tanaman setelah sebelum kering.

5. Berhembusnya angin baik panas, dingin, kering, basah, angin timur dan angin barat, angin utara dan angin selatan, sesuai kebutuhan manusia.
6. Awan yang berjalan diantara langit dan bumi, terkadang bergantian antara suatu negeri dengan lainnya, hingga terjadi hujan di sebuah tempat dan tidak terjadi di tempat lain, sesuai kehendak Allah yang maha perkasa lagi maha bijaksana.

Al-Maraghi dalam tafsirnya menjelaskan ayat tersebut secara terperinci mengenai fenomena yang menunjukkan kekuasaan Allah yaitu:

1. Langit, yamg benda-bendanya terdiri dari berbagai jenis atau kelompok. Setiap kelompok mempunyai tatanan tersendiri secara teratur, dan setiap satuan dari kelompok tersebut mempunyai tatanan yang sama pula. Semua ini menunjukkan bahwa penciptannya adalah esa (satu), tidak ada yang dapat menyamainya, baik dalam hal penciptaan, pengaturan, dan penataan. Diantara benda-benda langit itu, yang paling dekat dengan manusia adalah tata surya yang mempunyai matahari yang bersinar hingga adanya kehidupan di bumi ini. Kemudian planit-planit yang besar dan kecil yang jaraknya saling berbeda. Setiap planit tersebut beredar pada peredarannyasecara tetap. Stabilitas edar planit tersebut dipelihara dengan sunnatu ‘ilah yang kita kenal dengan gaya tarik (grafitasi).
2. Bumi, bentuk, materi segala sesuatu yang ada di dalamnya berupa benda-benda padat, tetumbuhan dan aneka marga satwa, manfaat setiap benda yang saling berbeda, semuanya menunjukkan bahwa penciptanya maha berilmu, maha bijaksana dan maha mengetahui.

3. Silih bergantinya siang, malam dan bergilirnya antara keduanya dalam hal panjang dan pendeknya waktu sesuai dengan letak perbedaan negara dan musim. Pada semuanya itu terkandung manfaat dan maslahat bagi umat manusia, di samping menunjukkan bahwa penciptannya adalah satu, maha pengasih terhadap hamba-hambanya.
4. *Al-fulk* adalah kata yang pengertiannya seperti perahu atau banyak perahu. Bukti keesaan Allah melalui masalah ini membutuhkan pengetahuan tentang tabiat air laut dan kaidah-kaidah gaya tarik, tabiat udara. Angin, awan dan listrik yang merupakan penggerak utama kapal-kapal di masa sekarang.
5. *"wa maaanzalallah minassamaai mimmaaiin"*

   Di dalam ayat ini, Allah menjelaskan bagaimana menurunkan hujan:

   "Allah, dialah yang mengirim angin, lalu angin itu menggerakkan awan dan Allah membentangkannya di langit menurut yang dikehendaki-Nya, yang menjadikan nya bergumpal-gumpal, lalu kamu lihat hujan keluar dari celah-celah nya,..(ar Rum, 30:48)"

   Gambaran singkat ini, kemudian dijelaskan oleh para ahli ilmu alam mengatakan:

   Timbulnya hujan itu karena terjadi adanya penguapan air disebabkan oleh panasnya udara yang menyengat permukaan laut. Sehingga terjadilah pergeseran molekul-molekul zat air yang kemudian menjadi uap karena gesekan dengan panas. Ketika uap tersebut naik ke atas terbentuklah mendung yang semakin menebal. Karena beratnya, maka mendung itu berubah menjadi hujan yang jatuh ke bumi. Itulah yang dinamakan hujan. Karena air inilah timbul kehidupan dengan berbagai tumbuhan dipermukaan bumi, yang kemudian dimanfaatkan hewan sebagai sumber kehidupan. Setiap bumi yang tidak terkena air hujan atau dilewati arus air, maka belahan

tersebut tidak mempunyai margasatwa (fauna) dan tumbuhan (flora) yang hidup didalamnya.

Turunnya hujan dan bentuk yang dapat kita saksikan atau yang menjadi sebab kehidupan bagi tumbuhan dan hewan, merupakan bukti yang menunjukkan bahwa Allah maha esa dan maha pencipta. Dan jika ditinjau dari segi kemanfaatannya., maka kenyataan tersebut merupakan rahmat illahi yang berlaku umum.

6. Dalam mengendalikan arah angin ini, sudah barang tentu sesuai kodrat Allah dan sunnatu 'ilah yang diciptakan oleh yang maha bijaksana, fungsinya adalah untuk mengawinkan antara serbuk jantan dan betina yang terdapat di dalam tumbuhan. Juga ada jenis angin yang berfungsi sebagaimana tersebut di atas, tetapi hanya untuk menggugurkan. Semua ini tidak lain menunjukkan kesatuan dari sumbernya dan menunjukkan kasih sayang Allah yang telah menciptakan segalannya, disamping mengatur.
7. Pada mendung yang berkelompok dengan ketebalannya di udara itu untuk kepentingan turunnya hujan di berbagai negara. Cara turunnya pun teratur, disamping mendung itu merupakan pemandangan indah dilihat dari berbagai belahan bumi.

Pada korelasinnya maka dalam penciptaan langit, bumi, pergantian siang dan malam, terdapat perbedaan yang nyata, pada semua gejala ini terdapat petunjuk bagi orang-orang yang berpikir untuk mengetahui watak dan rahasia-rahasianya. Dengan demikian dapat dibedakan antara bermanfaat dan yang membahayakan, disamping dapat diketahui betapa teliti dan halusnya kekuasaan

yang maha menciptakan semua ini. yang Akhirnya akan sampai pada kesimpulan bahwa yang menciptakan semua inilah yang berhak untuk disembah dan di taati

BAB III

PEMBAHASAN

3.1 Penurunan Runge Kutta Orde lima

Dalam sub bab ini akan dibahas tentang penurunan metode runge kutta orde lima. Tujuan dari penurunan ini adalah untuk mengetahui asal mula koefisien yang ada pada metode runge kutta orde lima. Pada penurunan ini menggunakan aturan deret taylor dan aturan rantai.

Diberikan rumus umum runge kutta orde lima.

$$y_{i+1} = y_i + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4 + a_5k_5)h \tag{3.1}$$

dimana:

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + p_1h, y_i + q_{11}k_1h)$

$k_3 = f(x_i + p_2h, y_i + q_{21}k_1h + q_{22}k_2h)$

$k_4 = f(x_i + p_3h, y_i + q_{31}k_1h + q_{32}k_2h + q_{33}k_3h)$

$k_5 = f(x_i + p_4h, y_i + q_{41}k_1h + q_{42}k_2h + q_{43}k_3h + q_{44}k_4h)$

Karena akan dilakukan penurunan metode runge kutta orde lima maka deret taylor dipotong sampai orde kelima.

$$y_{i+1} = y_i + f(x_i, y_i)h + f'(x_i, y_i)\frac{h^2}{2!} + f''(x_i, y_i)\frac{h^3}{3!} + f'''(x_i, y_i)\frac{h^4}{4!} + f^{(4)}(x_i, y_i)\frac{h^5}{5!} \tag{3.2}$$

Setelah itu mencari penurunan pertama, kedua, ketiga, dan keempat dari deret taylor diatas, dengan dimisalkan $f(x, y) = \frac{dy}{dx}$, maka

$$f'(x,y) = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}\frac{dy}{dx} = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$f''(x,y) = \frac{\partial}{\partial x}\big(f'(x,y)\big) + \frac{\partial}{\partial y}\big(f'(x,y)\big)f(x,y)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} f(x,y)\right)$$

$$+ \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y} f(x,y)\right)f(x,y)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+ \left(\frac{\partial^2 f(x,y)}{\partial y \partial x} + \frac{\partial^2 f(x,y)}{\partial y^2} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}\right)f(x,y)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+ \frac{\partial^2 f(x,y)}{\partial y^2}\big(f(x,y)\big)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + 2\frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x} + \frac{\partial^2 f(x,y)}{\partial y^2}\big(f(x,y)\big)^2$$

$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)$$

$$f'''(x,y) = \frac{\partial}{\partial x}\big(f''(x,y)\big) + \frac{\partial}{\partial y}\big(f''(x,y)\big)f(x,y)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial^2 f(x,y)}{\partial x^2} + 2\frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}\right.$$

$$\left. + \frac{\partial^2 f(x,y)}{\partial y^2}\big(f(x,y)\big)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\right)$$

$$+ \frac{\partial}{\partial y}\left(\frac{\partial^2 f(x,y)}{\partial x^2} + 2\frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}\right.$$

$$+\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\Bigg)f(x,y)$$

$$=\frac{\partial^3 f(x,y)}{\partial x^3}+2\frac{\partial^3 f(x,y)}{\partial x^2\partial y}f(x,y)\ +2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\left(f(x,y)\right)^2+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial\left(f(x,y)\right)^2}{\partial x}$$

$$+2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial x}+\left(\frac{\partial^3 f(x,y)}{\partial y\partial x^2}\right.$$

$$+2\frac{\partial^3 f(x,y)}{\partial y^2\partial x}f(x,y)+2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}$$

$$+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x\partial y}+\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial\left(f(x,y)\right)^2}{\partial y}$$

$$+2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\left.\frac{\partial f(x,y)}{\partial y}\right)f(x,y)$$

$$=\frac{\partial^3 f(x,y)}{\partial x^3}+2\frac{\partial^3 f(x,y)}{\partial x^2\partial y}f(x,y)\ +2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\left(f(x,y)\right)^2+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial\left(f(x,y)\right)^2}{\partial x}$$

$$+2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial x}+\frac{\partial^3 f(x,y)}{\partial y\partial x^2}f(x,y)$$

$$+2\frac{\partial^3 f(x,y)}{\partial y^2\partial x}f(x,y)f(x,y)\ +2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}f(x,y)+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)$$

$$+\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2 f(x,y)+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial\left(f(x,y)\right)^2}{\partial y}f(x,y)$$

$$+2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$= \frac{\partial^3 f(x,y)}{\partial x^3} + 2\frac{\partial^3 f(x,y)}{\partial x^2 \partial y} f(x,y) + 2\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x} + \frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+ \frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^3 f(x,y)}{\partial x \partial y^2}\left(f(x,y)\right)^2 + \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial \left(f(x,y)\right)^2}{\partial x}$$

$$+2\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y) + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial x} + \frac{\partial^3 f(x,y)}{\partial y \partial x^2} f(x,y)$$

$$+2\frac{\partial^3 f(x,y)}{\partial y^2 \partial x}\left(f(x,y)\right)^2 + 2\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$+ \frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y) + \frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$

$$+ \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial \left(f(x,y)\right)^2}{\partial y} f(x,y) + 2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$= \frac{\partial^3 f(x,y)}{\partial x^3} + 3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y} f(x,y) + 3\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}$$

$$+3\frac{\partial^3 f(x,y)}{\partial x \partial y^2}\left(f(x,y)\right)^2 + \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial \left(f(x,y)\right)^2}{\partial x} + 5\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial x} + \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x} f(x,y) + \frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$

$$+ \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial \left(f(x,y)\right)^2}{\partial y} f(x,y) + 2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$f^{(4)}(x,y) = \frac{\partial}{\partial x}\left(f'''(x,y)\right) + \frac{\partial}{\partial y}\left(f'''(x,y)\right) f(x,y)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial^3 f(x,y)}{\partial x^3} + 3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y} f(x,y) + 3\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x}\right.$$
$$+ \frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2} + 3\frac{\partial^3 f(x,y)}{\partial x \partial y^2}\left(f(x,y)\right)^2$$
$$+ 3f(x,y)\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x} + 5\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y)$$
$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial x} + \frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$
$$+ 4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$
$$\left. + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial y} f(x,y)\right)$$
$$+ \frac{\partial}{\partial y}\left(\frac{\partial^3 f(x,y)}{\partial x^3} + 3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y} f(x,y) + 3\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x}\right.$$
$$+ \frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2} + 3\frac{\partial^3 f(x,y)}{\partial x \partial y^2}\left(f(x,y)\right)^2$$
$$+ 3f(x,y)\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x} + 5\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y)$$
$$+ \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial x} + \frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$
$$+ 4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$
$$\left. + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 \frac{\partial f(x,y)}{\partial y} f(x,y)\right) f(x,y)$$

$$= \frac{\partial^4 f(x,y)}{\partial x^4} + 3\frac{\partial^4 f(x,y)}{\partial x^3 \partial y} f(x,y) + \frac{\partial f(x,y)}{\partial x} 3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y} + 3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y}\frac{\partial f(x,y)}{\partial x}$$
$$+3\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}$$
$$+3\frac{\partial^4 f(x,y)}{\partial x^2 \partial y^2}\left(f(x,y)\right)^2 + 3\frac{\partial^3 f(x,y)}{\partial x \partial y^2} 2f(x,y)\frac{\partial f(x,y)}{\partial x}$$
$$+3\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2} f(x,y) + 3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial x \partial y^2} f(x,y)$$
$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x} + 5\frac{\partial^3 f(x,y)}{\partial x^2 \partial y}\frac{\partial f(x,y)}{\partial y} f(x,y)$$

$$+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+\frac{\partial^2 f(x,y)}{\partial x^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2+\frac{\partial f(x,y)}{\partial x}2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}$$

$$+\frac{\partial^4 f(x,y)}{\partial y^3\partial x}\left(f(x,y)\right)^3+\frac{\partial^3 f(x,y)}{\partial y^3}\ 3f(x,y)\frac{\partial f(x,y)}{\partial x}$$

$$+4\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\left(f(x,y)\right)^2$$

$$+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}2f(x,y)\frac{\partial f(x,y)}{\partial x}+2\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\left(\frac{\partial^4 f(x,y)}{\partial x^3\partial y}\right.$$

$$+3\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}f(x,y)+3\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial y}+3\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial x}$$

$$+3\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x\partial y}+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^2\partial y}$$

$$+3\frac{\partial^4 f(x,y)}{\partial x\partial y^3}\left(f(x,y)\right)^2+3\frac{\partial^3 f(x,y)}{\partial x\partial y^2}2f(x,y)\frac{\partial f(x,y)}{\partial y}$$

$$+3\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}f(x,y)+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}f(x,y)$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}+5\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}f(x,y)+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}$$

$$+\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2+\frac{\partial f(x,y)}{\partial x}2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}$$

$$+\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^3+\frac{\partial^3 f(x,y)}{\partial y^3}\ 3f(x,y)\frac{\partial f(x,y)}{\partial y}$$

$$+4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2$$

$$+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}2f(x,y)\frac{\partial f(x,y)}{\partial y}+2\ \frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\ \frac{\partial^2 f(x,y)}{\partial y^2}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\ \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}\Bigg)f(x,y)$$

$$=\frac{\partial^4 f(x,y)}{\partial x^4}+3\frac{\partial^4 f(x,y)}{\partial x^3\partial y}f(x,y)\ +\frac{\partial f(x,y)}{\partial x}3\ \frac{\partial^3 f(x,y)}{\partial x^2\partial y}+3\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+3\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\ \frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}$$

$$+3\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}\left(f(x,y)\right)^2+3\frac{\partial^3 f(x,y)}{\partial x\partial y^2}2f(x,y)\frac{\partial f(x,y)}{\partial x}$$

$$+3\frac{\partial^2 f(x,y)}{\partial y^2}\ \frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)+3\frac{\partial f(x,y)}{\partial x}\ \frac{\partial^3 f(x,y)}{\partial x\partial y^2}f(x,y)$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}+5\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+\frac{\partial^2 f(x,y)}{\partial x^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2+\frac{\partial f(x,y)}{\partial x}2\ \frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}+\frac{\partial^4 f(x,y)}{\partial y^3\partial x}\left(f(x,y)\right)^3$$

$$+\frac{\partial^3 f(x,y)}{\partial y^3}\ 3f(x,y)\frac{\partial f(x,y)}{\partial x}+4\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2$$

$$+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\left(f(x,y)\right)^2+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}2f(x,y)\frac{\partial f(x,y)}{\partial x}$$

$$+2\ \frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\ \frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\ \frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^4 f(x,y)}{\partial x^3\partial y}f(x,y)$$

$$+3\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}\left(f(x,y)\right)^2+3\ \frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+3\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial x}f(x,y)+3\left(\frac{\partial^2 f(x,y)}{\partial x\partial y}\right)^2 f(x,y)$$

$$+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^2\partial y}f(x,y)$$

$$+3\frac{\partial^4 f(x,y)}{\partial x\partial y^3}\left(f(x,y)\right)^3+6\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+3\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)+5\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)$$

$$+\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)+2\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^4+3\frac{\partial^3 f(x,y)}{\partial y^3}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+4\left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)^2\left(f(x,y)\right)^3+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$

$$+8\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+2\frac{\partial^2 f(x,y)}{\partial y^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2\left(f(x,y)\right)^2$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+\left(\frac{\partial f(x,y)}{\partial y}\right)^4 f(x,y)$$

$$=\frac{\partial^4 f(x,y)}{\partial x^4}+4\frac{\partial^4 f(x,y)}{\partial x^3\partial y}f(x,y)+4\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial x}+4\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}$$

$$+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}+6\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}\left(f(x,y)\right)^2+12\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial x}f(x,y)$$

$$+4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)+3\left(\frac{\partial f(x,y)}{\partial x}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}$$

$$+9\frac{\partial^3 f(x,y)}{\partial x^2 \partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+8\left(\frac{\partial^2 f(x,y)}{\partial x \partial y}\right)^2 f(x,y)$$

$$+7\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^2 f(x,y)}{\partial x^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2+4\frac{\partial^4 f(x,y)}{\partial x \partial y^3}\left(f(x,y)\right)^3$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}f(x,y)+12\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2$$

$$+15\frac{\partial^3 f(x,y)}{\partial x \partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2+13\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)$$

$$+9\frac{\partial^2 f(x,y)}{\partial x \partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2+\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^4$$

$$+3\frac{\partial^3 f(x,y)}{\partial y^3}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2+4\left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)^2\left(f(x,y)\right)^3$$

$$+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3+11\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y\right.$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^4 f(x,y)$$

Kemudian subtitusi hasil turunan ke persamaan 3.2 maka

$$y_{i+1}=f(x_i,y_i)h+\left(\frac{\partial f(x,y)}{\partial x}+\frac{\partial f(x,y)}{\partial y}f(x,y)\right)\frac{h^2}{2}$$

$$+\left(\frac{\partial^2 f(x,y)}{\partial x^2}+2\frac{\partial^2 f(x,y)}{\partial x \partial y}f(x,y)+\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2\right.$$

$$\left.+\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\right)\frac{h^3}{6}$$

$$+\left(\frac{\partial^3 f(x,y)}{\partial x^3}+3\frac{\partial^3 f(x,y)}{\partial x^2 \partial y}f(x,y)+3\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\right.$$

$$\left.+3\frac{\partial^3 f(x,y)}{\partial x \partial y^2}\left(f(x,y)\right)^2\right.$$

$$+3f(x,y)\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}+5\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial x}$$
$$+\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$

$$+4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}f(x,y)\Bigg)\frac{h^4}{24}$$

$$+\left(\frac{\partial^4 f(x,y)}{\partial x^4}+4\frac{\partial^4 f(x,y)}{\partial x^3\partial y}f(x,y)+4\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial x}+4\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\right.$$
$$\left.+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}\right.$$

$$+6\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}\left(f(x,y)\right)^2+12\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial x}f(x,y)+4\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)$$

$$+3\left(\frac{\partial f(x,y)}{\partial x}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}+9\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)+8\left(\frac{\partial^2 f(x,y)}{\partial x\partial y}\right)^2 f(x,y)$$

$$+7\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial^2 f(x,y)}{\partial x^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2+4\frac{\partial^4 f(x,y)}{\partial x\partial y^3}\left(f(x,y)\right)^3$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}f(x,y)+12\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2$$
$$+15\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$

$$+13\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)+9\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)$$
$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}$$

$$+3\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2+\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^4+3\frac{\partial^3 f(x,y)}{\partial y^3}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2$$
$$+4\left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)^2\left(f(x,y)\right)^3+4\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3$$

$$+11\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2+\left(\frac{\partial f(x,y)}{\partial y}\right)^4 f(x,y)\Bigg)\frac{h^5}{120}$$

$$y_{i+1}=f(x_i,y_i)h+\left(\frac{\partial f(x,y)}{\partial x}\frac{h^2}{2}+\frac{\partial f(x,y)}{\partial y}f(x,y)\frac{h^2}{2}\right)$$
$$+\left(\frac{\partial^2 f(x,y)}{\partial x^2}\frac{h^3}{6}+\frac{\partial^2 f(x,y)}{\partial x\partial y}f(x,y)\frac{h^3}{3}+\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}\frac{h^3}{6}+\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2\frac{h^3}{6}\right.$$
$$\left.+\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\frac{h^3}{6}\right)$$

$$+\left(\frac{\partial^3 f(x,y)}{\partial x^3}\frac{h^4}{24}+\frac{\partial^3 f(x,y)}{\partial x^2\partial y}f(x,y)\frac{h^4}{8}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial x}\frac{h^4}{8}+\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\frac{h^4}{24}\right.$$
$$\left.+\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\left(f(x,y)\right)^2\frac{h^4}{8}\right)$$

$$+f(x,y)\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}\frac{h^4}{8}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)\frac{5h^4}{24}+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial x}\frac{h^4}{24}$$
$$+\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3\frac{h^4}{24}$$

$$+\left(\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2\frac{h^4}{6}+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}f(x,y)\frac{h^4}{24}\right)$$

$$+\left(\frac{\partial^4 f(x,y)}{\partial x^4}\frac{h^5}{120}+\frac{\partial^4 f(x,y)}{\partial x^3\partial y}f(x,y)\frac{h^5}{30}+\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial x}\frac{h^5}{30}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\frac{h^5}{30}\right.$$
$$\left.+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}\frac{h^5}{120}\right.$$

$$+\frac{\partial^4 f(x,y)}{\partial x^2\partial y^2}\left(f(x,y)\right)^2\frac{h^5}{20}+\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial x}f(x,y)\frac{h^5}{10}+\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)\frac{h^5}{30}$$

$$+\left(\frac{\partial f(x,y)}{\partial x}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\frac{h^5}{40}+\frac{\partial^3 f(x,y)}{\partial x^2\partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)\frac{9h^5}{120}+\left(\frac{\partial^2 f(x,y)}{\partial x\partial y}\right)^2 f(x,y)\frac{h^5}{15}$$

$$+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}\frac{7h^5}{120}+\frac{\partial^2 f(x,y)}{\partial x^2}\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{h^5}{120}+\frac{\partial^4 f(x,y)}{\partial x\partial y^3}\left(f(x,y)\right)^3\frac{h^5}{30}$$

$$+\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}f(x,y)\frac{h^5}{40}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2\frac{h^5}{10}$$
$$+\frac{\partial^3 f(x,y)}{\partial x\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2\frac{h^5}{8}$$

$$+\frac{\partial f(x,y)}{\partial x}\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}f(x,y)\frac{13h^5}{120}+\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\frac{3h^5}{40}$$
$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}+\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^2\frac{h^5}{40}$$
$$+\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^4\frac{h^5}{120}+\frac{\partial^3 f(x,y)}{\partial y^3}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2\frac{h^5}{40}$$

$$+\left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)^2\left(f(x,y)\right)^3\frac{h^5}{30}+\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3\frac{h^5}{30}$$

$$+\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2\frac{11h^5}{120}+\left(\frac{\partial f(x,y)}{\partial y}\right)^4 f(x,y)\frac{h^5}{120}\Bigg) \qquad (3.3)$$

Selanjutnya ubah k_2, k_3, k_4, k_5, k_6 kederet Taylor dua variabel orde dua

Untuk $k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h)$

$$f(x_i + p_1 h, y_i + q_{11} k_1 h)$$

$$= f + p_1 h \frac{\partial f}{\partial x} + q_{11} f h \frac{\partial f}{\partial y} + \frac{1}{2} p_1^2 h^2 \frac{\partial^2 f}{\partial x^2} + p_1 q_{11} h^2 f \frac{\partial^2 f}{\partial x \partial y}$$

$$+ \frac{1}{2} q_{11}^2 h^2 f^2 \frac{\partial^2 f}{\partial y^2} + \frac{1}{6} p_1^3 h^3 \frac{\partial^3 f}{\partial x^3} + \frac{1}{2} q_{11} p_1^2 h^3 f \frac{\partial^2 f}{\partial x^2} \frac{\partial f}{\partial y} + \frac{1}{2} p_1 q_{11}^2 f^2 h^3 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2}$$

$$+ \frac{1}{6} q_{11}^3 h^3 f^3 \frac{\partial^3 f}{\partial y^3} + \frac{1}{24} p_1^4 h^4 \frac{\partial^4 f}{\partial x^4} + \frac{1}{6} q_{11} p_1^3 h^4 f \frac{\partial f}{\partial y} \frac{\partial^3 f}{\partial x^3} + \frac{1}{4} p_1^2 q_{11}^2 h^4 f \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2}$$

$$+ \frac{1}{6} p_1 q_{11} h^4 f \frac{\partial f}{\partial x} \frac{\partial^3 f}{\partial y^3} + \frac{1}{24} q_{11}^4 h^4 f \frac{\partial^4 f}{\partial y^4}$$

Untuk $k_3 = f(x_i + p_2 h, y_i + q_{21} k_1 h + q_{22} k_2 h)$

$$f(x_i + p_2 h, y_i + q_{21} k_1 h + q_{22} k_2 h)$$

$$= f + \left(p_2 \frac{\partial f}{\partial x} + q_{21} f \frac{\partial f}{\partial x} + q_{22} f \frac{\partial f}{\partial x} + q_{21} f \frac{\partial^2 f}{\partial y^2} + q_{22} f \frac{\partial^2 f}{\partial y^2} \right) h$$

$$+ \left(q_{22} p_1 \left(\frac{\partial f}{\partial x} \right)^2 + p_2 q_{21} f \frac{\partial^2 f}{\partial x \partial y} + q_{22} q_{11} f \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} + q_{22} q_{11} f \frac{\partial^2 f}{\partial y^2} \frac{\partial f}{\partial y} + \frac{1}{2} p_2^2 \frac{\partial^2 f}{\partial x} \right.$$

$$\left. + p_2 q_{22} f \frac{\partial^2 f}{\partial x \partial y} + p_1 q_{22} \frac{\partial^2 f}{\partial y^2} \frac{\partial f}{\partial x} \right) h^2$$

$$+ \left(\frac{1}{9} p_2^3 \frac{\partial^3 f}{\partial x^3} + \frac{1}{9} q_{22}^3 f^3 \frac{\partial^3 f}{\partial y^3} + \frac{1}{2} p_2 q_{22}^2 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + \frac{1}{2} p_2^2 q_{22} f \frac{\partial^2 f}{\partial x^2} \frac{\partial f}{\partial y} \right.$$

$$\left. + \frac{1}{2} q_{22} q_{11}^2 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} \right.$$

$$+ p_1 p_2 q_{22} \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x \partial y} + \frac{1}{2} p_1^2 q_{22} \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x^2} + \frac{1}{2} p_1^2 q_{22} \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} + \frac{1}{2} q_{22} q_{11}^2 f^2 \left(\frac{\partial^2 f}{\partial y^2} \right)$$

$$+ \frac{1}{3} q_{21}^2 q_{22} f^3 \frac{\partial^3 f}{\partial y^3}$$

$$+ \frac{1}{3} q_{22}^2 q_{21} f^3 \frac{\partial^3 f}{\partial y^3} + \frac{1}{9} q_{21}^3 f^3 \frac{\partial^3 f}{\partial y^3} + p_1 q_{11} q_{22} f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x \partial y} + p_2 q_{22} q_{11} f \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y}$$

$$+ q_{22} p_1 q_{11} f \frac{\partial^2 f}{\partial y^2} \frac{\partial^2 f}{\partial x \partial y} + p_2 q_{21} q_{22} f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2}$$

$$\left. + \frac{1}{2} p_2 q_{21}^2 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + \frac{1}{2} p_2^2 q_{21} f \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x^2} \right) h^3$$

$$+\left(\frac{1}{6}q_{21}^3q_{22}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{21}^2q_{22}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{21}q_{22}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{22}p_1^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}\right.$$
$$+\frac{1}{6}q_{22}p_1^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{22}q_{11}p_1^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+p_2q_{11}q_{22}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{11}q_{22}p_1^2f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{21}q_{22}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{24}q_{22}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{24}q_{21}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{24}p_2^4\frac{\partial^4 f}{\partial x^4}+\frac{1}{6}p_2^3q_{21}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{6}p_2^3q_{22}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{6}p_2q_{21}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_2q_{22}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}p_2^2q_{21}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{4}p_2^2q_{22}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{3}q_{11}q_{22}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{3}p_1q_{22}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2p_2q_{22}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}q_{11}^3q_{22}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{11}q_{22}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+\frac{1}{2}p_2q_{11}^2q_{22}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_1q_{11}^2q_{22}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_1p_2^2q_{22}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2^2q_{11}q_{22}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}p_2q_{21}^2q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{22}^2q_{21}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{21}q_{22}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_1q_{11}^2q_{22}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{22}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{3}p_1q_{21}^2q_{22}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{3}q_{21}^2q_{22}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{2}{3}p_1q_{21}q_{22}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{2}{3}q_{11}q_{21}q_{22}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$\left.+p_2q_{21}q_{22}q_{11}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right)h^4$$

$$+\left(\frac{1}{24}p_1^4 q_{22}\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial x^4}+\frac{1}{24}p_1^4 q_{22}\frac{\partial^2 f}{\partial y^2}\frac{\partial^4 f}{\partial x^4}+\frac{1}{2}p_2 q_{11} q_{22}^3 f^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}\right.$$
$$+\frac{1}{2}p_1 p_2 q_{21}^2 q_{22} f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{2}{3}p_1 q_{11} q_{21} q_{22}^2 f^3\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}\dots\dots\dots\dots\dots\dots\dots$$
$$\left.+p_2 q_{11} q_{21} q_{22} f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}\right)h^5$$

Untuk $k_4 = f(x_i + p_3 h, y_i + q_{31}k_1 h + q_{32}k_2 h + q_{33}k_3 h)$

$$f(x_i + p_3 h, y_i + q_{31}k_1 h + q_{32}k_2 h + q_{33}k_3 h)$$
$$= f + \left(q_{31} f\frac{\partial f}{\partial y}+q_{32} f\frac{\partial f}{\partial y}+q_{33} f\frac{\partial f}{\partial y}\right)h$$
$$\left(\frac{1}{2}p_3^2\frac{\partial^2 f}{\partial x^2}+q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right.$$
$$\left.+q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right.$$
$$+\frac{1}{2}q_{33}^2 f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{32}^2 f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2 f^2\frac{\partial^2 f}{\partial y^2}+q_{31}q_{32}f^2\frac{\partial^2 f}{\partial y^2}+q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}+p_3 q_{31}f\frac{\partial^2 f}{\partial x\partial y}+p_3 q_{32}f\frac{\partial^2 f}{\partial x\partial y}+p_3 q_{33}f\frac{\partial^2 f}{\partial x\partial y}$$
$$\left.+q_{11}q_{32}f\left(\frac{\partial f}{\partial y}\right)^2\right.$$
$$\left.+p_1 q_{32}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_2 q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\right)h^2+\left(p_3 q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2 q_{32}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\right.$$
$$+\frac{1}{2}p_2^2 q_{33}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{22}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_2 q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_2 q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1 q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1 q_{11}q_{32}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}p_3^3\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{32}q_{33}^2 f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{32}^2 f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{32}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{31}^2q_{33}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}^2q_{31}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}^2q_{31}f^3\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{32}^2f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{32}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{31}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{32}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{32}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{33}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1p_3q_{32}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_2p_3q_{33}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}q_{32}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{33}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{31}q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{11}^2q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{22}q_{33}\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{6}q_{31}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_3^2q_{32}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{31}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3q_{31}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{31}q_{32}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{31}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{11}q_{32}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{21}q_{33}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{22}q_{33}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{32}q_{33}f^3\frac{\partial^3 f}{\partial y^3}\Big)h^3+\Big(\frac{1}{6}p_1^3q_{32}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{9}p_2^3q_{33}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{24}q_{32}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{31}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{32}^2q_{33}q_{21}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}^2q_{33}q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{33}q_{21}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{33}q_{22}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}^2q_{33}p_2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{32}^2q_{33}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}^2q_{33}p_1f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_2^2q_{22}q_{33}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2^2q_{21}q_{33}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1^2q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_1^2q_{22}q_{33}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{11}^2q_{22}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{21}^2q_{22}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{3}q_{22}^2q_{21}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{32}q_{33}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{11}q_{32}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{11}q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+2\,q_{11}q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{11}q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{21}q_{22}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}q_{32}^2q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{32}q_{33}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}q_{33}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{32}^3q_{33}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{11}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{31}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}p_2^2p_3q_{33}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{21}^2q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{22}^2q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}^2q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_1^2q_{32}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{33}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{11}^2q_{31}q_{32}f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}q_{11}^2q_{32}q_{33}f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}^2q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}^2q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{21}q_{22}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3+\frac{1}{2}p_1^2p_3q_{32}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}p_3q_{32}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_3q_{22}q_{32}q_{33}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1p_3q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_2p_3q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_3^2q_{32}q_{33}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{33}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{33}^2q_{32}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}^2q_{32}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}^2q_{32}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1q_{33}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{33}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2p_2q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2p_2q_{32}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{11}q_{32}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_3q_{31}q_{33}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{31}q_{32}q_{33}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{32}q_{33}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{31}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{31}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{11}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_3^3q_{33}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{6}p_3^3q_{32}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3q_{21}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{22}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2p_3q_{31}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_3^2q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3^2q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_1p_3q_{31}q_{32}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{24}p_3^4\frac{\partial^4 f}{\partial x^4}$$

$$+\frac{1}{2}p_3q_{31}q_{32}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3q_{32}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{11}q_{31}q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_3q_{11}q_{32}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2p_3q_{21}q_{33}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2p_3q_{22}q_{33}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+\frac{1}{2}p_3q_{11}^2q_{32}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1p_3q_{22}q_{33}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+2q_{21}q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{22}q_{33}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{22}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_2q_{21}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{22}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{11}q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{21}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_2q_{21}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{22}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_1^2q_{31}q_{32}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1^2q_{32}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{31}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{32}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{4}p_3^2q_{33}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{4}p_3^2q_{31}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{6}p_3q_{31}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{4}p_3^2q_{32}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+2q_{11}q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{21}q_{32}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{22}q_{31}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+2p_1q_{22}q_{32}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{11}q_{31}q_{32}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{21}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{21}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{22}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_1q_{21}q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{31}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+2p_1q_{22}q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_1q_{11}q_{32}^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1p_3q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{31}q_{32}q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{31}q_{33}q_{32}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}p_1q_{11}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_2q_{22}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{22}q_{33}f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}p_2q_{21}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{11}^2q_{22}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{21}q_{33}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{33}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2q_{32}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_2^2q_{33}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}^2q_{32}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{21}^2q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$
$$+\frac{1}{2}q_{22}^2q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$

$$+\frac{1}{2}q_{21}q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{32}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{9}q_{22}^3q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{9}q_{21}^3q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3p_1f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}^2q_{32}q_{33}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{31}^3q_{33}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}^3q_{32}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}p_3q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{24}q_{33}^4f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{21}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{22}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_3q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1p_3q_{32}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2p_3q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3q_{31}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{31}q_{32}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{31}q_{33}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{31}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{32}q_{31}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{31}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{33}q_{31}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}q_{31}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1q_{32}^2q_{31}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{31}q_{32}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{31}q_{33}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{33}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{33}^2q_{31}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}^2q_{31}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{31}q_{33}^2q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}q_{31}^2q_{32}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}q_{32}^3f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{4}q_{31}^2q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}\Bigg)h^4$$

$$+\Bigg(\frac{1}{3}q_{11}q_{22}^3q_{33}f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_2^3q_{33}q_{21}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{6}p_2^3q_{33}q_{22}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}\dots\dots\dots\dots\dots\dots\dots$$
$$+\frac{1}{2}p_1^2q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2\Bigg)h^5$$

Untuk $k_5 = f(x_i + p_4 h, y_i + q_{41}k_1 h + q_{42}k_2 h + q_{43}k_3 h + q_{44}k_4 h)$

$$f(x_i + p_4 h, y_i + q_{41}k_1 h + q_{42}k_2 h + q_{43}k_3 h + q_{44}k_4 h)f$$

$$+\left(q_{41}f\frac{\partial f}{\partial y} + q_{42}f\frac{\partial f}{\partial y} + q_{43}f\frac{\partial f}{\partial y} + q_{44}f\frac{\partial f}{\partial y} + p_4\frac{\partial f}{\partial x}\right)h$$

$$+\left(\frac{1}{2}q_{44}^2 f^2\frac{\partial^2 f}{\partial y^2} + \frac{1}{2}q_{42}^2 f^2\frac{\partial^2 f}{\partial y^2}q_{41}q_{42}f^2\frac{\partial^2 f}{\partial y^2} + q_{11}q_{42}f\left(\frac{\partial f}{\partial y}\right)^2 + q_{31}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\right.$$

$$+q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2 + q_{33}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2 + p_1 q_{42}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + p_2 q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + p_3 q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}p_4^2\frac{\partial^2 f}{\partial x^2} + q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2} + q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2} + q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2} + q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2} + \frac{1}{2}q_{41}^2 f^2\frac{\partial^2 f}{\partial y^2} + \frac{1}{2}q_{43}^2 f^2\frac{\partial^2 f}{\partial y^2}$$

$$+ q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}$$

$$+q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2} + p_4 q_{41}f\frac{\partial^2 f}{\partial x \partial y} + p_4 q_{42}f\frac{\partial^2 f}{\partial x \partial y} + p_4 q_{43}f\frac{\partial^2 f}{\partial x \partial y}$$

$$\left.+q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2} + p_4 q_{44}f\frac{\partial^2 f}{\partial x \partial y}\right)h^2 + \frac{1}{6}q_{44}^2 f^3\frac{\partial^3 f}{\partial y^3} + \frac{1}{6}p_4^3\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{2}q_{33}^2 q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1 q_{32}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{2}q_{11}^2 q_{42}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2} + q_{44}^2 q_{31}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+ q_{44}^2 q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{44}^2 q_{33}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2} + p_3 q_{44}^2 f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2} + p_1 p_4 q_{42}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial y} + p_2 p_4 q_{43}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial y}$$

$$+q_{21}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2 + q_{22}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2 + p_3 p_4 q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial y}$$

$$+q_{21}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{42}^2q_{11}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{42}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{43}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{44}q_{32}f\left(\frac{\partial f}{\partial y}\right)^3$$

$$+\frac{1}{2}q_{32}^2q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{44}^2p_4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{41}q_{42}q_{43}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{32}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+q_{21}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{22}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{22}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}++p_1q_{22}q_{43}\frac{\partial f}{\partial y}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{11}q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{31}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{32}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{42}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{42}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{44}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{42}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{44}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{41}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{41}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{41}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{42}q_{43}q_{44}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{21}q_{43}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{22}q_{43}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{41}q_{42}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{41}q_{42}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{41}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{31}q_{32}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}^2q_{44}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{43}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{43}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_3^2q_{44}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1^2q_{42}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_2^2q_{43}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{6}q_{42}^3f^3\frac{\partial^3 f}{\partial y^3}+q_{41}q_{42}q_{44}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{41}q_{43}q_{44}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_4^2q_{41}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_4^2q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}q_{43}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{22}q_{43}\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{33}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2\Bigg)h^3$$
$$+\left(\frac{1}{2}q_{31}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right.$$
$$+p_1q_{41}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2p_4q_{42}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+q_{21}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+2q_{22}q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{32}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{22}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{22}q_{42}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{44}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{31}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{32}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{33}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{31}q_{32}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2q_{21}q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{22}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{41}q_{43}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{32}q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}q_{31}q_{33}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3p_4q_{44}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{32}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}q_{32}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{31}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{31}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_3p_4^2q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}q_{32}^2q_{33}\,q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{31}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{21}q_{33}^2q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{33}^2q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{43}^2q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{43}^2q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}^2q_{32}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{22}q_{42}^2q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{32}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{32}^2q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_2q_{43}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4^2q_{33}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{11}^2q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_1q_{22}q_{33}q_{44}\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{33}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{32}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1p_4^2q_{42}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+p_3q_{44}^2q_{41}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{31}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_4^2q_{11}q_{42}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2^2q_{22}q_{43}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{41}q_{42}q_{43}q_{44}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_4q_{21}q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{33}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2^2q_{21}q_{43}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}p_2p_4^2q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_1^2q_{22}q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+p_4q_{22}q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_4q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{4}p_4^2q_{43}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}q_{44}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{4}p_4^2q_{44}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{11}q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_1^2q_{22}q_{43}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{21}q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{11}^2q_{22}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{3}q_{21}^2q_{22}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2p_4q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{3}q_{22}^2q_{21}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{11}q_{41}q_{42}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{11}q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{42}^2q_{41}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2q_{11}q_{42}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+p_1q_{41}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{31}q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{31}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{32}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{31}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_4q_{44}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{33}q_{44}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{33}q_{44}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4p_3q_{31}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+p_4p_3q_{32}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+\frac{1}{2}p_4q_{42}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_3q_{33}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+p_4q_{44}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_4q_{44}q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{41}q_{44}q_{31}q_{32}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4q_{41}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{44}q_{31}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4p_2q_{22}q_{43}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_1p_4q_{22}q_{43}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_4^2q_{42}q_{43}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{44}q_{31}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_3^2q_{44}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2p_3q_{43}q_{44}\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_4^2q_{42}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{44}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{44}q_{32}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{43}q_{44}q_{31}q_{32}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{43}q_{44}q_{31}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{43}q_{44}q_{32}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_2^2q_{41}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{41}q_{42}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{41}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{33}^2q_{32}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{11}q_{32}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_4q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}^2q_{42}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{41}q_{42}q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+2p_1q_{22}q_{42}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{22}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{33}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{24}p_4^4\frac{\partial^4 f}{\partial x^4}$$
$$+p_1q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{21}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{22}q_{33}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{33}q_{44}\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_4q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{11}q_{41}q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{24}q_{43}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{22}^2q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_2^2q_{43}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}^2q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_2q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_4q_{41}q_{42}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{31}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}^2q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{33}q_{44}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{44}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{22}q_{43}^2f^2\left(\frac{\partial f}{\partial y}\right)^3$$

$$+\frac{1}{2}q_{11}^2q_{42}q_{43}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_2q_{22}^2q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{43}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{33}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}^2q_{43}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$

$$+\frac{1}{2}p_2^2q_{43}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{33}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}q_{41}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{22}q_{33}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^3$$

$$+p_2q_{41}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_4^2q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_4^2q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_4^2q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4^2q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{22}q_{33}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{22}q_{44}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1p_3q_{32}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{44}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4q_{11}^2q_{42}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4p_1q_{11}q_{42}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+p_3q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{21}q_{22}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{31}q_{32}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_4q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{32}q_{41}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{42}q_{43}\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{11}q_{44}^2q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}q_{41}^2q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{21}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3p_4q_{42}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{4}p_4^2q_{41}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_4q_{41}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3q_{33}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}^2q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{22}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^3$$
$$+p_2q_{21}q_{33}q_{44}f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial f}{\partial y}\right)^2+p_3q_{41}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{21}q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{11}q_{42}^2q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{33}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{44}^2q_{32}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_3q_{33}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_2q_{33}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_3q_{31}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$
$$+p_3q_{32}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_1q_{41}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{22}q_{44}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}q_{31}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{22}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{32}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2q_{41}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+2q_{21}q_{33}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{21}q_{44}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_3q_{22}q_{44}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{22}q_{43}^2q_{42}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{31}q_{32}q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{42}q_{44}^2q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{4}q_{41}^2q_{42}^2f^6\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{41}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{44}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{44}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{44}^2q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{42}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{43}^2q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{43}^2q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{44}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{41}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_3q_{31}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{32}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{33}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{22}q_{43}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{43}q_{41}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{31}q_{33}q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1p_4q_{22}q_{43}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}$$

$$+p_4q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{32}q_{43}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_3^2p_4q_{44}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{22}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{32}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+q_{31}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{21}q_{43}q_{44}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+p_3q_{22}q_{43}q_{44}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+q_{32}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{41}q_{31}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{33}q_{42}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+2q_{22}q_{33}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}p_1q_{41}^2q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1q_{11}^2q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+2q_{11}q_{22}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{41}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{43}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{31}q_{42}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_4q_{21}q_{43}q_{42}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+p_1q_{31}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+2p_1q_{32}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{41}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+2q_{11}q_{32}q_{42}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{32}q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{32}q_{33}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{11}q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{22}q_{43}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_2q_{22}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_4^2q_{41}q_{42}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{21}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{31}q_{33}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{31}q_{32}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4q_{31}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3p_4q_{44}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{41}^2q_{42}q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{42}^2q_{41}q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{42}q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{41}^2q_{42}q_{43}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{42}^2q_{44}q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{44}q_{42}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{44}^2q_{42}q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{22}q_{41}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1p_4q_{42}q_{44}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{33}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{33}q_{32}q_{31}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{22}\,q_{41}q_{42}q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}q_{44}q_{33}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{8}p_4^3q_{42}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}p_2q_{21}^2\,q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_3q_{33}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_2q_{43}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{42}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{42}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{42}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{32}q_{42}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_2q_{33}^2q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{4}q_{41}^2q_{43}^2f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{41}^2q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}p_4q_{41}q_{42}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{33}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3^2q_{44}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{33}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1q_{33}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{32}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{21}q_{43}q_{42}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{41}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{21}\,q_{41}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}\,q_{41}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1p_4q_{42}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{21}q_{41}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{31}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{21}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{32}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{22}q_{41}q_{43}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{42}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2q_{42}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{44}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{42}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{21}q_{42}q_{43}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+q_{21}q_{42}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{42}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}q_{43}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2q_{43}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{41}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{43}q_{42}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4^2q_{43}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_4^2q_{43}q_{41}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1q_{42}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{32}q_{33}q_{42}q_{43}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{32}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{11}q_{42}q_{43}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}\,q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+p_1q_{11}q_{42}^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{42}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{44}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{2}p_2^2p_4q_{43}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{11}^2q_{41}q_{42}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{8}p_4^3q_{41}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{8}p_4^3q_{44}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+p_3q_{42}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_3q_{42}q_{44}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}\ p_3q_{44}q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{31}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+2q_{11}q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{32}q_{44}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}^2q_{42}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{9}q_{22}^3q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{9}q_{21}^3q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2p_4q_{21}q_{43}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2q_{21}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+p_2q_{22}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}p_2q_{43}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{22}q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{22}q_{43}q_{41}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2q_{42}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{11}^2q_{42}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{32}q_{43}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_1q_{22}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}p_1^2q_{42}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}^2q_{43}^2f^2\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{21}q_{41}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+2p_1q_{22}q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_1q_{42}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{11}\,q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{24}q_{41}^4f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{24}q_{44}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{42}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{22}q_{42}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{21}q_{43}q_{42}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}\,q_{31}q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{21}q_{43}q_{44}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{42}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_3q_{44}q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{32}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+q_{21}q_{31}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+q_{22}q_{32}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{41}^2q_{33}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{22}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{21}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+q_{22}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{31}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{31}\,q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{11}q_{33}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{6}q_{31}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{33}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{31}\,q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{21}q_{43}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_2q_{33}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{43}^2q_{42}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_1q_{32}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{33}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{4}p_4^2q_{42}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{8}p_4^3q_{43}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{11}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_4q_{33}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{32}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+2q_{43}^2q_{21}q_{22}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_4^2q_{41}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{43}^2q_{11}q_{22}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{22}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{6}q_{32}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2q_{32}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_4q_{42}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{33}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{44}^2q_{43}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{11}q_{22}q_{43}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{6}p_1^3q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{9}p_2^3q_{43}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+p_4q_{21}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{22}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+q_{33}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{43}q_{42}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{43}q_{41}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+p_3p_4q_{41}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{31}q_{43}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{43}q_{22}q_{11}^2f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{41}^2q_{44}q_{31}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{42}^2q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{44}^2q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_2q_{43}q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}p_4q_{44}^3f^4\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{42}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+p_4q_{32}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{33}q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}^2q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{11}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{22}q_{43}q_{44}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{21}q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3q_{31}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{42}q_{43}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{32}q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{22}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{21}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{22}q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_2^2q_{33}q_{44}\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{31}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{42}q_{43}q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{44}q_{43}q_{41}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{41}q_{43}q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{41}q_{44}q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{31}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2q_{41}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{32}^2q_{41}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{32}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{32}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2p_2q_{33}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\Big)h^4$$
$$+\Big(2\,p_1q_{21}q_{22}q_{43}^2f\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{11}q_{22}q_{21}^2q_{43}f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2^2p_1q_{22}q_{43}\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2q_{11}^2q_{42}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\cdots\ldots\ldots\ldots+\frac{1}{2}p_2^2p_4q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2}\Big)h^5$$

Subtitusikan k_1, k_2, k_3, k_4, k_5, yang sudah diubah ke deret Taylor dua variabel ke persamaan

$$y_{i+1} = y_i + (a_1k_1 + a_2k_2 + a_3k_3 + a_4k_4 + a_5k_5)h$$
$$= y_i + a_1k_1h + a_2k_2h + a_3k_3h + a_4k_4h + a_5k_5h$$
$$= y_i + a_1hf + a_2h\Big(f + p_1h\frac{\partial f}{\partial x} + q_{11}fh\frac{\partial f}{\partial y} + \frac{1}{2}p_1^2h^2\frac{\partial^2 f}{\partial x^2} + p_1q_{11}h^2f\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}q_{11}^2h^2f^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{6}p_1^3h^3\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{11}p_1^2h^3f\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial y}+\frac{1}{2}p_1q_{11}^2f^2h^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}q_{11}^3h^3f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{24}p_1^4h^4\frac{\partial^4 f}{\partial x^4}$$
$$+\frac{1}{6}q_{11}p_1^3h^4f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{4}p_1^2q_{11}^2h^4f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{6}p_1q_{11}h^4f\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{24}q_{11}^4h^4f\frac{\partial^4 f}{\partial y^4}\Big)$$

$$+a_3h\left(f+\left(p_2\frac{\partial f}{\partial x}+q_{21}f\frac{\partial f}{\partial x}+q_{22}f\frac{\partial f}{\partial x}+q_{21}f\frac{\partial^2 f}{\partial y^2}+q_{22}f\frac{\partial^2 f}{\partial y^2}\right)h\right.$$

$$+\left(q_{22}p_1\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{21}f\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{11}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{11}f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}+\frac{1}{2}p_2^2\frac{\partial^2 f}{\partial x}\right.$$

$$\left.+p_2q_{22}f\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\right)h^2$$

$$+\left(\frac{1}{9}p_2^3\frac{\partial^3 f}{\partial x^3}+\frac{1}{9}q_{22}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{22}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{22}f\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial y}\right.$$

$$+\frac{1}{2}q_{22}q_{11}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{22}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_1^2q_{22}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1^2q_{22}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{22}q_{11}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)$$

$$+\frac{1}{3}q_{21}^2q_{22}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{3}q_{22}^2q_{21}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{9}q_{21}^3f^3\frac{\partial^3 f}{\partial y^3}+p_1q_{11}q_{22}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{11}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}p_1q_{11}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$\left.+p_2q_{21}q_{22}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2q_{21}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{21}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\right)h^3$$

$$+\left(\frac{1}{6}q_{21}^3q_{22}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{21}^2q_{22}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{21}q_{22}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{22}p_1^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}\right.$$

$$+\frac{1}{6}q_{22}p_1^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{2}q_{22}q_{11}p_1^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+p_2q_{11}q_{22}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{11}q_{22}p_1^2f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{21}q_{22}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{24}q_{22}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{21}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}p_2^4\frac{\partial^4 f}{\partial x^4}$$

$$+\frac{1}{6}p_2^3q_{21}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{6}p_2^3q_{22}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{6}p_2q_{21}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_2q_{22}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}p_2^2q_{21}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}p_2^2q_{22}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{3}q_{11}q_{22}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{3}p_1q_{22}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}q_{11}^3q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2p_2q_{22}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}q_{11}^3q_{22}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{11}q_{22}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+\frac{1}{2}p_2q_{11}^2q_{22}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_1q_{11}^2q_{22}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_1p_2^2q_{22}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2^2q_{11}q_{22}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_2q_{21}^2q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{22}^2q_{21}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{21}q_{22}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{11}^2q_{22}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{22}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{3}p_1q_{21}^2q_{22}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{3}q_{21}^2q_{22}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{2}{3}p_1q_{21}q_{22}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{2}{3}q_{11}q_{21}q_{22}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{21}q_{22}q_{11}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\Bigg)h^4$$

$$+\Bigg(\frac{1}{24}p_1^4q_{22}\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial x^4}+\frac{1}{24}p_1^4q_{22}\frac{\partial^2 f}{\partial y^2}\frac{\partial^4 f}{\partial x^4}+\frac{1}{2}p_2q_{11}q_{22}^3f^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1p_2q_{21}^2q_{22}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{2}{3}p_1q_{11}q_{21}q_{22}^2f^3\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}\dots\dots\dots\dots\dots\dots\dots$$
$$+p_2q_{11}q_{21}q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}\Bigg)h^5\Bigg)$$

$$+a_4h\Bigg(f+\left(q_{31}f\frac{\partial f}{\partial y}+q_{32}f\frac{\partial f}{\partial y}+q_{33}f\frac{\partial f}{\partial y}\right)h$$

$$\left(\frac{1}{2}p_3^2\frac{\partial^2 f}{\partial x^2}+q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\right.$$

$$\left.+q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\right.$$

$$+\frac{1}{2}q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2f^2\frac{\partial^2 f}{\partial y^2}+q_{31}q_{32}f^2\frac{\partial^2 f}{\partial y^2}+q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}+q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}f\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{32}f\frac{\partial^2 f}{\partial x\partial y}+p_3q_{33}f\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{32}f\left(\frac{\partial f}{\partial y}\right)^2+p_1q_{32}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_2q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\Big)h^2$$

$$+\Big(p_3q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{32}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2^2q_{33}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{11}q_{22}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{11}q_{32}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}p_3^3\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{32}q_{33}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{32}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{32}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{31}^2q_{33}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{31}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{33}^2q_{31}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{32}^2f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{31}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{21}q_{32}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{31}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{32}q_{33}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{32}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{33}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1p_3q_{32}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_2p_3q_{33}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{31}q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{6}q_{32}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{33}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{31}q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{11}^2q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{22}q_{33}\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{6}q_{31}^3f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_3^2q_{32}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_3^2q_{31}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3q_{31}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{31}q_{32}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{11}q_{32}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{21}q_{33}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{22}q_{33}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{32}q_{33}f^3\frac{\partial^3 f}{\partial y^3}\Big)h^3$$

$$+\Big(\frac{1}{6}p_1^3q_{32}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{9}p_2^3q_{33}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{24}q_{32}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{31}^4f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{32}^2q_{33}q_{21}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}^2q_{33}q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{33}q_{21}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{33}q_{22}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{33}p_2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{32}^2q_{33}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}^2q_{33}p_1f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{22}q_{33}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}p_2^2q_{21}q_{33}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_1^2q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1^2q_{22}q_{33}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{11}^2q_{22}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{3}q_{21}^2q_{22}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{3}q_{22}^2q_{21}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{32}q_{33}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_1^2q_{11}q_{32}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{11}q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+2\,q_{11}q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{32}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{11}q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{21}q_{22}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}q_{32}^2q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{32}q_{33}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}q_{33}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{32}^3q_{33}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{11}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{31}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}p_2^2p_3q_{33}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}q_{21}^2q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{22}^2q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}^2q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1^2q_{32}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_2^2q_{33}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{11}^2q_{31}q_{32}f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{11}^2q_{32}q_{33}f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{21}^2q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}^2q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{22}q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3+\frac{1}{2}p_1^2p_3q_{32}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}p_3q_{32}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_3q_{22}q_{32}q_{33}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1p_3q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_2p_3q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_3^2q_{32}q_{33}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{33}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{33}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{11}q_{33}^2q_{32}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}^2q_{32}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{33}^2q_{32}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1q_{33}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{33}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2p_2q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2p_2q_{32}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_3^2q_{11}q_{32}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_3q_{31}q_{33}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{31}q_{32}q_{33}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{32}q_{33}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{31}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{31}q_{32}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{11}q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_3^3q_{33}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_3^3q_{32}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{21}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{22}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_3q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_3q_{22}q_{31}q_{33}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2p_3q_{31}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3^2q_{21}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_3^2q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{21}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{22}q_{33}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_3q_{31}q_{32}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{24}p_3^4\frac{\partial^4 f}{\partial x^4}+\frac{1}{2}p_3q_{31}q_{32}^2f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_3q_{32}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{11}q_{31}q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1p_3q_{11}q_{32}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2p_3q_{21}q_{33}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+p_2p_3q_{22}q_{33}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+\frac{1}{2}p_3q_{11}^2q_{32}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1p_3q_{22}q_{33}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+2q_{21}q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{22}q_{33}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{22}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_2q_{21}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{22}q_{33}^2f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{11}q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{21}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{22}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{21}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_2q_{22}\,q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_1^2q_{31}q_{32}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{32}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{31}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_2^2q_{32}q_{33}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{4}p_3^2q_{33}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{4}p_3^2q_{31}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{6}p_3q_{31}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}p_3^2q_{32}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{21}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{11}q_{21}q_{31}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+2q_{11}q_{22}q_{32}q_{33}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{21}q_{32}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1q_{22}q_{31}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+2p_1q_{22}q_{32}q_{33}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{31}q_{32}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{21}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{21}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{22}q_{31}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{32}q_{33}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_1q_{21}q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{31}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+2p_1q_{22}q_{32}q_{33}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_1q_{11}q_{32}^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_3q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{31}q_{32}q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{31}q_{33}q_{32}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}p_1q_{11}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_2q_{22}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{22}q_{33}f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}p_2q_{21}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{22}q_{33}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{11}^2q_{22}q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{32}q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{21}q_{33}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{33}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2q_{32}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_2^2q_{33}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}^2q_{32}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{21}^2q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$
$$+\frac{1}{2}q_{22}^2q_{33}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$

$$+\frac{1}{2}q_{21}q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{32}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{9}q_{22}^3q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{9}q_{21}^3q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3p_1f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}^2q_{32}q_{33}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{31}^3q_{33}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}^3q_{32}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}p_3q_{33}^3f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{24}q_{33}^4f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{21}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{22}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{21}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_3q_{22}q_{33}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1p_3q_{32}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2p_3q_{33}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3q_{31}^2q_{32}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{31}q_{32}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{31}q_{33}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_1q_{31}^2q_{32}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{11}q_{32}q_{31}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{31}^2q_{33}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{22}q_{31}^2q_{33}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{33}q_{31}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}q_{31}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1q_{32}^2q_{31}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{31}q_{32}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{31}q_{33}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{33}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{33}^2q_{31}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}^2q_{31}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{31}q_{33}^2q_{22}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}q_{31}^2q_{32}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}q_{32}^3f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{31}^2q_{33}^2f^4\frac{\partial^4 f}{\partial y^4}\Bigg)$$

$$+\Bigg(\frac{1}{3}q_{11}q_{22}^3q_{33}f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_2^3q_{33}q_{21}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{6}p_2^3q_{33}q_{22}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}\dots\dots\dots\dots\dots\dots\dots$$
$$+\frac{1}{2}p_1^2q_{11}q_{22}q_{33}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2\Bigg)h^5\Bigg)$$

$$+a_5h\left(f+\left(q_{41}f\frac{\partial f}{\partial y}+q_{42}f\frac{\partial f}{\partial y}+q_{43}f\frac{\partial f}{\partial y}+q_{44}f\frac{\partial f}{\partial y}+p_4\frac{\partial f}{\partial x}\right)h\right.$$
$$+\left(\frac{1}{2}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{42}^2f^2\frac{\partial^2 f}{\partial y^2}q_{41}q_{42}f^2\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}f\left(\frac{\partial f}{\partial y}\right)^2+q_{31}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\right.$$
$$+q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{33}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2+p_1q_{42}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_2q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_3q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+\frac{1}{2}p_4^2\frac{\partial^2 f}{\partial x^2}$$
$$+q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}+q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}+q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{41}^2f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}+q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}+q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}+p_4q_{41}f\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{42}f\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{43}f\frac{\partial^2 f}{\partial x\partial y}+q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}+p_4q_{44}f\frac{\partial^2 f}{\partial x\partial y}\Big)h^2+\frac{1}{6}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_4^3\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{2}q_{33}^2q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{32}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{11}^2q_{42}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{44}^2q_{31}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{44}^2q_{32}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{44}^2q_{33}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{44}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1p_4q_{42}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_2p_4q_{43}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3p_4q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{42}^2q_{11}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{42}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{43}^2f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{44}q_{32}f\left(\frac{\partial f}{\partial y}\right)^3$$
$$+\frac{1}{2}q_{32}^2q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{31}^2q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{44}^2p_4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{42}q_{43}f^3\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{31}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+q_{21}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{22}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{11}q_{22}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$++p_1q_{22}q_{43}\frac{\partial f}{\partial y}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{31}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{32}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{42}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{42}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{44}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{42}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{44}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{41}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{41}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}q_{42}^2f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}q_{41}^3f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{42}q_{43}q_{44}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{43}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{22}q_{43}f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{41}q_{42}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{41}q_{42}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{41}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{31}q_{32}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{32}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{6}q_{43}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}^2q_{44}f^3\frac{\partial^3 f}{\partial y^3}+q_{21}q_{43}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{43}^2f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}p_3^2q_{44}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1^2q_{42}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2^2q_{43}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{6}q_{42}^3f^3\frac{\partial^3 f}{\partial y^3}$$
$$+q_{41}q_{42}q_{44}f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{43}q_{44}f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_4^2q_{41}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_4^2q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}q_{43}q_{41}^2f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}q_{44}^2f^3\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{22}q_{43}\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{33}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2\Big)h^3$$

$$+\left(\frac{1}{2}q_{31}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right.$$
$$+p_1q_{41}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2p_4q_{42}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{21}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+2q_{22}q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{32}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{22}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}q_{42}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{44}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{31}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{32}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}q_{33}^2q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{31}q_{32}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2q_{21}q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{22}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{41}q_{43}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{32}q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}q_{31}q_{33}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3p_4q_{44}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{32}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}q_{32}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{31}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{43}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{31}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_3p_4^2q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}q_{32}^2q_{33}\,q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{31}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{21}q_{33}^2q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{33}^2q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{43}^2q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{43}^2q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}^2q_{32}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{22}q_{42}^2q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{32}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{32}^2q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_2q_{43}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4^2q_{33}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{11}^2q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_1q_{22}q_{33}q_{44}\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{33}q_{41}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{32}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1p_4^2q_{42}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+p_3q_{44}^2q_{41}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{31}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_4^2q_{11}q_{42}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2^2q_{22}q_{43}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{41}q_{42}q_{43}q_{44}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_4q_{21}q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{33}q_{44}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2^2q_{21}q_{43}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}p_2p_4^2q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_1^2q_{22}q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+p_4q_{22}q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_4q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{4}p_4^2q_{43}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}q_{44}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{4}p_4^2q_{44}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{11}q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_1^2q_{22}q_{43}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{21}q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{11}^2q_{22}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{3}q_{21}^2q_{22}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2p_4q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{3}q_{22}^2q_{21}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{11}q_{41}q_{42}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{11}q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{42}^2q_{41}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2q_{11}q_{42}f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+p_1q_{41}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{31}q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{31}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{32}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{31}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_4q_{44}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{33}q_{44}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{33}q_{44}^2f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4p_3q_{31}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+p_4p_3q_{32}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+\frac{1}{2}p_4q_{42}q_{41}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_3q_{33}q_{44}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+p_4q_{44}q_{33}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_4q_{44}q_{32}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{41}q_{44}q_{31}q_{32}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4q_{41}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{44}q_{31}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4p_2q_{22}q_{43}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_1p_4q_{22}q_{43}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_4^2q_{42}q_{43}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{44}q_{31}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_3^2q_{44}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2p_3q_{43}q_{44}\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_4^2q_{42}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{44}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{44}q_{32}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{43}q_{44}q_{31}q_{32}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{43}q_{44}q_{31}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{43}q_{44}q_{32}q_{33}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_2^2q_{41}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{41}q_{42}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{41}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{33}^2q_{32}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{11}q_{32}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_4q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}^2q_{42}q_{11}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{41}q_{42}q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+2p_1q_{22}q_{42}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{22}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{33}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{24}p_4^4\frac{\partial^4 f}{\partial x^4}$$
$$+p_1q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{21}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{22}q_{33}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{33}q_{44}\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_4q_{31}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{33}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{11}q_{41}q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{24}q_{43}^4 f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{22}^2 q_{43}^2 f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_2^2 q_{43}^2 f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}^2 q_{43}^2 f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_2 q_{42}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1 p_4 q_{41}q_{42}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3 q_{31}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}^2 q_{43}^2 f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{33}q_{44}^3 f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3 q_{44}^3 f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{22}q_{43}^2 f^2\left(\frac{\partial f}{\partial y}\right)^3$$

$$+\frac{1}{2}q_{11}^2 q_{42}q_{43}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_2 q_{22}^2 q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2 p_4 q_{43}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{33}q_{42}^2 q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}q_{42}^2 q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}^2 q_{43}^2 f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$
$$+\frac{1}{2}p_2^2 q_{43}^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3 q_{33}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1 q_{22}q_{41}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{11}q_{22}q_{33}q_{44}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^3$$

$$+p_2 q_{41}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2 q_{42}^2 f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_4^2 q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_4^2 q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_4^2 q_{21}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4^2 q_{21}q_{43}f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{22}q_{33}^2 q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4 q_{22}q_{44}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1 p_3 q_{32}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4 q_{21}q_{44}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4 q_{11}^2 q_{42}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4 p_1 q_{11}q_{42}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+p_3q_{32}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{21}q_{22}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{31}q_{32}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}q_{33}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_4q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{32}q_{41}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{42}q_{43}\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{11}q_{44}^2q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}q_{41}^2q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{21}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3p_4q_{42}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{4}p_4^2q_{41}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_4q_{41}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3q_{33}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}^2q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{22}q_{33}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^3$$
$$+p_2q_{21}q_{33}q_{44}f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial f}{\partial y}\right)^2+p_3q_{41}q_{43}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{21}q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{11}q_{42}^2q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{33}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{44}^2q_{32}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{33}q_{44}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_3q_{33}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_2q_{33}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_3q_{31}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$
$$+p_3q_{32}q_{44}^2f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_1q_{41}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{22}q_{44}q_{43}f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}q_{31}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{22}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{22}q_{32}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2q_{41}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+2q_{21}q_{33}q_{44}q_{43}f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{21}q_{44}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_3q_{22}q_{44}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{22}q_{43}^2q_{42}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{31}q_{32}q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{42}q_{44}^2q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{4}q_{41}^2q_{42}^2f^6\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{41}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{44}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{44}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{44}^2q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{42}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{42}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{43}^2q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{43}^2q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{44}^3q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{41}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_3q_{31}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{32}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{33}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{22}q_{43}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{43}q_{41}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{31}q_{33}q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1p_4q_{22}q_{43}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}$$

$$+p_4q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{32}q_{43}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{44}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_3^2p_4q_{44}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{22}q_{43}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{32}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+q_{31}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{21}q_{43}q_{44}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+p_3q_{22}q_{43}q_{44}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+q_{32}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{41}q_{31}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{33}q_{42}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+2q_{22}q_{33}q_{43}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}p_1q_{41}^2q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1q_{11}^2q_{42}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+2q_{11}q_{22}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{41}\,q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{43}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{31}q_{42}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_4q_{21}q_{43}q_{42}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+p_1q_{31}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+2p_1q_{32}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{41}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+2q_{11}q_{32}q_{42}q_{44}f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{32}q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{32}q_{33}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{11}q_{42}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{22}q_{43}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_2q_{22}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_4^2q_{41}q_{42}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{21}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{31}q_{33}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{31}q_{32}q_{44}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_4q_{31}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3p_4q_{44}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{41}^2q_{42}q_{44}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{42}^2q_{41}q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{42}q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{41}^2q_{42}q_{43}f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{42}^2q_{44}q_{43}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{44}q_{42}f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{44}^2q_{42}q_{41}f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{22}q_{41}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1p_4q_{42}q_{44}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{33}q_{41}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{33}q_{32}q_{31}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{22}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{22}\,q_{41}q_{42}q_{43}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}q_{44}q_{33}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{8}p_4^3q_{42}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}p_2q_{21}^2\,q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_3q_{33}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_2q_{43}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{42}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{42}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{42}^2f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{32}q_{42}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_2q_{33}^2q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{4}q_{41}^2q_{43}^2f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{41}^2q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}p_4q_{41}q_{42}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{33}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3^2q_{44}^2f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{33}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1q_{33}q_{42}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{32}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{21}q_{43}q_{42}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{41}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{41}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{41}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1p_4q_{42}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{21}q_{41}q_{43}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{31}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{21}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{32}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{22}q_{41}q_{43}f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{42}q_{43}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2q_{42}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{44}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{42}^2q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{21}q_{42}q_{43}f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+q_{21}q_{42}q_{43}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{42}q_{44}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}q_{43}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2q_{43}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{41}q_{43}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{43}q_{42}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4^2q_{43}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_4^2q_{43}q_{41}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1q_{42}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{32}q_{33}q_{42}q_{43}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{32}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{11}q_{42}q_{43}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}\,q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+p_1q_{11}q_{42}^2f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{42}q_{43}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{44}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{2}p_2^2p_4q_{43}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{22}q_{41}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{11}^2q_{41}q_{42}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{8}p_4^3q_{41}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{8}p_4^3q_{44}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+p_3q_{42}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_3q_{42}q_{44}\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}\;p_3q_{44}q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{31}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+2q_{11}q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{32}q_{44}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}^2q_{42}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{9}q_{22}^3\,q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{9}q_{21}^3\,q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2p_4q_{21}q_{43}f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2q_{21}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+p_2q_{22}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}p_2q_{43}q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_2q_{22}q_{43}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{22}q_{43}q_{41}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2q_{42}^2f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{11}^2q_{42}^2f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{32}q_{43}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_1q_{22}q_{43}^2f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}p_1^2q_{42}q_{43}f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}^2q_{43}^2f^2\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{21}q_{41}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+2p_1q_{22}q_{42}q_{43}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_1q_{42}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{11}\,q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{24}q_{41}^4f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{24}q_{44}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{42}^4f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{22}q_{42}^2q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{21}q_{43}q_{42}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}\,q_{31}q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{21}q_{43}q_{44}^2f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{42}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_3q_{44}q_{32}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{32}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+q_{21}q_{31}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+q_{22}q_{32}q_{33}q_{44}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{41}^2q_{33}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{11}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{22}q_{41}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{21}q_{42}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{22}q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{31}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{33}q_{41}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{31}\,q_{33}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{33}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{6}q_{31}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{33}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{31}q_{42}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{31}\,q_{32}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{21}q_{43}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_2q_{33}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{11}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{43}^2q_{42}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4p_1q_{32}q_{44}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{33}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{4}p_4^2q_{42}^2f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{8}p_4^3q_{43}f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{11}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_4q_{33}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{32}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{42}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+2q_{43}^2q_{21}q_{22}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_4^2q_{41}q_{44}f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{43}^2q_{11}q_{22}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{22}q_{43}^2f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{6}q_{32}^2q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2q_{32}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1q_{42}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_4q_{42}^3f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{33}q_{42}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{44}^2q_{43}q_{33}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{11}q_{22}q_{43}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{42}q_{43}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_1^3q_{42}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{9}p_2^3q_{43}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+p_4q_{21}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{22}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{22}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+q_{33}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{43}q_{42}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{43}q_{41}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{11}q_{32}q_{44}f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+p_3p_4q_{41}q_{44}f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+q_{31}q_{43}q_{44}^2f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{43}q_{22}q_{11}^2f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{41}^2q_{44}q_{31}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}q_{41}q_{42}q_{44}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{42}^2q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}q_{44}^2f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_3q_{44}^2q_{43}f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2q_{43}q_{41}^2f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{41}q_{43}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{41}q_{44}f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}p_4q_{44}^3f^4\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{42}^3f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}q_{42}q_{43}f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+p_4q_{32}q_{44}^2f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{33}q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{42}q_{43}q_{44}f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}^2q_{43}^2f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{11}q_{41}q_{42}q_{43}f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{22}q_{43}q_{44}^3f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{21}q_{43}^2f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3q_{31}^2q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{42}q_{43}^3f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{32}q_{41}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{22}q_{43}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{22}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{21}q_{33}q_{44}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{22}q_{33}q_{42}q_{44}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_2^2q_{33}q_{44}\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}q_{31}^2q_{43}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{21}q_{42}q_{43}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{42}q_{43}q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{44}q_{43}q_{41}^2f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{41}q_{43}q_{44}^2f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{41}q_{44}q_{42}^2f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{31}^2q_{44}^2f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2q_{41}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{32}^2q_{41}q_{44}f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{32}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{22}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{33}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{32}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2p_2q_{33}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{43}q_{44}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\left(2\,p_1q_{21}q_{22}q_{43}^2f\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{11}q_{22}q_{21}^2q_{43}f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}\right.$$
$$+\frac{1}{2}p_2^2p_1q_{22}q_{43}\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2q_{11}^2q_{42}q_{43}f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$\left.+\cdots\ldots\ldots\ldots+\frac{1}{2}p_2^2p_4q_{22}q_{43}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2}\right)h^5\Bigg)$$

$$=y_i+a_1hf+a_2hf+a_2p_1h^2\frac{\partial f}{\partial x}+a_2q_{11}fh^2\frac{\partial f}{\partial y}+a_2\frac{1}{2}p_1^2h^3\frac{\partial^2 f}{\partial x^2}$$
$$+a_2p_1q_{11}h^3f\frac{\partial^2 f}{\partial x\partial y}$$

$$+a_2\frac{1}{2}q_{11}^2h^3f^2\frac{\partial^2 f}{\partial y^2}+a_2\frac{1}{6}p_1^3h^4\frac{\partial^3 f}{\partial x^3}+a_2\frac{1}{2}q_{11}p_1^2h^4f\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial y}$$
$$+a_2\frac{1}{2}p_1q_{11}^2f^2h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+a_2\frac{1}{6}q_{11}^3h^4f^3\frac{\partial^3 f}{\partial y^3}+a_2\frac{1}{24}p_1^4h^5\frac{\partial^4 f}{\partial x^4}+a_2\frac{1}{6}q_{11}p_1^3h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+a_2\frac{1}{4}p_1^2q_{11}^2h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+a_2\frac{1}{6}p_1q_{11}h^5f\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+a_2\frac{1}{24}q_{11}^4h^5f\frac{\partial^4 f}{\partial y^4}fa_3h+a_3h^2p_2\frac{\partial f}{\partial x}+a_3h^2q_{21}f\frac{\partial f}{\partial x}$$
$$+a_3h^2q_{22}f\frac{\partial f}{\partial x}$$

$$+a_3h^2q_{21}f\frac{\partial^2 f}{\partial y^2}+a_3h^2q_{22}f\frac{\partial^2 f}{\partial y^2}a_3h^3q_{22}p_1\left(\frac{\partial f}{\partial x}\right)^2+a_3h^3p_2q_{21}f\frac{\partial^2 f}{\partial x\partial y}$$
$$+a_3h^3q_{22}q_{11}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+a_3h^3q_{22}q_{11}f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y}+a_3h^3\frac{1}{2}p_2^2\frac{\partial^2 f}{\partial x^2}+a_3h^3p_2q_{22}f\frac{\partial^2 f}{\partial x\partial y}$$
$$+a_3h^3p_1q_{22}\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}$$

$$\frac{1}{9}p_2^3a_3h^4\frac{\partial^3 f}{\partial x^3}+\frac{1}{9}q_{22}^3a_3h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{22}^2a_3h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_2^2q_{22}a_3h^4f\frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}q_{22}q_{11}^2a_3h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{22}a_3h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_1^2q_{22}a_3h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_1^2q_{22}a_3h^4\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{22}q_{11}^2a_3h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{21}^2q_{22}a_3h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{3}q_{22}^2q_{21}a_3h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{9}q_{21}^3a_3h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{11}q_{22}a_3h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{11}a_3h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{22}p_1q_{11}a_3h^4f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{21}q_{22}a_3h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2q_{21}^2a_3h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{21}a_3h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{6}q_{21}^3q_{22}a_3h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{21}^2q_{22}^2a_3h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{21}q_{22}^3a_3h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{22}p_1^3a_3h^5\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{6}q_{22}p_1^3a_3h^5\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{22}q_{11}p_1^2a_3h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+p_2q_{11}q_{22}^2a_3h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{11}q_{22}p_1^2a_3h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{21}q_{22}a_3h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{24}q_{22}^2a_3h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{24}q_{21}^2a_3h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}p_2^4a_3h^5\frac{\partial^4 f}{\partial x^4}+\frac{1}{6}p_2^3q_{21}a_3h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{6}p_2^3q_{22}a_3h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{6}p_2q_{21}^3a_3h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_2q_{22}^3a_3h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}p_2^2q_{21}^2a_3h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{4}p_2^2q_{22}^2a_3h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{3}q_{11}q_{22}^2a_3h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{3}p_1q_{22}^3a_3h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}q_{11}^3q_{22}a_3h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{11}^2q_{22}a_3h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{3}q_{21}^2q_{22}q_{11}a_3h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2p_2q_{22}a_3h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{6}q_{11}^3q_{22}a_3h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1p_2q_{11}q_{22}a_3h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+\frac{1}{2}p_1q_{11}^2q_{22}a_3h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_1p_2^2q_{22}a_3h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_2^2q_{11}q_{22}a_3h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_2q_{21}^2q_{22}a_3h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{22}^2q_{21}a_3h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2^2q_{21}q_{22}a_3h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{11}^2q_{22}a_3h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1p_2q_{22}^2a_3h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{3}p_1q_{21}^2q_{22}a_3h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{2}{3}p_1q_{21}q_{22}^2a_3h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{2}{3}q_{11}q_{21}q_{22}^2a_3h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2q_{21}q_{22}q_{11}a_3h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{24}p_1^4q_{22}a_3h^6\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial x^4}+\frac{1}{24}p_1^4q_{22}a_3h^6\frac{\partial^2 f}{\partial y^2}\frac{\partial^4 f}{\partial x^4}$$
$$+\frac{1}{2}p_2q_{11}q_{22}^3a_3h^6f^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1p_2q_{21}^2q_{22}a_3h^6f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{2}{3}p_1q_{11}q_{21}q_{22}^2a_3h^6f^3\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+p_2q_{11}q_{21}q_{22}a_3h^6f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial x^3}+a_4h\,f+q_{31}a_4h^2f\frac{\partial f}{\partial y}+q_{32}a_4h^2f\frac{\partial f}{\partial y}$$
$$+q_{33}a_4h^2f\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}p_3^2a_4h^3\frac{\partial^2 f}{\partial x^2}+q_{21}q_{33}a_4h^3f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{33}a_4h^3f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$
$$+q_{21}q_{33}a_4h^3f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{33}a_4h^3f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{33}^2a_4h^3f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{32}^2a_4h^3f^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{31}^2a_4h^3f^2\frac{\partial^2 f}{\partial y^2}$$

$$+q_{31}q_{32}a_4h^3f^2\frac{\partial^2 f}{\partial y^2}+q_{31}q_{33}a_4h^3f^2\frac{\partial^2 f}{\partial y^2}+q_{32}q_{33}a_4h^3f^2\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{31}a_4h^3f\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{32}a_4h^3f\frac{\partial^2 f}{\partial x\partial y}+p_3q_{33}a_4h^3f\frac{\partial^2 f}{\partial x\partial y}+q_{11}q_{32}a_4h^3f\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_1q_{32}a_4h^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_2q_{33}a_4h^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_3q_{31}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{32}a_4h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_2^2q_{33}a_4h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{11}q_{22}q_{33}a_4h^4f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{22}q_{33}a_4h^4f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_2q_{21}q_{33}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2q_{22}q_{33}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}q_{33}a_4h^4\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{32}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{6}p_3^3a_4h^4\frac{\partial^3 f}{\partial x^3}+\frac{1}{2}q_{32}q_{33}^2a_4h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{33}q_{32}^2a_4h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}^2a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{33}^2a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{32}a_4h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{31}^2q_{33}a_4h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{31}a_4h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{33}^2q_{31}a_4h^4f^3\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}^2a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{33}^2a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{32}^2a_4h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{31}q_{33}a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{32}q_{33}a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}q_{31}q_{33}a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{32}q_{33}a_4h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{32}^2a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{33}^2a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1p_3q_{32}a_4h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_2p_3q_{33}a_4h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{32}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{6}q_{32}^3a_4h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{6}q_{33}^3a_4h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{33}^2a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3q_{32}^2a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{31}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{32}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{31}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{32}q_{33}a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{31}q_{32}a_4h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{11}^2q_{32}a_4h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{22}q_{33}a_4h^4\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{6}q_{31}^3a_4h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{33}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{32}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{31}a_4h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_3q_{31}^2a_4h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{32}q_{33}a_4h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{21}q_{33}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{22}q_{33}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{31}q_{32}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}q_{33}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_2q_{31}q_{33}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{32}q_{33}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_3q_{11}q_{32}a_4h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{21}q_{33}a_4h^4f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{22}q_{33}a_4h^4f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{32}q_{33}a_4h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}p_1^3q_{32}a_4h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{9}p_2^3q_{33}a_4h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{24}q_{32}^4a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{31}^4a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{32}^2q_{33}q_{21}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{33}q_{22}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}^2q_{33}q_{21}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^2q_{33}q_{22}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{33}p_2a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{32}^2q_{33}q_{11}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{32}^2q_{33}p_1a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2^2q_{22}q_{33}a_4h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_2^2q_{21}q_{33}a_4h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1^2q_{22}q_{33}a_4h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_1^2q_{22}q_{33}a_4h^5\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}q_{11}^2q_{22}q_{33}a_4h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{21}^2q_{22}q_{33}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{3}q_{22}^2q_{21}q_{33}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3q_{32}q_{33}^2a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{11}q_{32}a_4h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_3q_{11}q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{22}q_{31}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2\,q_{11}q_{22}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{21}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{32}q_{33}a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+$$

$$+p_1q_{11}q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{11}q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{21}q_{22}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}q_{32}^2q_{33}^2a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{32}q_{33}^3a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}q_{33}^3a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{32}^3q_{33}a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+p_3q_{11}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{31}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{2}p_2^2p_3q_{33}a_4h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{21}^2q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{22}^2q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{11}^2q_{32}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_1^2q_{32}^2a_4h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{33}^2a_4h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{11}^2q_{31}q_{32}a_4h^5f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}q_{11}^2q_{32}q_{33}a_4h^5f^3\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}^2q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}^2q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{21}q_{22}q_{33}^2a_4h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3+\frac{1}{2}p_1^2p_3q_{32}a_4h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}p_3q_{32}^3a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}a_4h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_3q_{22}q_{32}q_{33}a_4h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{32}^2a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{21}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{22}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1p_3q_{32}q_{33}a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_2p_3q_{32}q_{33}a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_3^2q_{32}q_{33}a_4h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{33}^2q_{32}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}^2q_{32}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{33}^2q_{32}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}^2q_{32}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}^2q_{32}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1q_{33}^2q_{32}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2q_{33}^2q_{32}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2p_2q_{33}a_4h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2p_2q_{32}a_4h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_3^2q_{11}q_{32}a_4h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_3q_{31}q_{33}^2a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2q_{31}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{31}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{11}q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_3^3q_{33}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{6}p_3^3q_{32}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3q_{21}q_{31}q_{33}a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_3q_{22}q_{31}q_{33}a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{21}q_{31}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_3q_{22}q_{31}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2p_3q_{31}q_{33}a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_3^2q_{21}q_{33}a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_3^2q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_3^2q_{21}q_{33}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3^2q_{22}q_{33}a_4h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_1p_3q_{31}q_{32}a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{24}p_3^4a_4h^5\frac{\partial^4 f}{\partial x^4}$$

$$+\frac{1}{2}p_3q_{31}q_{32}^2a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3q_{32}^2q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_3q_{11}q_{31}q_{32}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_3q_{11}q_{32}a_4h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_2p_3q_{21}q_{33}a_4h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+p_2p_3q_{22}q_{33}a_4h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$

$$+\frac{1}{2}p_3q_{11}^2q_{32}a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1p_3q_{22}q_{33}a_4h^5\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+2q_{21}q_{22}q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{22}q_{33}^2a_4h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{22}q_{33}^2a_4h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2q_{22}q_{33}^2a_4h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1q_{11}q_{32}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{21}q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{22}q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{22}\,q_{33}^2a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{21}\,q_{33}^2a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_2q_{22}\,q_{33}^2a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_1^2q_{31}q_{32}a_4h^5f\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}+\frac{1}{2}p_1^2q_{32}q_{33}a_4h^5f\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}$$
$$+\frac{1}{2}p_2^2q_{31}q_{33}a_4h^5f\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}$$

$$+\frac{1}{2}p_2^2q_{32}q_{33}a_4h^5f\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}+p_1p_2q_{32}q_{33}a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{4}p_3^2q_{33}^2a_4h^5f^2\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}$$

$$+\frac{1}{4}p_3^2q_{31}^2a_4h^5f^2\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}+\frac{1}{6}p_3q_{31}^3a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2f}{\partial y^2}+\frac{1}{4}p_3^2q_{32}^2a_4h^5f^2\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}$$

$$+q_{11}q_{21}q_{31}q_{33}a_4h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2f}{\partial y^2}\right)^2+2q_{11}q_{22}q_{32}q_{33}a_4h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2f}{\partial y^2}\right)^2$$

$$+p_1q_{21}q_{32}q_{33}a_4h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2f}{\partial y^2}\right)^2+p_1q_{22}q_{31}q_{33}a_4h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2f}{\partial y^2}\right)^2$$
$$+2p_1q_{22}q_{32}q_{33}a_4h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2f}{\partial y^2}\right)^2$$

$$+q_{11}q_{21}q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2f}{\partial y^2}+p_1q_{11}q_{31}q_{32}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}$$
$$+p_1q_{11}q_{32}q_{33}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}$$

$$+p_2q_{21}q_{31}q_{33}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}+p_2q_{21}q_{32}q_{33}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}$$
$$+p_2q_{22}q_{31}q_{33}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}$$

$$+p_2q_{22}q_{32}q_{33}a_4h^5f^2\frac{\partial^2f}{\partial y^2}\frac{\partial^2f}{\partial x\partial y}+p_1q_{21}q_{32}q_{33}a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_1q_{22}q_{31}q_{33}a_4h^5f\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+2p_1q_{22}q_{32}q_{33}a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_1q_{11}q_{32}^2a_4h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_3q_{22}q_{33}a_4h^5\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{31}q_{32}q_{33}^2a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{31}q_{33}q_{32}^2a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}p_1q_{11}^2q_{32}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_2q_{22}^2q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{11}q_{22}q_{33}a_4h^5f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_2q_{21}^2q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1p_2q_{22}q_{33}a_4h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{11}^2q_{22}q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{31}q_{32}q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{21}q_{33}^3a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{33}^3a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2q_{33}^3a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1^2q_{32}^2a_4h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_2^2q_{33}^2a_4h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}^2q_{32}^2a_4h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}q_{21}^2q_{33}^2a_4h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3+\frac{1}{2}q_{22}^2q_{33}^2a_4h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3+\frac{1}{2}q_{21}q_{33}^3a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{22}q_{33}^3a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{32}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{9}q_{22}^3q_{33}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{9}q_{21}^3q_{33}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3q_{11}a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{32}^3p_1a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{31}^2q_{32}q_{33}a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}^3q_{33}a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{31}^3q_{32}a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}p_3q_{33}^3a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{24}q_{33}^4a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+p_3q_{21}q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{22}q_{33}^2a_4h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{21}q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_3q_{22}q_{33}^2a_4h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1p_3q_{32}^2a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_2p_3q_{33}^2a_4h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}p_3q_{31}^2q_{32}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_3q_{31}^2q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_3^2q_{31}q_{32}a_4h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_3^2q_{31}q_{33}a_4h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1q_{31}^2q_{32}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}q_{32}q_{31}^2a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2q_{31}^2q_{33}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{21}q_{31}^2q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{31}^2q_{33}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{21}q_{33}q_{31}^2a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{22}q_{33}q_{31}^2a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1q_{32}^2q_{31}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{31}q_{32}^2a_4h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{31}q_{33}^2a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{31}q_{33}^2a_4h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2q_{33}^2q_{31}a_4h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{33}^2q_{31}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{31}q_{33}^2q_{22}a_4h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{4}q_{31}^2q_{32}^2a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{31}q_{32}^3a_4h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{31}^2q_{33}^2a_4h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{3}q_{11}q_{22}^3q_{33}a_4h^6f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_2^3q_{33}q_{21}a_4h^6f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{6}p_2^3q_{33}q_{22}a_4h^6f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial x^3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+\frac{1}{2}p_1^2q_{11}q_{22}q_{33}a_4h^6f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+a_5h\,f+q_{41}a_5h^2\,f\frac{\partial f}{\partial y}+q_{42}a_5h^2f\frac{\partial f}{\partial y}+q_{43}a_5h^2f\frac{\partial f}{\partial y}+q_{44}a_5h^2f\frac{\partial f}{\partial y}$$
$$+p_4a_5h^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}q_{44}^2a_5h^3f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{42}^2a_5h^3f^2\frac{\partial^2 f}{\partial y^2}+q_{41}q_{42}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{42}a_5h^3f\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{31}q_{44}a_5h^3f\left(\frac{\partial f}{\partial y}\right)^2+q_{32}q_{44}a_5h^3f\left(\frac{\partial f}{\partial y}\right)^2+q_{33}q_{44}a_5h^3f\left(\frac{\partial f}{\partial y}\right)^2$$
$$+p_1q_{42}a_5h^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_2q_{43}a_5h^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_3q_{44}a_5h^3\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+\frac{1}{2}p_4^2a_5h^3\frac{\partial^2 f}{\partial x^2}+q_{22}q_{43}a_5h^3f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{42}q_{44}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}$$

$$+q_{42}q_{43}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}+q_{21}q_{43}a_5h^3f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+q_{22}q_{43}a_5h^3f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$
$$+q_{21}q_{43}a_5h^3f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{41}^2a_5h^3f^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{43}^2a_5h^3f^2\frac{\partial^2 f}{\partial y^2}+q_{41}q_{43}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{41}q_{44}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{41}a_5h^3f\frac{\partial^2 f}{\partial x\partial y}+p_4q_{42}a_5h^3f\frac{\partial^2 f}{\partial x\partial y}+p_4q_{43}a_5h^3f\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{43}q_{44}a_5h^3f^2\frac{\partial^2 f}{\partial y^2}$$

$$p_4 q_{44} a_5 h^3 f \frac{\partial^2 f}{\partial x \partial y} + \frac{1}{6} q_{44}^2 a_5 h^4 f^3 \frac{\partial^3 f}{\partial y^3} + \frac{1}{6} p_4^3 a_5 h^4 \frac{\partial^3 f}{\partial x^3} + \frac{1}{2} q_{33}^2 q_{44} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2}$$

$$+ p_1 q_{32} q_{44} a_5 h^4 \frac{\partial f}{\partial x} \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{2} q_{11}^2 q_{42} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + q_{44}^2 q_{31} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + q_{44}^2 q_{32} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2}$$

$$+ q_{44}^2 q_{33} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + p_3 q_{44}^2 a_5 h^4 f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + p_1 p_4 q_{42} a_5 h^4 f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x \partial y} + p_2 p_4 q_{43} a_5 h^4 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x \partial y}$$

$$+ q_{21} q_{43} q_{44} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + q_{22} q_{43} q_{44} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + p_3 p_4 q_{44} a_5 h^4 f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial x \partial y}$$

$$+ q_{21} q_{41} q_{43} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + q_{22} q_{41} q_{43} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + q_{42}^2 q_{11} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2}$$

$$+ q_{21} q_{42} q_{43} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + q_{22} q_{42} q_{43} a_5 h^4 f^2 \left(\frac{\partial^2 f}{\partial y^2}\right)^2 + p_1 q_{42}^2 a_5 h^4 f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + q_{21} q_{43}^2 a_5 h^4 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + q_{22} q_{43}^2 a_5 h^4 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + p_2 q_{43}^2 a_5 h^4 f \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2}$$

$$+ q_{11} q_{44} q_{32} a_5 h^4 f \left(\frac{\partial f}{\partial y}\right)^3 + \frac{1}{2} q_{32}^2 q_{44} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + \frac{1}{2} q_{31}^2 q_{44} a_5 h^4 f^2 \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial y^2} + \frac{1}{2} q_{44}^2 p_4 a_5 h^4 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2}$$

$$+ q_{41} q_{42} q_{43} a_5 h^4 f^3 \frac{\partial^3 f}{\partial y^3} + q_{21} q_{42} q_{43} a_5 h^4 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + q_{22} q_{42} q_{43} a_5 h^4 f^2 \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} + p_3 q_{33} q_{44} a_5 h^4 f \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y}$$

$$+q_{11}q_{42}q_{43}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}q_{43}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{31}q_{42}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{32}q_{42}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{33}q_{42}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{42}q_{43}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{42}q_{44}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{33}q_{44}a_5h^4f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{42}q_{43}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{42}q_{44}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{33}q_{44}a_5h^4f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{21}q_{33}q_{44}a_5h^4f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{22}q_{33}q_{44}a_5h^4f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{11}q_{22}q_{43}a_5h^4f\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{11}q_{22}q_{43}a_5h^4f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_2q_{21}q_{43}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_2q_{22}q_{43}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_1q_{22}q_{43}a_5h^4\frac{\partial f}{\partial y}\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{31}q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{32}q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{33}q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{43}q_{44}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{43}q_{44}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_1q_{11}q_{42}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{21}q_{43}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{22}q_{43}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{31}q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{32}q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{43}q_{44}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{42}q_{43}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{42}q_{44}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{43}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+p_4q_{41}q_{44}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{41}q_{42}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{42}q_{44}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{42}q_{43}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{44}q_{42}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{42}q_{41}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}q_{42}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{44}q_{41}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{41}q_{43}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4^2q_{43}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{41}q_{44}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{41}q_{42}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}q_{41}^3a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{42}^2a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{42}q_{43}q_{44}a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}^2a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{33}q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{43}a_5h^4f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{22}q_{43}a_5h^4f\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{21}q_{41}q_{43}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{41}q_{43}a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{41}q_{42}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{31}q_{41}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{32}q_{41}q_{44}f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{33}q_{41}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{41}q_{42}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{41}q_{43}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{41}q_{44}a_5h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{31}q_{32}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{31}q_{33}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{32}q_{33}q_{44}a_5h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{32}q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{6}q_{43}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{43}^2q_{44}a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+q_{21}q_{43}^2a_5h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}q_{43}^2a_5h^4f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_3^2q_{44}a_5h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_1^2q_{42}a_5h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_2^2q_{43}a_5h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{6}q_{42}^3a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+q_{41}q_{42}q_{44}a_5h^4f^3\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{41}^2a_5h^4f^2\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}+q_{41}q_{43}q_{44}a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4^2q_{44}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_4^2q_{41}a_5h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}p_4^2q_{42}f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}+\frac{1}{2}q_{43}q_{41}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{43}q_{44}^2a_5h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{22}q_{43}a_5h^4\frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)^2+p_2q_{33}q_{44}a_5h^4\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+a_5h^5$$
$$+\frac{1}{2}q_{31}q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{31}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{41}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2p_4q_{42}a_5h^5\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+q_{21}q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+2q_{22}q_{33}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{31}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{22}q_{32}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{32}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{31}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{32}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{22}q_{32}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{31}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{22}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{31}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_3q_{22}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}q_{42}q_{43}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3^2q_{44}^2a_5h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{31}^2q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}q_{32}^2q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{33}^2q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_3q_{21}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_3q_{31}q_{32}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{32}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+2q_{21}q_{33}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{22}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{21}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{32}q_{33}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}q_{31}q_{33}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3p_4q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{32}q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{31}q_{32}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{31}q_{41}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{33}q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{33}q_{31}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3p_4^2q_{44}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}q_{32}^2q_{33}\,q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{32}q_{41}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4^2q_{31}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{21}q_{33}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{33}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{21}q_{43}^2q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{43}^2q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{31}^2q_{32}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{42}^2q_{43}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4^2q_{32}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{11}q_{32}^2q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_2q_{43}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4^2q_{33}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}q_{11}^2q_{32}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_1q_{22}q_{33}q_{44}a_5h^5\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{33}q_{41}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{32}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_1p_4^2q_{42}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+p_3q_{44}^2q_{41}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{31}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_4^2q_{11}q_{42}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}p_2^2q_{22}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{41}q_{42}q_{43}q_{44}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+p_4q_{21}q_{43}^2a_5h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_3^2q_{33}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}p_2^2q_{21}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{2}p_2p_4^2q_{43}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_1^2q_{22}q_{43}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+p_4q_{22}q_{43}^2a_5h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_4q_{43}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{32}^2q_{43}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{4}p_4^2q_{43}^2a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}q_{44}^3a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{33}^2q_{43}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{4}p_4^2q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_1q_{11}q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{21}q_{31}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_1^2q_{22}q_{43}a_5h^5\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+p_4q_{21}q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{11}^2q_{22}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_4q_{22}q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{21}^2q_{22}q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2p_4q_{43}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{3}q_{22}^2q_{21}q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_1q_{11}q_{41}q_{42}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{11}q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{32}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}^2q_{41}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_1^2q_{11}q_{42}a_5h^5f\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_1q_{41}q_{42}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}q_{41}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{11}q_{32}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+q_{31}q_{32}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+q_{31}q_{33}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+q_{32}q_{33}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{31}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_4q_{44}q_{41}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{33}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{21}q_{33}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{22}q_{33}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_4p_3q_{31}q_{44}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_4p_3q_{32}q_{44}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+\frac{1}{2}p_4q_{42}q_{41}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4p_3q_{33}q_{44}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_4q_{44}q_{33}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_4q_{44}q_{32}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{41}q_{44}q_{31}q_{32}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_4q_{41}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{44}q_{31}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4p_2q_{22}q_{43}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_1p_4q_{22}q_{43}a_5h^5\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_4^2q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{41}q_{44}q_{31}q_{33}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_3^2q_{44}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2p_3q_{43}q_{44}a_5h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_4^2q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_2^2q_{44}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{41}q_{44}q_{32}q_{33}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{43}q_{44}q_{31}q_{32}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{43}q_{44}q_{31}q_{33}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{43}q_{44}q_{32}q_{33}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_2^2q_{41}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1^2q_{41}q_{42}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_3^2q_{41}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{33}^2q_{32}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{21}q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{32}q_{41}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+p_4q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{41}^2q_{42}q_{11}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{42}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+2p_1q_{22}q_{42}q_{43}a_5h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{21}q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1q_{11}q_{32}q_{44}a_5h^5f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{22}q_{43}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{33}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{24}p_4^4a_5h^5\frac{\partial^4 f}{\partial x^4}+p_1q_{42}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{21}q_{41}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{22}q_{33}q_{44}a_5h^5f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{43}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{22}q_{33}q_{44}a_5h^5\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2+p_4q_{31}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{33}q_{43}q_{44a_5h^5}f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{11}q_{41}q_{42}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{24}q_{43}^4a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{22}^2q_{43}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}p_2^2q_{43}^2a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}^2q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{42}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1p_4q_{41}q_{42}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{31}q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}^2q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{33}q_{44}^3a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3q_{44}^3a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{22}q_{43}^2a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^3+\frac{1}{2}q_{11}^2q_{42}q_{43}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_2q_{22}^2q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{43}q_{44}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{33}q_{42}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{32}q_{42}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}^2q_{43}^2a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^3$$
$$+\frac{1}{2}p_2^2q_{43}^2a_5h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_3q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{22}q_{41}q_{43}a_5h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{11}q_{22}q_{33}q_{44}a_5h^5\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^3$$

$$+p_2q_{41}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}^2a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_4^2q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}$$

$$+\frac{1}{2}p_4^2q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_4^2q_{21}q_{43}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}q_{31}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_4^2q_{21}q_{43}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x^2}+q_{22}q_{33}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{22}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_1p_3q_{32}q_{44}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{21}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}p_4q_{11}^2q_{42}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4p_1q_{11}q_{42}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+p_3q_{32}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{21}q_{22}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{31}q_{32}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{31}q_{33}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{33}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1p_4q_{42}q_{43}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{2}q_{32}q_{41}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1p_2q_{42}q_{43}a_5h^5\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+\frac{1}{2}q_{11}q_{44}^2q_{42}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{21}q_{41}^2q_{43}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{41}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3p_4q_{42}q_{44}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+\frac{1}{4}p_4^2q_{41}^2a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{6}p_4q_{41}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{11}q_{41}q_{42}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_3q_{33}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{32}^2q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{22}q_{33}q_{44}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^3$$
$$+p_2q_{21}q_{33}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x\partial y}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+p_3q_{41}q_{43}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{21}q_{42}q_{43}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+q_{11}q_{42}^2q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{33}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{44}^2q_{32}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{33}q_{44}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_3q_{33}q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_2q_{33}q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$
$$+p_3q_{31}q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$+p_3q_{32}q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}+p_1q_{41}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1q_{22}q_{44}q_{43}a_5h^5f\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{31}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{22}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{22}q_{32}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_2q_{41}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+2q_{21}q_{33}q_{44}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_3q_{21}q_{44}q_{43}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2+p_3q_{22}q_{44}q_{43}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+q_{22}q_{43}^2q_{42}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{31}q_{32}q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{42}q_{44}^2q_{33}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{4}q_{41}^2q_{42}^2a_5h^5f^6\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{41}^3q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{44}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{41}^3q_{42}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{44}^3q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{44}^3q_{42}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{44}^2q_{42}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{41}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{42}^3q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{6}q_{43}^3q_{42}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{42}^3q_{44}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{43}^2q_{42}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{6}q_{43}^3q_{44}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{4}q_{43}^2q_{44}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{43}^3q_{41}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{6}q_{44}^3q_{41}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+p_3q_{41}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{31}q_{44}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{44}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_3q_{33}q_{44}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1q_{22}q_{43}q_{44}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_1q_{22}q_{43}q_{41}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{31}q_{33}q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+p_1p_4q_{22}q_{43}a_5h^5\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial f}{\partial x}$$
$$+p_4q_{11}q_{32}q_{44}a_5h^5f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{32}q_{43}q_{44}a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+q_{22}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_2q_{21}q_{44}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}p_3^2p_4q_{44}a_5h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}+p_4q_{11}q_{42}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{22}q_{43}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{21}q_{32}q_{43}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+q_{31}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{21}q_{43}q_{44}a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+p_3q_{22}q_{43}q_{44}a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+q_{32}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{41}q_{31}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{22}q_{33}q_{42}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+2q_{22}q_{33}q_{43}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}p_1q_{41}^2q_{42}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_1q_{11}^2q_{42}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2q_{11}q_{22}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{41}q_{42}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{41}\,q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{43}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{21}q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{31}q_{42}q_{44}a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_4q_{21}q_{43}q_{42}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+p_1q_{31}q_{42}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+2p_1q_{32}q_{42}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2p_4q_{41}q_{43}a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+2q_{11}q_{32}q_{42}q_{44}a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{32}q_{33}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{32}q_{33}q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_2q_{11}q_{42}q_{43}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_1q_{22}q_{43}^{a_5h^5 2}f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+p_2q_{22}q_{43}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}p_4^2q_{41}q_{42}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2q_{21}q_{43}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{33}q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{31}q_{32}q_{44}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+p_4q_{31}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3p_4q_{44}q_{43}a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{41}^2q_{42}q_{44}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{42}^2q_{41}q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{42}q_{41}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{41}^2q_{42}q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{42}^2q_{44}q_{43}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{43}^2q_{44}q_{42}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{44}^2q_{42}q_{41}a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{22}q_{41}^2q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_1p_4q_{42}q_{44}a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{33}q_{41}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{33}q_{32}q_{31}q_{44}a_5h^5a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}q_{11}q_{22}q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+q_{22}\,q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{41}q_{44}q_{33}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{8}p_4^3q_{42}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{2}p_2q_{21}^2\,q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2p_3q_{33}q_{44}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}p_2q_{43}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}q_{33}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{31}q_{42}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_2^2q_{42}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{11}q_{42}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{32}q_{42}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{33}^2q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{4}q_{41}^2q_{43}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{4}q_{41}^2q_{44}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}p_4q_{41}q_{42}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{33}q_{42}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{11}q_{42}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_3^2q_{44}^2a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{33}^2q_{44}^2a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+p_1q_{33}q_{42}q_{44a_5h^5}f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{32}^2q_{44}^2a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}q_{21}q_{43}q_{42}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{41}q_{43}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{21}\,q_{41}q_{43}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{22}\,q_{41}q_{43}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_1p_4q_{42}^2a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{21}q_{41}q_{43}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{31}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_3q_{21}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{32}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{22}q_{41}q_{43}a_5h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1q_{42}q_{43}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{43}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{43}^2q_{42}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{44}^2q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4q_{42}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{21}q_{42}q_{43}a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+q_{21}q_{42}q_{43}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3^2q_{42}q_{44}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}q_{43}^3a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{22}q_{43}^3a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_2q_{43}^3a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}q_{43}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{22}q_{43}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_4q_{41}q_{43}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_4q_{43}q_{42}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_4^2q_{43}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{2}p_4^2q_{43}q_{41}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}p_1q_{42}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{32}q_{33}q_{42}q_{43}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{11}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_3q_{32}q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}q_{11}q_{42}q_{43}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{32}\,q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{41}q_{43}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+p_1q_{11}q_{42}^2a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_2p_4q_{42}q_{43}a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{6}p_3^3q_{44}a_5h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$
$$+\frac{1}{2}p_2^2p_4q_{43}a_5h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{22}q_{41}q_{43}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{11}^2q_{41}q_{42}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{8}p_4^3q_{41}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{8}p_4^3q_{44}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+p_3q_{42}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1p_3q_{42}q_{44}a_5h^5\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$

$$\frac{1}{2}\ p_3q_{44}q_{41}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+q_{31}q_{41}q_{42}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+2q_{11}q_{22}q_{42}q_{43}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+\frac{1}{2}q_{32}q_{44}^3a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{11}^2q_{42}^2a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}\frac{1}{9}q_{22}^3\,q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{9}q_{21}^3\,q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{6}q_{11}^3q_{42}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{21}q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_2p_4q_{21}q_{43}a_5h^5f\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$$
$$+p_2q_{21}q_{43}^2a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+p_2q_{22}q_{43}^2a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}+\frac{1}{2}p_2q_{43}q_{42}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+p_1p_2q_{22}q_{43}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$

$$+\frac{1}{2}q_{22}q_{43}q_{41}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{42}^2a_5h^5f\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{11}^2q_{42}^2a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+q_{32}q_{43}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_1q_{22}q_{43}^2a_5h^5f\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$

$$+\frac{1}{2}p_1^2q_{42}q_{43}a_5h^5f\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{21}^2q_{43}^2a_5h^5f^2\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{22}q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}q_{21}q_{41}^2q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+2p_1q_{22}q_{42}q_{43}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+p_1q_{42}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{11}\,q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{24}q_{41}^4a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{24}q_{44}^4a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{24}q_{42}^4a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$

$$+\frac{1}{2}q_{22}q_{42}^2q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}q_{43}q_{42}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}\,q_{31}q_{32}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

$$+\frac{1}{2}q_{21}q_{43}q_{44}^2a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_3q_{42}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}p_3q_{44}q_{32}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{32}q_{33}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+q_{21}q_{31}q_{33}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$

$$+q_{22}q_{32}q_{33}q_{44}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}+\frac{1}{2}q_{41}^2q_{33}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4q_{11}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{11}q_{22}q_{41}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{21}q_{42}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+q_{22}q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{31}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_4q_{32}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+p_4q_{33}q_{41}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+q_{11}q_{42}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+p_4q_{31}\,q_{33}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+q_{11}q_{33}q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2+\frac{1}{6}q_{31}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}q_{33}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{31}q_{42}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+p_4q_{31}\,q_{32}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{2}q_{21}q_{43}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4p_2q_{33}q_{44}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+p_1q_{11}q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{21}q_{43}^2q_{42}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_4p_1q_{32}q_{44}a_5h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+\frac{1}{2}q_{33}^2q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{4}p_4^2q_{42}^2a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}$$

$$+\frac{1}{8}p_4^3q_{43}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{11}^2q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2p_4q_{33}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$++\frac{1}{2}q_{32}^2q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}q_{31}^2q_{42}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+2q_{43}^2q_{21}q_{22}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$+\frac{1}{2}p_4^2q_{41}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}+q_{43}^2q_{11}q_{22}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+q_{21}q_{22}q_{43}^2a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{6}q_{32}^2q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}p_1^2q_{32}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial^2 f}{\partial x^2}$$
$$+\frac{1}{2}p_1q_{42}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{6}p_4q_{42}^3a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_3q_{33}q_{42}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+q_{44}^2q_{43}q_{33}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{11}q_{22}q_{43}a_5h^5f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+q_{31}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{6}p_1^3q_{42}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$+\frac{1}{9}p_2^3q_{43}a_5h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}+p_4q_{21}q_{41}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+q_{22}q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{33}q_{41}q_{42}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{22}q_{43}q_{42}q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{22}q_{43}q_{41}q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{11}q_{32}q_{44}a_5h^5f\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial x\partial y}+p_3p_4q_{41}q_{44}a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+q_{31}q_{43}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{43}q_{22}q_{11}^2a_5h^5f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{41}^2q_{44}q_{31}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$
$$+q_{32}q_{41}q_{42}q_{44}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+q_{11}q_{21}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_1q_{42}^2q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}q_{22}q_{43}q_{44}^2a_5h^5f^3\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_3q_{44}^2q_{43}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}+p_4q_{22}q_{42}q_{43}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial x}$$
$$+\frac{1}{2}p_2q_{43}q_{41}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_2q_{22}q_{41}q_{43}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+p_3q_{32}q_{41}q_{44}a_5h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$
$$+\frac{1}{6}p_4q_{44}^3a_5h^5f^4\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{11}q_{42}^3a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}+q_{21}q_{33}q_{42}q_{43}a_5h^5f^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2\frac{\partial f}{\partial y}$$
$$+p_4q_{32}q_{44}^2a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$+q_{21}q_{42}q_{43}q_{44}a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+\frac{1}{2}q_{21}^2q_{43}^2a_5h^5f^2\left(\frac{\partial f}{\partial x}\right)^2\frac{\partial^2 f}{\partial y^2}$$
$$+q_{11}q_{41}q_{42}q_{43}a_5h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$+\frac{1}{2}q_{22}q_{43}q_{44}^3a_5h^5f^3\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial y^3}+p_2q_{22}q_{43}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$
$$+p_2q_{21}q_{43}^2a_5h^5f\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$+\frac{1}{2}p_3q_{31}^2q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_2q_{33}q_{41}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{42}q_{43}^3a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$+p_1q_{32}q_{41}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{11}q_{22}q_{43}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{22}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$+p_4q_{21}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}+q_{22}q_{33}q_{42}q_{44}a_5h^5f^2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}p_2^2q_{33}q_{44}a_5h^5\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial f}{\partial y}\right)^2$$
$$+\frac{1}{2}q_{31}^2q_{43}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2+q_{11}q_{21}q_{42}q_{43}a_5h^5f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{1}{2}q_{42}q_{43}q_{44}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{44}q_{43}q_{41}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{41}q_{43}q_{44}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}+\frac{1}{2}q_{41}q_{44}q_{42}^2a_5h^5f^4\frac{\partial^4 f}{\partial y^4}$$
$$+\frac{1}{2}q_{31}^2q_{44}^2a_5h^5f^2\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^2 f}{\partial y^2}+\frac{1}{2}q_{31}^2q_{41}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+\frac{1}{2}q_{32}^2q_{41}q_{44}a_5h^5f^3\left(\frac{\partial^2 f}{\partial y^2}\right)^2$$
$$+p_1q_{32}q_{43}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+p_4q_{22}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_4q_{21}q_{33}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}$$
$$+p_2q_{32}q_{43}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}+2p_2q_{33}q_{43}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_2q_{31}q_{43}q_{44}a_5h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$
$$+p_1q_{21}q_{22}q_{43}^2a_5h^6f\left(\frac{\partial f}{\partial x}\right)^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{3}q_{11}q_{22}q_{21}^2q_{43}a_5h^6f^3\left(\frac{\partial f}{\partial y}\right)^2\frac{\partial^3 f}{\partial y^3}$$
$$+\frac{1}{2}p_2q_{11}^2q_{42}q_{43}a_5h^6f^2\frac{\partial f}{\partial x}\left(\frac{\partial^2 f}{\partial y^2}\right)^2+\frac{1}{2}p_2^2p_4q_{22}q_{43}a_5h^6f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^2 f}{\partial x^2} \quad (3.4)$$

Samakan persamaan 3.3 dengan persamaan 3.4

$$f(x,y)h=(a_1+a_2+a_3+a_4+a_5)hf \quad (1a)$$

$$\frac{1}{2}\frac{\partial f(x,y)}{\partial x}h^2=(a_2p_1+a_3p_2+a_3q_{21}+a_3q_{22}+p_4a_5)h^2\frac{\partial f}{\partial x} \quad (2a)$$

$$\frac{1}{2}\frac{\partial f(x,y)}{\partial y} f(x,y)h^2 \tag{3a}$$
$$= (a_2 q_{11} + q_{31}a_4 + q_{32}a_4 + q_{33}a_4 + q_{41}a_5 + q_{42}a_5$$
$$+q_{43}a_5 + q_{44})fh^2 \frac{\partial f}{\partial y}$$

$$\frac{1}{6}\frac{\partial^2 f(x,y)}{\partial x^2}h^3 = (a_2\frac{1}{2}p_1^2 + a_3\frac{1}{2}p_2^2 + \frac{1}{2}p_3^2 a_4 + \frac{1}{2}p_4^2 a_5)h^3\frac{\partial^2 f}{\partial x^2} \tag{4a}$$

$$\frac{1}{3}\frac{\partial^2 f(x,y)}{\partial x \partial y} f(x,y)h^3 \tag{5a}$$
$$= (a_2 p_1 q_{11} + a_3 h^3 p_2 q_{21} + a_3 p_2 q_{22} + p_3 q_{31} a_4$$
$$+ p_3 q_{32} a_4$$
$$+p_3 q_{33} a_4 + p_4 q_{41} a_5 + p_4 q_{42} a_5 + p_4 q_{43} a_5 + p_4 q_{44} a_5)h^3 f \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{1}{6}\frac{\partial^2 f(x,y)}{\partial y^2}\left(f(x,y)\right)^2 h^3 \tag{6a}$$
$$= (a_2\frac{1}{2}q_{11}^2 + \frac{1}{2}q_{33}^2 a_4 + \frac{1}{2}q_{32}^2 a_4 + \frac{1}{2}q_{31}^2 a_4 + q_{31}q_{32}a_4$$
$$+q_{31}q_{33}a_4 + q_{32}q_{33}a_4 + \frac{1}{2}q_{44}^2 a_5 + \frac{1}{2}q_{42}^2 a_5 + q_{41}q_{42}a_5 + q_{42}q_{44}a_5$$
$$+ q_{42}q_{43}a_5$$
$$+\frac{1}{2}q_{41}^2 a_5 + \frac{1}{2}q_{43}^2 a_5 + q_{41}q_{43}a_5 + q_{41}q_{44}a_5 + q_{43}q_{44}a_5)h^3 f^2 \frac{\partial^2 f}{\partial y^2}$$

$$\frac{1}{6}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}h^3 \tag{7a}$$
$$= (p_1 q_{32} a_4 + p_2 q_{33} a_4 + p_1 q_{42} a_5 + p_2 q_{43} a_5$$
$$+ p_3 q_{44} a_5)h^3 \frac{\partial f}{\partial x}\frac{\partial f}{\partial y}$$

$$\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\frac{h^3}{6} = (q_{11}q_{32}a_4 + q_{11}q_{42}a_5 + q_{31}q_{44}a_5 \tag{8a}$$
$$+q_{32}q_{44}a_5 + q_{33}q_{44}a_5)h^3 f\left(\frac{\partial f}{\partial y}\right)^2$$

$$\frac{1}{24}\frac{\partial^3 f(x,y)}{\partial x^3}h^4 = (a_2\frac{1}{6}p_1^3 + \frac{1}{9}p_2^3 a_3 + \frac{1}{6}p_3^3 a_4 + \frac{1}{6}p_4^3 a_5)h^4\frac{\partial^3 f}{\partial x^3} \tag{9a}$$

$$\frac{1}{8}\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial x}h^4 = (p_1p_2q_{22}a_3 + p_1p_3q_{32}a_4 + p_2p_3q_{33}a_4 + p_2p_4q_{43}a_5)h^4\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial x \partial y} \quad (10a)$$

$$\frac{\partial f(x,y)}{\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\frac{h^4}{24} = (\frac{1}{2}p_1^2q_{32}a_4 + \frac{1}{2}p_2^2q_{33}a_4 + \frac{1}{2}p_3^2q_{44}a_5 + \frac{1}{2}p_1^2q_{42}a_5 + \frac{1}{2}p_2^2q_{43}a_5)h^4\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x^2} \quad (11a)$$

$$\frac{1}{8}f(x,y)\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial x}h^4 = (p_1q_{32}^2a_4 + p_2q_{33}^2a_4 + p_1q_{31}q_{32}a_4 + p_1q_{32}q_{33}a_4 + p_2q_{31}q_{33}a_4 + p_2q_{32}q_{33}a_4 + p_3q_{44}^2a_5 + p_1q_{42}^2a_5 + p_2q_{43}^2a_5 + p_1q_{42}q_{43}a_5 + p_1q_{42}q_{44}a_5 + p_2q_{42}q_{43}a_5 + p_3q_{42}q_{44}a_5 + p_2q_{43}q_{44}a_5 + p_3q_{43}q_{44}a_5 + p_1q_{41}q_{42}a_5 + p_2q_{41}q_{43}a_5 + p_3q_{41}q_{44}a_5)h^4f\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2} \quad (12a)$$

$$\frac{5}{24}\frac{\partial^2 f(x,y)}{\partial x \partial y}\frac{\partial f(x,y)}{\partial y}f(x,y)h^4 = (p_2q_{22}q_{11}a_3 + p_2q_{21}q_{33}a_4 + p_2q_{22}q_{33}a_4 + p_1q_{11}q_{32}a_4 + p_3q_{33}q_{44}a_5 + p_2q_{21}q_{43}a_5 + p_2q_{22}q_{43}a_5 + p_1q_{11}q_{42}a_5 + p_4q_{11}q_{42}a_5 + p_4q_{31}q_{44}a_5 + p_4q_{32}q_{44}a_5 + p_4q_{33}q_{44}a_5 + p_3q_{31}q_{44}a_5 + p_3q_{32}q_{44}a_5)h^4f\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} \quad (13a)$$

$$\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial f(x,y)}{\partial x}\frac{h^4}{24} = (p_1q_{32}q_{44}a_5)h^4\frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)^2 \quad (14a)$$

$$\frac{\partial^3 f(x,y)}{\partial y^3}\left(f(x,y)\right)^3\frac{h^4}{24} \tag{15a}$$

$$= (a_2\frac{1}{6}q_{11}^3 + \frac{1}{9}q_{22}^3a_3 + \frac{1}{3}q_{21}^2q_{22}a_3 + \frac{1}{3}q_{22}^2q_{21}a_3 + \frac{1}{9}q_{21}^3a_3$$

$$+\frac{1}{2}q_{32}q_{33}^2a_4 + \frac{1}{2}q_{33}q_{32}^2a_4 + \frac{1}{2}q_{31}^2q_{32}a_4 + \frac{1}{2}q_{31}^2q_{33}a_4 + \frac{1}{2}q_{32}^2q_{31}a_4$$

$$+\frac{1}{2}q_{33}^2q_{31}a_4 + \frac{1}{6}q_{32}^3a_4 + \frac{1}{6}q_{33}^3a_4 + \frac{1}{6}q_{31}^3a_4 + q_{31}q_{32}q_{33}a_4 + \frac{1}{6}q_{44}^2a_5$$

$$+q_{41}q_{42}q_{43}a_5 + \frac{1}{2}q_{42}q_{44}^2a_5 + \frac{1}{2}q_{42}q_{43}^2a_5 + \frac{1}{2}q_{44}q_{42}^2a_5 + \frac{1}{2}q_{42}q_{41}^2a_5$$

$$+\frac{1}{2}q_{43}q_{42}^2a_5$$

$$+\frac{1}{2}q_{44}q_{41}^2a_5 + \frac{1}{2}q_{41}q_{43}^2a_5 + \frac{1}{2}q_{41}q_{44}^2a_5 + \frac{1}{2}q_{41}q_{42}^2a_5 + \frac{1}{6}q_{41}^3a_5$$

$$+ q_{42}q_{43}q_{44}a_5$$

$$+\frac{1}{6}q_{43}^2a_5 + \frac{1}{2}q_{43}^2q_{44}a_5 + \frac{1}{6}q_{42}^3a_5 + q_{41}q_{42}q_{44}a_5 + q_{41}q_{43}q_{44}a_5$$

$$+\frac{1}{2}q_{43}q_{41}^2a_5$$

$$+\frac{1}{2}q_{43}q_{44}^2a_5 + p_2q_{33}q_{44}a_5)h^4f^3\frac{\partial^3 f}{\partial y^3}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial f(x,y)}{\partial y}\left(f(x,y)\right)^2\frac{h^4}{6} \tag{16a}$$

$$= (q_{11}q_{32}^2a_4 + q_{11}q_{31}q_{32}a_4 + \frac{1}{2}q_{11}^2q_{32}a_4$$

$$+q_{11}q_{32}q_{33}a_4 + \frac{1}{2}q_{33}^2q_{44}a_5 + \frac{1}{2}q_{11}^2q_{42}a_5 + q_{44}^2q_{31}a_5 + q_{44}^2q_{32}a_5$$

$$+ q_{44}^2q_{33}a_5$$

$$+q_{42}^2q_{11}a_5 + \frac{1}{2}q_{32}^2q_{44}a_5 + q_{11}q_{42}q_{43}a_5 + q_{11}q_{42}q_{43}a_5 + q_{31}q_{42}q_{44}a_5$$

$$+ q_{32}q_{42}q_{44}a_5$$

$$+q_{33}q_{42}q_{44}a_5 + q_{31}q_{43}q_{44}a_5 + q_{32}q_{43}q_{44}a_5 + q_{33}q_{43}q_{44}a_5$$

$$+ q_{11}q_{41}q_{42}a_5$$

$$+q_{31}q_{41}q_{44}a_5 + q_{32}q_{41}q_{44}a_5 + q_{33}q_{41}q_{44}a_5 + q_{31}q_{32}q_{44}a_5$$

$$+ q_{31}q_{33}q_{44}a_5$$

$$+q_{32}q_{33}q_{44}a_5)h^4f^2\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^4 f(x,y)}{\partial x^4}\frac{h^5}{120} = (a_2\frac{1}{24}p_1^4 + \frac{1}{24}p_2^4a_3 + \frac{1}{24}p_3^4a_4 + \frac{1}{24}p_4^4a_5)h^5\frac{\partial^4 f}{\partial x^4} \quad (17a)$$

$$\frac{\partial^2 f(x,y)}{\partial x\partial y}\frac{\partial^2 f(x,y)}{\partial x^2}\frac{h^5}{30} \quad (18a)$$
$$= (\frac{1}{2}p_1^2p_2q_{22}a_3 + \frac{1}{2}p_2^2p_3q_{33}a_4 + \frac{1}{2}p_1^2p_3q_{32}a_4$$
$$+\frac{1}{2}p_1^2p_4q_{42}a_5$$
$$+\frac{1}{2}p_3^2p_4q_{44}a_5 + \frac{1}{2}p_2^2p_4q_{43}a_5)h^5\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial x^3}\frac{h^5}{120} \quad (19a)$$
$$= (\frac{1}{6}p_2^3q_{22}a_3 + \frac{1}{6}p_1^3q_{32}a_4 + \frac{1}{9}p_2^3q_{33}a_4 + \frac{1}{6}p_3^3q_{44}a_5$$
$$+\frac{1}{9}p_2^3q_{43}a_5)h^5\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial x^3}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2}\frac{\partial^2 f(x,y)}{\partial x^2}f(x,y)\frac{h^5}{30} \quad (20a)$$
$$= (a_2\frac{1}{4}p_1^2q_{11}^2 + \frac{1}{2}p_1^2q_{32}^2a_4 + \frac{1}{2}p_2^2q_{33}^2a_4 + \frac{1}{2}p_1^2q_{31}q_{32}a_4$$
$$+\frac{1}{2}p_1^2q_{32}q_{33}a_4$$
$$+\frac{1}{2}p_2^2q_{31}q_{33}a_4 + \frac{1}{2}p_2^2q_{32}q_{33}a_4 + \frac{1}{2}p_3^2q_{44}q_{43}a_5 + \frac{1}{2}p_2^2q_{44}q_{43}a_5$$
$$+\frac{1}{2}p_2^2q_{41}q_{43}a_5$$
$$+\frac{1}{2}p_1^2q_{41}q_{42}a_5 + \frac{1}{2}p_3^2q_{41}q_{44}a_5 + \frac{1}{2}p_2^2q_{43}^2a_5 + \frac{1}{2}p_1^2q_{42}^2a_5$$
$$+\frac{1}{2}p_2^2q_{42}q_{43}a_5$$

$$+\frac{1}{2}p_1^2q_{42}q_{44}a_5+\frac{1}{2}p_3^2q_{44}^2a_5+\frac{1}{2}p_3^2q_{42}q_{44}a_5$$
$$+\frac{1}{2}p_1^2q_{42}q_{43}a_5)h^5f\frac{\partial^2f}{\partial x^2}\frac{\partial^2f}{\partial y^2}$$

$$\left(\frac{\partial f(x,y)}{\partial x}\right)^2\frac{\partial^2f(x,y)}{\partial y^2}\frac{h^5}{40} \tag{21a}$$
$$=(\frac{1}{2}p_1^2q_{32}^2a_4+\frac{1}{2}p_2^2q_{33}^2a_4+\frac{1}{2}p_3^2q_{44}^2a_5+p_2p_3q_{43}q_{44}a_5$$
$$+\frac{1}{2}p_2^2q_{43}^2a_5+p_1p_2q_{42}q_{43}a_5+p_1p_3q_{42}q_{44}a_5)h^5\frac{\partial^2f}{\partial y^2}\left(\frac{\partial f}{\partial x}\right)^2$$

$$\left(\frac{\partial^2f(x,y)}{\partial x\partial y}\right)^2f(x,y)\frac{h^5}{15} \tag{22a}$$
$$=(p_1p_2q_{11}q_{22}a_3+p_1p_3q_{11}q_{32}a_4+p_2p_3q_{21}q_{33}a_4$$
$$+p_2p_3q_{22}q_{33}a_4+p_4p_3q_{31}q_{44}a_5+p_4p_3q_{32}q_{44}a_5+p_4p_3q_{33}q_{44}a_5$$
$$+p_4p_2q_{22}q_{43}a_5+p_4p_1q_{11}q_{42}a_5+p_2p_4q_{21}q_{43}a_5)h^5f\left(\frac{\partial^2f}{\partial x\partial y}\right)^2$$

$$\frac{\partial^2f(x,y)}{\partial x\partial y}\frac{\partial f(x,y)}{\partial y}\frac{\partial f(x,y)}{\partial x}\frac{7h^5}{120} \tag{23a}$$
$$=(p_1p_2q_{22}q_{33}a_4+p_1p_3q_{32}q_{44}a_5+p_2p_3q_{33}q_{44}a_5$$
$$+p_1p_2q_{22}q_{43}a_5+p_4p_2q_{33}q_{44}a_5+p_4p_1q_{32}q_{44}a_5)h^5\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2f}{\partial x\partial y}$$

$$\frac{\partial f(x,y)}{\partial x}\frac{\partial^3f(x,y)}{\partial y^3}f(x,y)\frac{h^5}{40}=(a_2\frac{1}{6}p_1q_{11})h^5f\frac{\partial f}{\partial x}\frac{\partial^3f}{\partial y^3} \tag{24a}$$

$$\frac{\partial^2f(x,y)}{\partial x\partial y}\frac{\partial^2f(x,y)}{\partial y^2}\big(f(x,y)\big)^2\frac{h^5}{10} \tag{25a}$$
$$=\left(\frac{1}{2}p_2q_{11}^2q_{22}a_3+\frac{1}{2}p_3q_{11}^2q_{32}a_4+p_1q_{11}q_{32}^2a_4\right.$$

$$+p_2q_{21}q_{33}^2a_4+p_2q_{22}q_{33}^2a_4+p_1q_{11}q_{31}q_{32}+p_1q_{11}q_{32}q_{33}a_4 + p_2q_{21}q_{31}q_{33}a_4$$

$$+p_2q_{21}q_{32}q_{33}a_4+p_2q_{22}q_{31}q_{33}a_4+p_2q_{22}q_{32}q_{33}a_4+p_1q_{11}q_{41}q_{42}a_5$$

$$+p_3q_{31}q_{44}^2a_5+p_3q_{32}q_{44}^2a_5+p_3q_{33}q_{44}^2a_5+p_4q_{44}q_{33}^2a_5 + \frac{1}{2}p_4q_{44}q_{32}^2a_5$$

$$+\frac{1}{2}p_4q_{44}q_{31}^2a_5+p_3q_{31}q_{42}q_{44}a_5+\frac{1}{2}p_4q_{11}^2q_{42}a_5+p_2q_{21}q_{41}q_{43}a_5$$

$$+p_3q_{31}q_{44}q_{43}a_5+p_2q_{21}q_{44}q_{43}a_5+p_2q_{22}q_{43}^2a_5+p_2q_{21}q_{43}^2a_5$$

$$+p_1q_{11}\,q_{42}q_{43}a_5+p_3q_{32}q_{44}q_{43}a_5+p_3q_{33}q_{44}q_{43}a_5+p_2q_{22}q_{43}q_{44}a_5$$

$$+p_3q_{41}q_{31}q_{44}a_5+p_4q_{32}q_{33}q_{44}a_5+p_1q_{11}q_{42}^2a_5+p_3q_{32}q_{42}q_{44}a_5$$

$$+p_2q_{21}q_{42}q_{43}a_5+p_2q_{22}q_{42}q_{43}a_5+p_4q_{31}\,q_{33}q_{44}a_5+p_4q_{31}\,q_{32}q_{44}a_5$$

$$+p_1q_{11}q_{42}q_{44}a_5+p_3q_{33}q_{42}q_{44}a_5+p_2q_{22}q_{41}q_{43}a_5 + p_3q_{32}q_{41}q_{44}a_5)\,h^5f^2\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial x\partial y}$$

$$\frac{13}{120}\frac{\partial f(x,y)}{\partial x}\,\frac{\partial^2 f(x,y)}{\partial y^2}\,\frac{\partial f(x,y)}{\partial y}f(x,y)h^5 \tag{26a}$$

$$= (p_1q_{11}q_{32}^2a_4+p_1q_{32}q_{33}q_{44}a_5+p_2q_{31}q_{33}q_{44}a_5$$

$$+p_2q_{32}q_{33}q_{44}a_5+p_1q_{31}q_{32}q_{44}a_5+p_1q_{32}^2q_{44}a_5+p_1q_{44}^2q_{32}a_5 + p_3q_{33}q_{44}^2a_5$$

$$+p_2q_{33}q_{44}^2a_5+p_3q_{31}q_{44}^2a_5+p_3q_{32}q_{44}^2a_5+p_1q_{31}q_{42}q_{44}a_5 + 2p_1q_{32}q_{42}q_{44}a_5$$

$$+p_2q_{11}q_{42}q_{43}a_5+p_2q_{33}^2q_{44}a_5+p_2q_{33}q_{42}q_{44}a_5+p_3q_{11}q_{42}q_{44}a_5$$

$$+p_1q_{33}q_{42}q_{44}a_5+p_1q_{11}q_{42}^2a_5+p_2q_{33}q_{41}q_{44}a_5+p_1q_{32}q_{41}q_{44}a_5$$

$$+p_1q_{32}q_{43}q_{44}a_5+p_2q_{32}q_{43}q_{44}a_52p_2q_{33}q_{43}q_{44}a_5$$

$$+p_2q_{31}q_{43}q_{44}a_5)h^5f\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f(x,y)}{\partial x\partial y}\left(\frac{\partial f(x,y)}{\partial y}\right)^2 f(x,y)\frac{3h^5}{40} \tag{27a}$$

$$= (p_1q_{11}q_{32}q_{44}a_5+p_2q_{22}q_{33}q_{44}a_5+p_2q_{21}q_{33}q_{44}a_5$$

$$+p_4q_{11}q_{32}q_{44}a_5 + p_2q_{11}q_{22}q_{43}a_5 + p_3q_{11}q_{32}q_{44}a_5)h^5 f\left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial f(x,y)}{\partial x}\frac{\partial^3 f(x,y)}{\partial y^3}\big(f(x,y)\big)^2\frac{h^5}{40} \tag{28a}$$

$$= (\frac{1}{3}p_1q_{22}^3a_3 + \frac{1}{3}p_1q_{21}^2q_{22}a_3 + \frac{2}{3}p_1q_{21}q_{22}^2a_3$$

$$+\frac{1}{2}q_{32}^2q_{33}p_2 + q_{32}^2q_{33}p_1a_4 + \frac{1}{2}p_1q_{33}^2q_{32}a_4 + p_2q_{33}^2q_{32}a_4$$

$$+ p_2q_{31}q_{32}q_{33}a_4$$

$$+p_1q_{31}q_{32}q_{33}a_4 + \frac{1}{2}p_2q_{33}^3a_4 + \frac{1}{2}q_{32}^3p_1a_4 + \frac{1}{2}p_1q_{31}^2q_{32}a_4$$

$$+\frac{1}{2}p_2q_{31}^2q_{33}a_4$$

$$+p_1q_{32}^2q_{31}a_4 + p_2q_{33}^2q_{31}a_4 + p_1q_{41}q_{42}q_{43}a_5 + p_2q_{43}^2q_{44}a_5$$

$$+ p_3q_{44}^2q_{41}a_5$$

$$+p_1q_{41}q_{42}^2a_5 + p_3q_{42}q_{43}q_{44}a_5 + p_1q_{42}q_{43}q_{44}a_5 + p_2q_{42}q_{43}q_{44}a_5$$

$$+\frac{1}{2}p_3q_{44}^3a_5$$

$$+p_2q_{41}q_{43}q_{44}a_5 + p_3q_{41}q_{43}q_{44}a_5 + p_1q_{41}q_{42}q_{44}a_5 + p_2q_{41}q_{42}q_{43}a_5$$

$$+p_3q_{41}q_{42}q_{44}a_5 + \frac{1}{2}p_1q_{41}^2q_{42}a_5 + p_2q_{41}\,q_{43}^2a_5 + \frac{1}{2}p_3q_{43}^2q_{44}a_5$$

$$+\frac{1}{2}p_2q_{43}q_{44}^2a_5$$

$$+\frac{1}{2}p_1q_{42}q_{43}^2a_5 + \frac{1}{2}p_2q_{43}^3a_5 + \frac{1}{2}p_1q_{42}q_{44}^2a_5 + p_3q_{42}q_{44}^2a_5$$

$$+\frac{1}{2}\ p_3q_{44}q_{41}^2a_5$$

$$+\frac{1}{2}p_2q_{43}q_{42}^2a_5 + p_1q_{42}^2q_{44}a_5 + \frac{1}{2}p_3q_{42}^2q_{44}a_5 + \frac{1}{2}p_1q_{42}^2a_5$$

$$+ p_1q_{42}^2q_{43}a_5$$

$$+p_3q_{44}^2q_{43}a_5 + \frac{1}{2}p_2q_{43}q_{41}^2a_5 + p_2q_{42}q_{43}^3a_5)h^5f^2\frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y^3}$$

$$\frac{\partial^4 f(x,y)}{\partial y^4}\left(f(x,y)\right)^4 \frac{h^5}{120} \tag{29a}$$

$$= (\frac{1}{6}q_{21}^3 q_{22}a_3 + \frac{1}{4}q_{21}^2 q_{22}^2 a_3 + \frac{1}{6}q_{21}q_{22}^3 a_3 + \frac{1}{24}q_{22}^2 a_3 + \frac{1}{24}q_{21}^2 a_3$$

$$+\frac{1}{24}q_{32}^4 a_4 + \frac{1}{24}q_{31}^4 a_4 + \frac{1}{4}q_{32}^2 q_{33}^2 a_4 + \frac{1}{6}q_{32}q_{33}^3 a_4 + \frac{1}{6}q_{31}q_{33}^3 a_4 + \frac{1}{6}q_{32}^3 q_{33}a_4$$

$$+\frac{1}{2}q_{31}q_{32}q_{33}^2 a_4 + \frac{1}{2}q_{31}q_{33}q_{32}^2 a_4 + \frac{1}{2}q_{31}^2 q_{32}q_{33}a_4 + \frac{1}{6}q_{31}^3 q_{33}a_4 + \frac{1}{6}q_{31}^3 q_{32}a_4$$

$$+\frac{1}{24}q_{33}^4 a_4 + \frac{1}{4}q_{31}^2 q_{32}^2 a_4 + a_4 + \frac{1}{4}q_{31}^2 q_{33}^2 a_4 + q_{41}q_{42}q_{43}q_{44}a_5 + \frac{1}{24}q_{43}^4 a_5$$

$$+\frac{1}{6}q_{41}^3 q_{43}a_5 + \frac{1}{6}q_{41}^3 q_{44}a_5 + \frac{1}{6}q_{41}^3 q_{42}a_5 + \frac{1}{6}q_{44}^3 q_{43}a_5 + \frac{1}{2}q_{41}^2 q_{42}q_{44}a_5$$

$$+\frac{1}{2}q_{42}^2 q_{41}q_{43}a_5 + \frac{1}{2}q_{43}^2 q_{42}q_{41}a_5 + \frac{1}{2}q_{41}^2 q_{42}q_{43}a_5 + +\frac{1}{2}q_{42}^2 q_{44}q_{43}a_5$$

$$+\frac{1}{2}q_{43}^2 q_{44}q_{42}a_5 + \frac{1}{6}q_{44}^3 q_{42}a_5 + \frac{1}{4}q_{44}^2 q_{42}^2 a_5 + \frac{1}{6}q_{42}^3 q_{41}a_5 + \frac{1}{6}q_{42}^3 q_{43}a_5$$

$$+\frac{1}{6}q_{43}^3 q_{42}a_5 + \frac{1}{6}q_{42}^3 q_{44}a_5 + \frac{1}{4}q_{43}^2 q_{42}^2 a_5 + \frac{1}{6}q_{43}^3 q_{44}a_5 + \frac{1}{4}q_{43}^2 q_{44}^2 a_5$$

$$+\frac{1}{6}q_{43}^3 q_{41}a_5 + \frac{1}{6}q_{44}^3 q_{41}a_5 + \frac{1}{2}q_{44}^2 q_{42}q_{41}a_5 + \frac{1}{24}q_{41}^4 a_5$$

$$+\frac{1}{4}q_{41}^2 q_{43}^2 a_5 + \frac{1}{4}q_{41}^2 q_{44}^2 a_5 + \frac{1}{24}q_{44}^4 a_5 + \frac{1}{24}q_{42}^4 a_5 + \frac{1}{2}q_{42}q_{43}q_{44}^2 a_5$$

$$+\frac{1}{2}q_{44}q_{43}q_{41}^2 a_5 + \frac{1}{2}q_{41}q_{43}q_{44}^2 a_5 + \frac{1}{2}q_{41}q_{44}q_{42}^2 a_5)h^5 f^4 \frac{\partial^4 f}{\partial y^4}$$

$$\left(\frac{\partial^2 f(x,y)}{\partial y^2}\right)^2 (f(x,y))^3 \frac{h^5}{30} \tag{30a}$$

$$= (\frac{1}{2}q_{11}^2 q_{32}^2 a_4 + \frac{1}{2}q_{31}^2 q_{44}^2 a_5 + \frac{1}{2}q_{32}^2 q_{44}^2 a_5 + \frac{1}{2}q_{33}^2 q_{44}^2 a_5$$

$$\frac{1}{2}q_{32}^2 q_{43} q_{44} a_5 + \frac{1}{2}q_{33}^2 q_{43} q_{44} a_5 + q_{41} q_{44} q_{31} q_{32} a_5 + q_{41} q_{44} q_{31} q_{33} a_5$$

$$+ q_{41} q_{44} q_{32} q_{33} a_5 + q_{43} q_{44} q_{31} q_{32} a_5 + q_{43} q_{44} q_{31} q_{33} a_5$$

$$+ q_{43} q_{44} q_{32} q_{33} a_5$$

$$+ \frac{1}{2}q_{11}^2 q_{42} q_{43} a_5 + q_{31} q_{32} q_{42} q_{44} a_5 + q_{31} q_{33} q_{42} q_{44} a_5 + q_{31} q_{33} q_{44}^2 a_5$$

$$+ q_{31} q_{32} q_{44}^2 a_5 + q_{32} q_{33} q_{42} q_{43} a_5 + \frac{1}{2}q_{11}^2 q_{41} q_{42} a_5 + \frac{1}{2}q_{11}^2 q_{42}^2 a_5$$

$$+ \frac{1}{2}q_{33}^2 q_{42} q_{44} a_5 + \frac{1}{2}q_{11}^2 q_{42} q_{44} a_5 + \frac{1}{2}q_{32}^2 q_{42} q_{44} a_5 + \frac{1}{2}q_{31}^2 q_{42} q_{44} a_5$$

$$+ \frac{1}{2}q_{31}^2 q_{43} q_{44} a_5 + \frac{1}{2}q_{31}^2 q_{41} q_{44} a_5 + \frac{1}{2}q_{32}^2 q_{41} q_{44} a_5$$

$$+ q_{32} q_{33} q_{44}^2 a_5) h^5 f^3 \left(\frac{\partial^2 f}{\partial y^2}\right)^2$$

$$\frac{\partial f(x,y)}{\partial y}\frac{\partial^3 f(x,y)}{\partial y^3}(f(x,y))^3 \frac{h^5}{30} \tag{31a}$$

$$= (\frac{1}{3}q_{11} q_{22}^2 a_3 + \frac{1}{3}q_{21}^2 q_{22} q_{11} a_3 + \frac{2}{3}q_{11} q_{21} q_{22}^2 a_3$$

$$+ q_{32}^2 q_{33} q_{11} a_4 + \frac{1}{3}q_{21}^2 q_{22} q_{33} a_4 + \frac{1}{3}q_{22}^2 q_{21} q_{33} a_4 + \frac{1}{2}q_{11} q_{33}^2 q_{32} a_4$$

$$+ q_{11} q_{31} q_{32} q_{33} a_4$$

$$+ \frac{1}{2}q_{11} q_{32} q_{31}^2 a_4 + q_{11} q_{31} q_{32}^2 a_4 + + \frac{1}{2}q_{31} q_{33}^2 q_{44} a_5 + + \frac{1}{2}q_{32} q_{43}^2 q_{44} a_5$$

$$+ \frac{1}{2}q_{31} q_{32}^2 q_{44} a_5 + q_{31} q_{41} q_{44}^2 a_5 + \frac{1}{2}q_{33} q_{43}^2 q_{44} a_5 + \frac{1}{2}q_{33} q_{31}^2 q_{44} a_5$$

$$+ \frac{1}{2}q_{32}^2 q_{33}\, q_{44} a_5 + + q_{32} q_{41} q_{44}^2 a_5 + \frac{1}{2}q_{31}^2 q_{32} q_{44} a_5 + q_{33} q_{41} q_{44}^2 a_5$$

$$+ \frac{1}{2}q_{31} q_{44}^3 a_5$$

$$+ \frac{1}{3}q_{21}^2 q_{22} q_{43} a_5 + \frac{1}{3}q_{22}^2 q_{21} q_{43} a_5 + q_{11} q_{42}^2 q_{41} a_5 + \frac{1}{2}q_{33}^2 q_{32} q_{44} a_5$$

$$+\frac{1}{2}q_{41}^2q_{42}q_{11}a_5+q_{33}q_{42}q_{43}q_{44}a_5+\frac{1}{2}q_{33}q_{44}^3a_5+\frac{1}{2}q_{33}q_{42}^2q_{44}a_5$$
$$+\frac{1}{2}q_{32}q_{42}^2q_{44}$$
$$+\frac{1}{2}q_{32}q_{41}^2q_{44}a_5+\frac{1}{2}q_{11}q_{44}^2q_{42}a_5+q_{11}q_{41}q_{42}q_{44}a_5+q_{11}q_{42}^2q_{43}a_5$$
$$+q_{42}q_{44}^2q_{33}a_5+q_{31}q_{42}q_{44}^2a_5+q_{31}q_{41}q_{43}q_{44}a_5+q_{32}q_{41}q_{43}q_{44}a_5$$
$$+q_{33}q_{41}q_{43}q_{44}a_5+q_{33}q_{32}q_{31}q_{44}a_5+\frac{1}{2}q_{41}q_{44}q_{33}^2a_5+q_{32}q_{42}q_{44}^2a_5$$
$$+q_{11}q_{42}q_{43}q_{44}a_5+\frac{1}{2}q_{11}q_{42}q_{43}^2a_5+q_{32}\,q_{42}q_{43}q_{44}a_5$$
$$+q_{31}q_{41}q_{42}q_{44}a_5$$
$$+\frac{1}{2}q_{32}q_{44}^3a_5+\frac{1}{9}q_{22}^3\,q_{43}a_5+\frac{1}{9}q_{21}^3\,q_{43}a_5+\frac{1}{6}q_{11}^3q_{42}a_5$$
$$+q_{32}q_{43}q_{44}^2a_5a_5$$
$$+q_{11}q_{42}^2q_{44}+\frac{1}{6}q_{31}^2q_{44}a_5+\frac{1}{6}q_{33}^2q_{44}a_5+\frac{1}{2}q_{31}q_{42}^2q_{44}a_5$$
$$+\frac{1}{6}q_{32}^2q_{44}a_5q_{44}^2q_{43}q_{33}a_5+q_{31}q_{42}q_{43}q_{44}a_5+q_{33}q_{41}q_{42}q_{44}a_5$$
$$+q_{31}q_{43}q_{44}^2a_5$$
$$+\frac{1}{2}q_{41}^2q_{44}q_{31}a_5+q_{32}q_{41}q_{42}q_{44}a_5+\frac{1}{2}q_{11}q_{42}^3a_5$$
$$+q_{11}q_{41}q_{42}q_{43}a_5)h^5f^3\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}$$

$$\left(\frac{\partial f(x,y)}{\partial y}\right)^2\frac{\partial^2 f(x,y)}{\partial y^2}\big(f(x,y)\big)^2\frac{11h^5}{120} \tag{32a}$$
$$=(\frac{1}{2}q_{11}^2q_{32}^2a_4+q_{11}q_{32}q_{33}q_{44}a_5+q_{11}q_{32}^2q_{44}a_5$$
$$+\frac{1}{2}q_{11}^2q_{32}q_{44}a_5$$
$$+q_{31}q_{32}q_{44}^2a_5+q_{31}q_{33}q_{44}^2a_5+q_{32}q_{33}q_{44}^2a_5+q_{11}q_{32}q_{41}q_{44}a_5$$
$$+q_{11}q_{32}q_{43}q_{44}a_5+q_{11}q_{31}q_{42}q_{44}a_5+2q_{11}q_{32}q_{42}q_{44}a_5$$
$$+q_{11}\,q_{31}q_{32}q_{44}a_5+\frac{1}{2}q_{33}^2q_{44}^2a_5+\frac{1}{2}q_{11}^2q_{42}^2a_5+q_{11}q_{33}q_{42}q_{44}a_5$$
$$+\frac{1}{2}q_{31}^2q_{44}^2a_5)h^5f^2\frac{\partial^2 f}{\partial y^2}\left(\frac{\partial f}{\partial y}\right)^2$$

Maka akan diperoleh persamaan persamaan sebagai berikut

$$a_1 + a_2 + a_3 + a_4 + a_5 = 1 \quad (1b)$$

$$(a_2 p_1 + a_3(p_2 + q_{21} + q_{22}) + p_4 a_5) = \frac{1}{2} \quad (2b)$$

$$(a_2 q_{11} + (q_{31} + q_{32} + q_{33})a_4 + (q_{41} + q_{42} + q_{43} + q_{44})a_5) = \frac{1}{2} \quad (3b)$$

$$\left(a_2 \frac{1}{2} p_1^2 + a_3 \frac{1}{2} p_2^2 + \frac{1}{2} p_3^2 a_4 + \frac{1}{2} p_4^2 a_5\right) = \frac{1}{6} \quad (4b)$$

$$a_2 p_1 q_{11} + a_3 h^3 p_2 q_{21} + a_3 p_2 q_{22} + p_3 q_{31} a_4 + p_3 q_{32} a_4 + p_3 q_{33} a_4 + p_4 q_{41} a_5 + p_4 q_{42} a_5 + p_4 q_{43} a_5 + p_4 q_{44} a_5 = \frac{1}{3} \quad (5b)$$

$$(a_2 \frac{1}{2} q_{11}^2 + \frac{1}{2} q_{33}^2 a_4 + \frac{1}{2} q_{32}^2 a_4 + \frac{1}{2} q_{31}^2 a_4 + q_{31} q_{32} a_4 + q_{31} q_{33} a_4 + q_{32} q_{33} a_4 + \frac{1}{2} q_{44}^2 a_5 + \frac{1}{2} q_{42}^2 a_5 + q_{41} q_{42} a_5 + q_{42} q_{44} a_5 + q_{42} q_{43} a_5 + \frac{1}{2} q_{41}^2 a_5 + \frac{1}{2} q_{43}^2 a_5 + q_{41} q_{43} a_5 + q_{41} q_{44} a_5 + q_{43} q_{44} a_5) = \frac{1}{6} \quad (6b)$$

$$(p_1 q_{32} a_4 + p_2 q_{33} a_4 + p_1 q_{42} a_5 + p_2 q_{43} a_5 + p_3 q_{44} a_5) = \frac{1}{6} \quad (7b)$$

$$(q_{11} q_{32} a_4 + q_{11} q_{42} a_5 + q_{31} q_{44} a_5 + q_{32} q_{44} a_5 + q_{33} q_{44} a_5) = \frac{1}{6} \quad (8b)$$

$$\left(a_2 \tfrac{1}{6} p_1^3 + \tfrac{1}{9} p_2^3 a_3 + \tfrac{1}{6} p_3^3 a_4 + \tfrac{1}{6} p_4^3 a_5\right) = \tfrac{1}{24} \quad (9b)$$

$$(p_1 p_2 q_{22} a_3 + p_1 p_3 q_{32} a_4 + p_2 p_3 q_{33} a_4 + p_2 p_4 q_{43} a_5) = 1/8 \quad (10b)$$

$$\left(\frac{1}{2} p_1^2 q_{32} a_4 + \frac{1}{2} p_2^2 q_{33} a_4 + \frac{1}{2} p_3^2 q_{44} a_5 + \frac{1}{2} p_1^2 q_{42} a_5 + \frac{1}{2} p_2^2 q_{43} a_5\right) = \frac{1}{24} \quad (11b)$$

$$(p_1 q_{32}^2 a_4 + p_2 q_{33}^2 a_4 + p_1 q_{31} q_{32} a_4 + p_1 q_{32} q_{33} a_4 + p_2 q_{31} q_{33} a_4 + p_2 q_{32} q_{33} a_4 \quad (12b)$$

$$+p_3q_{44}^2\,a_5 + p_1q_{42}^2a_5 + p_2q_{43}^2a_5 + p_1q_{42}q_{43}a_5 + p_1q_{42}q_{44}a_5 + p_2q_{42}q_{43}a_5$$

$$+p_3q_{42}q_{44}a_5 + p_2q_{43}q_{44}a_5 + p_3q_{43}q_{44}a_5 + p_1q_{41}q_{42}a_5 + p_2q_{41}q_{43}a_5 + p_3q_{41}q_{44}a_5) = \frac{1}{8}$$

$$(p_2q_{22}q_{11}a_3 + p_2q_{21}q_{33}a_4 + p_2q_{22}q_{33}a_4 + p_1q_{11}q_{32}a_4 + p_3q_{33}q_{44}a_5 + p_2q_{21}q_{43}a_5 \quad (13b)$$

$$+p_2q_{22}q_{43}a_5 + p_1q_{11}q_{42}a_5 + p_4q_{11}q_{42}a_5 + p_4q_{31}q_{44}a_5 + p_4q_{32}q_{44}a_5 + p_4q_{33}q_{44}a_5$$

$$+p_3q_{31}q_{44}a_5 + p_3q_{32}q_{44}a_5) = \frac{5}{24}$$

$$(p_1q_{32}q_{44}a_5) = \frac{1}{24} \quad (14b)$$

$$(a_2\frac{1}{6}q_{11}^3 + \frac{1}{9}q_{22}^3a_3 + \frac{1}{3}q_{21}^2q_{22}a_3 + \frac{1}{3}q_{22}^2q_{21}a_3 + \frac{1}{9}q_{21}^3a_3 + \frac{1}{2}q_{32}q_{33}^2a_4 \quad (15b)$$

$$+\frac{1}{2}q_{33}q_{32}^2a_4 + \frac{1}{2}q_{31}^2q_{32}a_4 + \frac{1}{2}q_{31}^2q_{33}a_4 + \frac{1}{2}q_{32}^2q_{31}a_4 + \frac{1}{2}q_{33}^2q_{31}a_4 + \frac{1}{6}q_{32}^3a_4$$

$$+\frac{1}{6}q_{33}^3a_4 + \frac{1}{6}q_{31}^3a_4 + q_{31}q_{32}q_{33}a_4 + \frac{1}{6}q_{44}^2a_5 + q_{41}q_{42}q_{43}a_5 + \frac{1}{2}q_{42}q_{44}^2a_5$$

$$+\frac{1}{2}q_{42}q_{43}^2a_5 + \frac{1}{2}q_{44}q_{42}^2a_5 + \frac{1}{2}q_{42}q_{41}^2a_5 + \frac{1}{2}q_{43}q_{42}^2a_5 + \frac{1}{2}q_{41}q_{43}^2a_5 + \frac{1}{2}q_{41}q_{44}^2a_5$$

$$+\frac{1}{2}q_{41}q_{42}^2a_5 + \frac{1}{6}q_{41}^3a_5 + q_{42}q_{43}q_{44}a_5 + \frac{1}{6}q_{43}^2a_5 + \frac{1}{2}q_{43}^2q_{44}a_5 + \frac{1}{6}q_{42}^3a_5$$

$$+q_{41}q_{42}q_{44}a_5 + q_{41}q_{43}q_{44}a_5 + \frac{1}{2}q_{43}q_{41}^2a_5 + \frac{1}{2}q_{43}q_{44}^2a_5 + p_2q_{33}q_{44}a_5) = \frac{1}{24}$$

$$(q_{11}q_{32}^2a_4 + q_{11}q_{31}q_{32}a_4 + \frac{1}{2}q_{11}^2q_{32}a_4 + q_{11}q_{32}q_{33}a_4 + \frac{1}{2}q_{33}^2q_{44}a_5 + \frac{1}{2}q_{11}^2q_{42}a_5 \quad (16b)$$

$$+q_{44}^2q_{31}a_5 + q_{44}^2q_{32}a_5 + q_{44}^2q_{33}a_5 + q_{42}^2q_{11}a_5 + \frac{1}{2}q_{32}^2q_{44}a_5 + q_{11}q_{42}q_{43}a_5$$

$$+q_{11}q_{42}q_{43}a_5 + q_{31}q_{42}q_{44}a_5 + q_{32}q_{42}q_{44}a_5 + q_{33}q_{42}q_{44}a_5 + q_{31}q_{43}q_{44}a_5$$

$$+q_{32}q_{43}q_{44}a_5 + q_{33}q_{43}q_{44}a_5 + q_{11}q_{41}q_{42}a_5 + q_{31}q_{41}q_{44}a_5 + q_{32}q_{41}q_{44}a_5$$

$$+q_{33}q_{41}q_{44}a_5 + q_{31}q_{32}q_{44}a_5 + q_{31}q_{33}q_{44}a_5 + q_{32}q_{33}q_{44}a_5) = \frac{1}{6}$$

$$\left(a_2\frac{1}{24}p_1^4 + \frac{1}{24}p_2^4a_3 + \frac{1}{24}p_3^4a_4 + \frac{1}{24}p_4^4a_5\right) = \frac{1}{120} \quad (17b)$$

$$\left(\frac{1}{2}p_1^2p_2q_{22}a_3 + \frac{1}{2}p_2^2p_3q_{33}a_4 + \frac{1}{2}p_1^2p_3q_{32}a_4 + \frac{1}{2}p_1^2p_4q_{42}a_5 + \frac{1}{2}p_3^2p_4q_{44}a_5 + \frac{1}{2}p_2^2p_4q_{43}a_5\right) = \frac{1}{30} \quad (18b)$$

$$\left(\frac{1}{6}p_2^3q_{22}a_3 + \frac{1}{6}p_1^3q_{32}a_4 + \frac{1}{9}p_2^3q_{33}a_4 + \frac{1}{6}p_3^3q_{44}a_5 + \frac{1}{9}p_2^3q_{43}a_5\right) = \frac{1}{120} \quad (19b)$$

$$(a_2\frac{1}{4}p_1^2q_{11}^2 + \frac{1}{2}p_1^2q_{32}^2a_4 + \frac{1}{2}p_2^2q_{33}^2a_4 + \frac{1}{2}p_1^2q_{31}q_{32}a_4 + \frac{1}{2}p_1^2q_{32}q_{33}a_4 + \frac{1}{2}p_2^2q_{31}q_{33}a_4 \quad (20b)$$

$$+\frac{1}{2}p_2^2q_{32}q_{33}a_4 + \frac{1}{2}p_3^2q_{44}q_{43}a_5 + \frac{1}{2}p_2^2q_{44}q_{43}a_5 + \frac{1}{2}p_2^2q_{41}q_{43}a_5 + \frac{1}{2}p_1^2q_{41}q_{42}a_5 + \frac{1}{2}p_3^2q_{41}q_{44}a_5 + \frac{1}{2}p_2^2q_{43}^2a_5 + \frac{1}{2}p_1^2q_{42}^2a_5 + \frac{1}{2}p_2^2q_{42}q_{43}a_5$$

$$+\frac{1}{2}p_1^2q_{42}q_{44}a_5 + \frac{1}{2}p_3^2q_{44}^2a_5 + \frac{1}{2}p_3^2q_{42}q_{44}a_5 + \frac{1}{2}p_1^2q_{42}q_{43}a_5) = \frac{1}{30}$$

$$\left(\frac{1}{2}p_1^2q_{32}^2a_4 + \frac{1}{2}p_2^2q_{33}^2a_4 + \frac{1}{2}p_3^2q_{44}^2a_5 + p_2p_3q_{43}q_{44}a_5 + \frac{1}{2}p_2^2q_{43}^2a_5 + p_1p_2q_{42}q_{43}a_5 + p_1p_3q_{42}q_{44}a_5\right) = \frac{1}{40} \quad (21b)$$

$$(p_1p_2q_{11}q_{22}a_3 + p_1p_3q_{11}q_{32}a_4 + p_2p_3q_{21}q_{33}a_4 + p_2p_3q_{22}q_{33}a_4 + p_4p_3q_{31}q_{44}a_5 + p_4p_3q_{32}q_{44}a_5 + p_4p_3q_{33}q_{44}a_5 + p_4p_2q_{22}q_{43}a_5 + p_4p_1q_{11}q_{42}a_5 + p_2p_4q_{21}q_{43}a_5) = \frac{1}{15} \quad (22b)$$

$$(p_1p_2q_{22}q_{33}a_4 + p_1p_3q_{32}q_{44}a_5 + p_2p_3q_{33}q_{44}a_5 + p_1p_2q_{22}q_{43}a_5 + p_4p_2q_{33}q_{44}a_5 + p_4p_1q_{32}q_{44}a_5) = \frac{7}{120} \quad (23b)$$

$$\left(a_2\frac{1}{6}p_1q_{11}\right) = \frac{1}{40} \quad (24b)$$

$$\left(\frac{1}{2}p_2q_{11}^2q_{22}a_3 + \frac{1}{2}p_3q_{11}^2q_{32}a_4 + p_1q_{11}q_{32}^2a_4 + p_2q_{21}q_{33}^2a_4 + p_2q_{22}q_{33}^2a_4 + p_1q_{11}q_{31}q_{32} + p_1q_{11}q_{32}q_{33}a_4 + p_2q_{21}q_{31}q_{33}a_4 + p_2q_{21}q_{32}q_{33}a_4 + p_2q_{22}q_{31}q_{33}a_4 + p_2q_{22}q_{32}q_{33}a_4 + p_1q_{11}q_{41}q_{42}a_5 + p_3q_{31}q_{44}^2a_5 + p_3q_{32}q_{44}^2a_5 + p_3q_{33}q_{44}^2a_5 + p_4q_{44}q_{33}^2a_5 + \frac{1}{2}p_4q_{44}q_{32}^2a_5 + \frac{1}{2}p_4q_{44}q_{31}^2a_5 + p_3q_{31}q_{42}q_{44}a_5 + \frac{1}{2}p_4q_{11}^2q_{42}a_5 + p_2q_{21}q_{41}q_{43}a_5 + p_3q_{31}q_{44}q_{43}a_5 + p_2q_{21}q_{44}q_{43}a_5 + p_2q_{22}q_{43}^2a_5 + p_2q_{21}q_{43}^2a_5 + p_1q_{11}\,q_{42}q_{43}a_5 + p_3q_{32}q_{44}q_{43}a_5 + p_3q_{33}q_{44}q_{43}a_5 + p_2q_{22}q_{43}q_{44}a_5 + p_3q_{41}q_{31}q_{44}a_5 + p_4q_{32}q_{33}q_{44}a_5 + p_1q_{11}q_{42}^2a_5 + p_3q_{32}q_{42}q_{44}a_5 + p_2q_{21}q_{42}q_{43}a_5 + p_2q_{22}q_{42}q_{43}a_5 + p_4q_{31}\,q_{33}q_{44}a_5 + p_4q_{31}\,q_{32}q_{44}a_5 + p_1q_{11}q_{42}q_{44}a_5 + p_3q_{33}q_{42}q_{44}a_5 + p_2q_{22}q_{41}q_{43}a_5 + p_3q_{32}q_{41}q_{44}a_5) = \frac{1}{10} \quad (25b)$$

$$(p_1q_{11}q_{32}^2a_4 + p_1q_{32}q_{33}q_{44}a_5 + p_2q_{31}q_{33}q_{44}a_5 + p_2q_{32}q_{33}q_{44}a_5 + p_1q_{31}q_{32}q_{44}a_5 + p_1q_{32}^2q_{44}a_5 + p_1q_{44}^2q_{32}a_5 + p_3q_{33}q_{44}^2a_5 + p_2q_{33}q_{44}^2a_5 + p_3q_{31}q_{44}^2a_5 + p_3q_{32}q_{44}^2a_5 + p_1q_{31}q_{42}q_{44}a_5 + 2p_1q_{32}q_{42}q_{44}a_5 + p_2q_{11}q_{42}q_{43}a_5 + p_2q_{33}^2q_{44}a_5 + p_2q_{33}q_{42}q_{44}a_5 + p_3q_{11}q_{42}q_{44}a_5 + p_1q_{33}q_{42}q_{44}a_5 + p_1q_{11}q_{42}^2a_5 + p_2q_{33}q_{41}q_{44}a_5 + p_1q_{32}q_{41}q_{44}a_5 + p_1q_{32}q_{43}q_{44}a_5 + p_2q_{32}q_{43}q_{44}a_5 \quad (26b)$$

$$+2p_2q_{33}q_{43}q_{44}a_5 + p_2q_{31}q_{43}q_{44}a_5) = \frac{3}{120}$$

$$(p_1q_{11}q_{32}q_{44}a_5 + p_2q_{22}q_{33}q_{44}a_5 + p_2q_{21}q_{33}q_{44}a_5 + p_4q_{11}q_{32}q_{44}a_5 \quad (27b)$$
$$+ p_2q_{11}q_{22}q_{43}a_5 + p_3q_{11}q_{32}q_{44}a_5) = \frac{3}{40}$$

$$(\frac{1}{3}p_1q_{22}^3a_3 + \frac{1}{3}p_1q_{21}^2q_{22}a_3 + \frac{2}{3}p_1q_{21}q_{22}^2a_3 + \frac{1}{2}q_{32}^2q_{33}p_2 + q_{32}^2q_{33}p_1a_4 \quad (28b)$$
$$+\frac{1}{2}p_1q_{33}^2q_{32}a_4 + p_2q_{33}^2q_{32}a_4 + p_2q_{31}q_{32}q_{33}a_4 + p_1q_{31}q_{32}q_{33}a_4$$
$$+\frac{1}{2}p_2q_{33}^3a_4$$
$$+\frac{1}{2}q_{32}^3p_1a_4 + \frac{1}{2}p_1q_{31}^2q_{32}a_4 + \frac{1}{2}p_2q_{31}^2q_{33}a_4 + p_1q_{32}^2q_{31}a_4$$
$$+ p_2q_{33}^2q_{31}a_4$$
$$+p_1q_{41}q_{42}q_{43}a_5 + p_2q_{43}^2q_{44}a_5 + p_3q_{44}^2q_{41}a_5 + p_1q_{41}q_{42}^2a_5$$
$$+ p_3q_{42}q_{43}q_{44}a_5$$
$$+p_2q_{41}q_{43}q_{44}a_5 + p_3q_{41}q_{43}q_{44}a_5 + p_1q_{41}q_{42}q_{44}a_5 + p_2q_{41}q_{42}q_{43}a_5$$
$$+p_3q_{41}q_{42}q_{44}a_5 + \frac{1}{2}p_1q_{41}^2q_{42}a_5 + p_2q_{41}\,q_{43}^2a_5 + \frac{1}{2}p_3q_{43}^2q_{44}a_5$$
$$+\frac{1}{2}p_2q_{43}q_{44}^2a_5$$
$$+\frac{1}{2}p_1q_{42}q_{43}^2a_5 + \frac{1}{2}p_2q_{43}^3a_5 + \frac{1}{2}p_1q_{42}q_{44}^2a_5 + p_3q_{42}q_{44}^2a_5$$
$$+\frac{1}{2}\ p_3q_{44}q_{41}^2a_5$$
$$+\frac{1}{2}p_2q_{43}q_{42}^2a_5 + p_1q_{42}^2q_{44}a_5 + \frac{1}{2}p_3q_{42}^2q_{44}a_5 + \frac{1}{2}p_1q_{42}^2a_5$$
$$+ p_1q_{42}^2q_{43}a_5$$
$$+p_1q_{42}q_{43}q_{44}a_5 + p_2q_{42}q_{43}q_{44}a_5 + \frac{1}{2}p_3q_{44}^3a_5 + p_3q_{44}^2q_{43}a_5$$
$$+\frac{1}{2}p_2q_{43}q_{41}^2a_5 + p_2q_{42}q_{43}^3a_5) = \frac{1}{40}$$

$$(\frac{1}{6}q_{21}^3q_{22}a_3 + \frac{1}{4}q_{21}^2q_{22}^2a_3 + \frac{1}{6}q_{21}q_{22}^3a_3 + \frac{1}{24}q_{22}^2a_3 + \frac{1}{24}q_{21}^2a_3 \quad (29b)$$
$$+\frac{1}{24}q_{32}^4a_4 + \frac{1}{24}q_{31}^4a_4 + \frac{1}{4}q_{32}^2q_{33}^2a_4 + \frac{1}{6}q_{32}q_{33}^3a_4 + \frac{1}{6}q_{31}q_{33}^3a_4$$
$$+\frac{1}{6}q_{32}^3q_{33}a_4$$

$$+\frac{1}{2}q_{31}q_{32}q_{33}^2a_4+\frac{1}{2}q_{31}q_{33}q_{32}^2a_4+\frac{1}{2}q_{31}^2q_{32}q_{33}a_4+\frac{1}{6}q_{31}^3q_{33}a_4+\frac{1}{6}q_{31}^3q_{32}a_4$$

$$+\frac{1}{24}q_{33}^4a_4+\frac{1}{4}q_{31}^2q_{32}^2a_4+a_4+\frac{1}{4}q_{31}^2q_{33}^2a_4+q_{41}q_{42}q_{43}q_{44}a_5+\frac{1}{24}q_{43}^4a_5$$

$$+\frac{1}{6}q_{41}^3q_{43}a_5+\frac{1}{6}q_{41}^3q_{44}a_5+\frac{1}{6}q_{41}^3q_{42}a_5+\frac{1}{6}q_{44}^3q_{43}a_5+\frac{1}{2}q_{41}^2q_{42}q_{44}a_5$$

$$+\frac{1}{2}q_{42}^2q_{41}q_{43}a_5+\frac{1}{2}q_{43}^2q_{42}q_{41}a_5+\frac{1}{2}q_{41}^2q_{42}q_{43}a_5++\frac{1}{2}q_{42}^2q_{44}q_{43}a_5$$

$$+\frac{1}{2}q_{43}^2q_{44}q_{42}a_5+\frac{1}{6}q_{44}^3q_{42}a_5+\frac{1}{4}q_{44}^2q_{42}^2a_5+\frac{1}{6}q_{42}^3q_{41}a_5+\frac{1}{6}q_{42}^3q_{43}a_5$$

$$+\frac{1}{6}q_{43}^3q_{42}a_5+\frac{1}{6}q_{42}^3q_{44}a_5+\frac{1}{4}q_{43}^2q_{42}^2a_5+\frac{1}{6}q_{43}^3q_{44}a_5+\frac{1}{4}q_{43}^2q_{44}^2a_5$$

$$+\frac{1}{6}q_{43}^3q_{41}a_5+\frac{1}{6}q_{44}^3q_{41}a_5+\frac{1}{2}q_{44}^2q_{42}q_{41}a_5+\frac{1}{24}q_{41}^4a_5$$

$$+\frac{1}{4}q_{41}^2q_{43}^2a_5+\frac{1}{4}q_{41}^2q_{44}^2a_5+\frac{1}{24}q_{44}^4a_5+\frac{1}{24}q_{42}^4a_5+\frac{1}{2}q_{42}q_{43}q_{44}^2a_5$$

$$+\frac{1}{2}q_{44}q_{43}q_{41}^2a_5+\frac{1}{2}q_{41}q_{43}q_{44}^2a_5+\frac{1}{2}q_{41}q_{44}q_{42}^2a_5)=\frac{1}{20}$$

(30b)

$$(\frac{1}{2}q_{11}^2q_{32}^2a_4+\frac{1}{2}q_{31}^2q_{44}^2a_5+\frac{1}{2}q_{32}^2q_{44}^2a_5+\frac{1}{2}q_{33}^2q_{44}^2a_5$$

$$\frac{1}{2}q_{32}^2q_{43}q_{44}a_5+\frac{1}{2}q_{33}^2q_{43}q_{44}a_5+q_{41}q_{44}q_{31}q_{32}a_5+q_{41}q_{44}q_{31}q_{33}a_5$$

$$+q_{41}q_{44}q_{32}q_{33}a_5+q_{43}q_{44}q_{31}q_{32}a_5+q_{43}q_{44}q_{31}q_{33}a_5$$

$$+q_{43}q_{44}q_{32}q_{33}a_5$$

$$+\frac{1}{2}q_{11}^2q_{42}q_{43}a_5+q_{31}q_{32}q_{42}q_{44}a_5+q_{31}q_{33}q_{42}q_{44}a_5+q_{31}q_{33}q_{44}^2a_5$$

$$+q_{31}q_{32}q_{44}^2a_5+q_{32}q_{33}q_{42}q_{43}a_5+\frac{1}{2}q_{11}^2q_{41}q_{42}a_5+\frac{1}{2}q_{11}^2q_{42}^2a_5$$

$$+\frac{1}{2}q_{33}^2q_{42}q_{44}a_5+\frac{1}{2}q_{11}^2q_{42}q_{44}a_5+\frac{1}{2}q_{32}^2q_{42}q_{44}a_5+\frac{1}{2}q_{31}^2q_{42}q_{44}a_5$$

$$+\frac{1}{2}q_{31}^2q_{43}q_{44}a_5+\frac{1}{2}q_{31}^2q_{41}q_{44}a_5+\frac{1}{2}q_{32}^2q_{41}q_{44}a_5+q_{32}q_{33}q_{44}^2a_5)$$

$$=\frac{1}{30}$$

$$= (\frac{1}{3}q_{11}q_{22}^2a_3 + \frac{1}{3}q_{21}^2q_{22}q_{11}a_3 + \frac{2}{3}q_{11}q_{21}q_{22}^2a_3 \tag{31b}$$

$$+q_{32}^2q_{33}q_{11}a_4 + \frac{1}{3}q_{21}^2q_{22}q_{33}a_4 + \frac{1}{3}q_{22}^2q_{21}q_{33}a_4 + \frac{1}{2}q_{11}q_{33}^2q_{32}a_4$$
$$+ q_{11}q_{31}q_{32}q_{33}a_4$$

$$+\frac{1}{2}q_{11}q_{32}q_{31}^2a_4 + q_{11}q_{31}q_{32}^2a_4 + +\frac{1}{2}q_{31}q_{33}^2q_{44}a_5 + +\frac{1}{2}q_{32}q_{43}^2q_{44}a_5$$

$$+\frac{1}{2}q_{31}q_{32}^2q_{44}a_5 + q_{31}q_{41}q_{44}^2a_5 + \frac{1}{2}q_{33}q_{43}^2q_{44}a_5 + \frac{1}{2}q_{33}q_{31}^2q_{44}a_5$$

$$+\frac{1}{2}q_{32}^2q_{33}\,q_{44}a_5 + +q_{32}q_{41}q_{44}^2a_5 + \frac{1}{2}q_{31}^2q_{32}q_{44}a_5 + q_{33}q_{41}q_{44}^2a_5$$
$$+\frac{1}{2}q_{31}q_{44}^3a_5$$

$$+\frac{1}{3}q_{21}^2q_{22}q_{43}a_5 + \frac{1}{3}q_{22}^2q_{21}q_{43}a_5 + q_{11}q_{42}^2q_{41}a_5 + \frac{1}{2}q_{33}^2q_{32}q_{44}a_5$$

$$+\frac{1}{2}q_{41}^2q_{42}q_{11}a_5 + q_{33}q_{42}q_{43}q_{44}a_5 + \frac{1}{2}q_{33}q_{44}^3a_5 + \frac{1}{2}q_{33}q_{42}^2q_{44}a_5$$
$$+\frac{1}{2}q_{32}q_{42}^2q_{44}$$

$$+\frac{1}{2}q_{32}q_{41}^2q_{44}a_5 + \frac{1}{2}q_{11}q_{44}^2q_{42}a_5 + q_{11}q_{41}q_{42}q_{44}a_5 + q_{11}q_{42}^2q_{43}a_5$$

$$+q_{42}q_{44}^2q_{33}a_5 + q_{31}q_{42}q_{44}^2a_5 + q_{31}q_{41}q_{43}q_{44}a_5 + q_{32}q_{41}q_{43}q_{44}a_5$$

$$+q_{33}q_{41}q_{43}q_{44}a_5 + q_{33}q_{32}q_{31}q_{44}a_5 + \frac{1}{2}q_{41}q_{44}q_{33}^2a_5 + q_{32}q_{42}q_{44}^2a_5$$

$$+q_{11}q_{42}q_{43}q_{44}a_5 + \frac{1}{2}q_{11}q_{42}q_{43}^2a_5 + q_{32}\,q_{42}q_{43}q_{44}a_5 + q_{31}q_{41}q_{42}q_{44}a_5$$

$$+\frac{1}{2}q_{32}q_{44}^3a_5 + \frac{1}{9}q_{22}^3\,q_{43}a_5 + \frac{1}{9}q_{21}^3\,q_{43}a_5 + \frac{1}{6}q_{11}^3q_{42}a_5$$
$$+ q_{32}q_{43}q_{44}^2a_5a_5$$

$$+q_{11}q_{42}^2q_{44} + \frac{1}{6}q_{31}^2q_{44}a_5 + \frac{1}{6}q_{33}^2q_{44}a_5 + \frac{1}{2}q_{31}q_{42}^2q_{44}a_5$$

$$+\frac{1}{6}q_{32}^2q_{44}a_5q_{44}^2q_{43}q_{33}a_5 + q_{31}q_{42}q_{43}q_{44}a_5 + q_{33}q_{41}q_{42}q_{44}a_5$$
$$+ q_{31}q_{43}q_{44}^2a_5$$

$$+\frac{1}{2}q_{41}^2q_{44}q_{31}a_5 + q_{32}q_{41}q_{42}q_{44}a_5 + \frac{1}{2}q_{11}q_{42}^3a_5 + q_{11}q_{41}q_{42}q_{43}a_5)$$
$$= \frac{1}{30}$$

$$\left(\frac{1}{2}q_{11}^2q_{32}^2a_4 + q_{11}q_{32}q_{33}q_{44}a_5 + q_{11}q_{32}^2q_{44}a_5 + \frac{1}{2}q_{11}^2q_{32}q_{44}a_5\right. \quad (32b)$$

$$+q_{31}q_{32}q_{44}^2a_5 + q_{31}q_{33}q_{44}^2a_5 + q_{32}q_{33}q_{44}^2a_5 + q_{11}q_{32}q_{41}q_{44}a_5$$

$$+q_{11}q_{32}q_{43}q_{44}a_5 + q_{11}q_{31}q_{42}q_{44}a_5 + 2q_{11}q_{32}q_{42}q_{44}a_5$$

$$+q_{11}\,q_{31}q_{32}q_{44}a_5 + \frac{1}{2}q_{33}^2q_{44}^2a_5 + \frac{1}{2}q_{11}^2q_{42}^2a_5 + q_{11}q_{33}q_{42}q_{44}a_5$$

$$\left.+\frac{1}{2}q_{31}^2q_{44}^2a_5\right) = \frac{11}{120}$$

Dari persamaan 24b didapatkan

$$a_2\frac{1}{6}p_1q_{11} = \frac{1}{40}$$

$$\frac{1}{6}a_2 = \frac{\frac{1}{40}}{p_1q_{11}}$$

$$\frac{1}{6}a_2 = \frac{1}{40p_1q_{11}}$$

$$a_2 = \frac{6}{40p_1q_{11}}$$

Dari persamaan 14b didapatkan

$$p_1q_{32}q_{44}a_5 = \frac{1}{24}$$

$$a_5 = \frac{\frac{1}{24}}{p_1q_{32}q_{44}}$$

$$a_5 = \frac{1}{24p_1q_{32}q_{44}}$$

Untuk mencari nilai a_2 dan a_5 maka dipilih nilai sebarang $p_1 = \frac{1}{2}, q_{11} = \frac{1}{4}, q_{32} = -\frac{1}{2}$ dan $q_{44} = \frac{1}{8}$ sehingga nilai a_2 dan a_5 menjadi

$$a_2 = \frac{6}{40 p_1 q_{11}}$$

$$a_2 = \frac{6}{40 \cdot \frac{1}{2} \cdot \frac{1}{4}}$$

$$a_2 = \frac{6}{5}$$

$$a_5 = \frac{1}{24 p_1 q_{32} q_{44}}$$

$$a_5 = \frac{1}{24 \cdot \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot \frac{1}{8}}$$

$$a_5 = \frac{1}{\left(-\frac{3}{4}\right)}$$

$$a_5 = -\frac{4}{3}$$

Dari persamaan 2b didapatkan

$$(a_2 p_1 + a_3(p_2 + q_{21} + q_{22}) + p_4 a_5) = \frac{1}{2}$$

Karena $a_2 = \frac{6}{5}$ dan $a_5 = -\frac{3}{4}$ maka nilai a_2 dan a_5 dapat disubtitusikan ke persamaan 2b sehingga

$$a_2 p_1 + a_3(p_2 + q_{21} + q_{22}) + p_4 a_5 = \frac{1}{2}$$

$$\frac{6}{5}p_1 + a_3(p_2 + q_{21} + q_{22}) + p_4\left(-\frac{3}{4}\right) = \frac{1}{2}$$

$$\frac{6p_1}{5} + a_3(p_2 + q_{21} + q_{22}) - \frac{3p_4}{4} = \frac{1}{2}$$

$$a_3(p_2 + q_{21} + q_{22}) = \frac{1}{2} - \frac{6p_1}{5} + \frac{3p_4}{4}$$

$$a_3(p_2 + q_{21} + q_{22}) = \frac{10 - 24p_1 + 15p_4}{20}$$

$$a_3 = \frac{\frac{10 - 24p_1 + 15p_4}{20}}{(p_2 + q_{21} + q_{22})}$$

Dipilih $p_1 = \frac{1}{2}, p_2 = \frac{1}{4}, q_{21} = \frac{1}{8}, q_{22} = \frac{1}{8}, p_4 = \frac{3}{4}$ maka nilai a_3 menjadi

$$a_3 = \frac{\frac{10 - 24p_1 + 15p_4}{20}}{(p_2 + q_{21} + q_{22})}$$

$$a_3 = \frac{10 - 24p_1 + 15p_4}{20}\left(\frac{1}{(p_2 + q_{21} + q_{22})}\right)$$

$$a_3 = \frac{10 - 24 \cdot \left(\frac{1}{2}\right) + 15 \cdot \left(\frac{3}{4}\right)}{20} \frac{1}{\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right)}$$

$$a_3 = \frac{10 - \frac{24}{2} + \frac{45}{4}}{20} \frac{1}{\frac{1}{2}}$$

$$a_3 = \frac{\frac{37}{4}}{20 \cdot \frac{1}{2}}$$

$$a_3 = \frac{\frac{37}{4}}{\frac{20}{2}}$$

$$a_3 = \frac{37}{4} \cdot \frac{2}{20}$$

$$a_3 = \frac{37}{40}$$

Dari persamaan 4b didapatkan

$$\left(a_2 \frac{1}{2} p_1^2 + a_3 \frac{1}{2} p_2^2 + \frac{1}{2} p_3^2 a_4 + \frac{1}{2} p_4^2 a_5\right) = \frac{1}{6}$$

Karena $a_2 = \frac{6}{5}, a_3 = \frac{37}{40}$ dan $a_5 = -\frac{3}{4}$ maka nilai a_2, a_3 dan a_5 dapat disubtitusikan ke persamaan 4b sehingga

$$a_2 \frac{1}{2} p_1^2 + a_3 \frac{1}{2} p_2^2 + \frac{1}{2} p_3^2 a_4 + \frac{1}{2} p_4^2 a_5 = \frac{1}{6}$$

$$\frac{6}{5}\frac{1}{2} p_1^2 + \frac{37}{40}\frac{1}{2} p_2^2 + \frac{1}{2} p_3^2 a_4 + \frac{1}{2} p_4^2 \left(-\frac{3}{4}\right) = \frac{1}{6}$$

$$\frac{3}{5} p_1^2 + \frac{37}{80} p_2^2 + \frac{1}{2} p_3^2 a_4 - \frac{3}{8} p_4^2 = \frac{1}{6}$$

$$\frac{1}{2} p_3^2 a_4 = \frac{1}{6} - \frac{3}{5} p_1^2 - \frac{37}{80} p_2^2 + \frac{3}{8} p_4^2$$

$$a_4 = \frac{\frac{1}{6} - \frac{3}{5} p_1^2 - \frac{37}{80} p_2^2 + \frac{3}{8} p_4^2}{\frac{1}{2} p_3^2}$$

Dipilih $p_1 = \frac{1}{2}, p_2 = \frac{1}{4}, p_3 = \frac{1}{2}, p_4 = \frac{3}{4}$ maka nilai a_4 menjadi

$$a_4 = \frac{\frac{1}{6} - \frac{3}{5}p_1^2 - \frac{37}{80}p_2^2 + \frac{3}{8}p_4^2}{\frac{1}{2}p_3^2}$$

$$a_4 = \frac{\frac{1}{6} - \frac{3}{5}\left(\frac{1}{2}\right)^2 - \frac{37}{80}\left(\frac{1}{4}\right)^2 + \frac{3}{8}\left(\frac{3}{4}\right)^2}{\frac{1}{2}\left(\frac{1}{2}\right)^2}$$

$$a_4 = \frac{\frac{763}{3840}}{\frac{1}{8}}$$

$$a_4 = \frac{763}{3840} \cdot \frac{8}{1}$$

$$a_4 = \frac{763}{480}$$

Dari persamaan 1b didapatkan

$$a_1 + a_2 + a_3 + a_4 + a_5 = 1$$

Karena $a_2 = \frac{6}{5}, a_3 = \frac{37}{40}, a_4 = \frac{763}{480}$ dan $a_5 = -\frac{3}{4}$ maka nilai a_2, a_3, a_4 dan a_5 dapat disubtitusikan ke persamaan 1b sehingga

$$a_1 + a_2 + a_3 + a_4 + a_5 = 1$$

$$a_1 + \frac{6}{5} + \frac{37}{40} + \frac{763}{480} - \frac{3}{4} = 1$$

$$a_1 = 1 - \frac{6}{5} - \frac{37}{40} - \frac{763}{480} + \frac{3}{4}$$

$$a_1 = -\frac{943}{480}$$

Dari persamaan 7b didapatkan

$$(p_1 q_{32} a_4 + p_2 q_{33} a_4 + p_1 q_{42} a_5 + p_2 q_{43} a_5 + p_3 q_{44} a_5) = \frac{1}{6}$$

Dengan mensubtitusikan nilai $p_{1,} q_{32}, a_4, p_2, a_5, p_3, q_{44}$ maka

$$(p_1 q_{32} a_4 + p_2 q_{33} a_4 + p_1 q_{42} a_5 + p_2 q_{43} a_5 + p_3 q_{44} a_5) = \frac{1}{6}$$

$$\left(\frac{1}{2}\left(-\frac{1}{2}\right)\frac{763}{480} + \frac{1}{4} q_{33} \frac{763}{480} + \frac{1}{2} q_{42}\left(-\frac{4}{3}\right) + \frac{6}{5} q_{43}\left(-\frac{4}{3}\right) + \frac{37}{40}\frac{1}{8}\left(-\frac{4}{3}\right)\right) = \frac{1}{6}$$

$$-\frac{353}{640} + q_{33}\frac{763}{1920} - \frac{2}{3} q_{42} - \frac{8}{5} q_{43} = \frac{1}{6}$$

$$q_{33}\frac{763}{1920} = \frac{1}{6} + \frac{2}{3} q_{42} + \frac{8}{5} q_{43} + \frac{353}{640}$$

$$q_{33} = \frac{\frac{1}{6} + \frac{2}{3} q_{42} + \frac{8}{5} q_{43} + \frac{353}{640}}{\frac{763}{1920}}$$

Dipilih nilai sebarang untuk $q_{42} = \frac{1}{2}, q_{43} = \frac{1}{4}$

$$q_{33} = \frac{\frac{1}{6} + \frac{2}{3}\frac{1}{2} + \frac{8}{5}\frac{1}{4}\frac{1}{4} + \frac{353}{640}}{\frac{763}{1920}}$$

$$q_{33} = \frac{2787}{763}$$

Karena nilai $q_{31} = q_{43}$ maka nilai $q_{31} = \frac{1}{4}$

Dari persamaan 3b didapatkan

$$(a_2 q_{11} + (q_{31} + q_{32} + q_{33}) a_4 + (q_{41} + q_{42} + q_{43} + q_{44}) a_5) = \frac{1}{2}$$

$$\left(-\frac{943}{480}\frac{1}{4}+(\frac{1}{4}+(-\frac{1}{2})+\frac{2787}{763})\frac{763}{480}+(q_{41}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8})\left(-\frac{4}{3}\right)\right)=\frac{1}{2}$$

$$\frac{4373}{1920}-\frac{4}{3}q_{41}=\frac{1}{2}$$

$$q_{41}=\frac{\frac{1}{2}-\frac{4373}{1920}}{-\frac{4}{3}}$$

$$q_{41}=\frac{3413}{2560}$$

Kemudian subtitusikan nilai koefisien $a_1, a_2, a_3, a_4, a_5, a_6$ pada persamaan 3.1, sehingga didapatkan metode Rungge kutta

$$y_{i+1}=y_i+\left(-\frac{943}{480}k_1+\frac{6}{5}k_2+\frac{37}{40}k_3+\frac{763}{480}k_4-\frac{4}{3}k_5\right)h$$

Dimana:

$k_1=f(x_i,y_i)$

$k_2=f(x_i+\frac{1}{2}h,y_i+\frac{1}{4}k_1h)$

$k_3=f(x_i+\frac{1}{4}h,y_i+\frac{1}{8}k_1h+\frac{1}{8}k_2h)$

$k_4=f(x_i+\frac{1}{2}h,y_i+\frac{1}{4}k_1h-\frac{1}{2}k_2h+\frac{2787}{763}k_3h)$

$k_5=f(x_i+\frac{3}{4}h,y_i+\frac{3413}{2560}k_1h+\frac{1}{2}k_2h+\frac{1}{4}k_3h+\frac{1}{8}k_4h)$

3.2 Penyelesaian persamaan differensial linier orde satu

Dalam subbab ini akan dibahas solusi numerik dan analitik serta simulasi dari persamaan differensial linier orde satu. Persamaan ini diselesaikan dengan menggunakan metode Runge Kutta orde lima. Persamaan ini diambil dari jurnal " Penyelesaian Persamaan Diferensial Linier Orde Satu dan Dua disertai Nilai Awal dengan menggunakan Metode Runge Kutta Orde Lima Butcher dan Felhberg (RKF45)" yang di tulis oleh Sagita dan Agus pada tahun 2018. Dalam jurnal tersebut diberikan persamaan diferensial linier orde satu sebagai berikut.

$$\frac{dy}{dx} = x + y;\ \mathrm{y}(1) = 0 \tag{3.5}$$

Persamaan (3.5) disubtitusikan pada persamaan Runge Kutta orde lima, sehingga menghasilkan

$$k_1 = f(x_i, y_i)$$

$$= x_i + y_i$$

$$k_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{4}k_1 h\right)$$

$$= x_i + \frac{1}{2}h + y_i + \frac{1}{4}(x_i + y_i)h$$

$$= x_i + \frac{1}{2}h + y_i + \frac{1}{4}x_i h + \frac{1}{4}y_i h$$

$$k_3 = f\left(x_i + \frac{1}{4}h, y_i + \frac{1}{8}k_1 h + \frac{1}{8}k_2 h\right)$$

$$= x_i + \frac{1}{4}h + y_i + \frac{1}{8}(x_i + y_i)h + \frac{1}{8}\left(x_i + \frac{1}{2}h + y_i + \frac{1}{4}x_i h + \frac{1}{4}y_i h\right)h$$

$$= x_i + \frac{1}{4}h + y_i + \frac{1}{8}x_i h + \frac{1}{8}y_i h + \frac{1}{8}x_i h + \frac{1}{16}h^2 + \frac{1}{8}y_i h + \frac{1}{32}x_i h^2 + \frac{1}{32}y_i h^2$$

$$= x_i + y_i + \frac{2}{8}x_i h + \frac{2}{8}y_i h + \frac{1}{32}x_i h^2 + \frac{1}{32}y_i h^2 + \frac{1}{4}h + \frac{1}{16}h^2$$

$$k_4 = f\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{4}k_1 h - \tfrac{1}{2}k_2 h + \tfrac{2787}{763}k_3 h\right)$$

$$= x_i + \frac{1}{2}h + y_i + \frac{1}{4}hx_i + \frac{1}{4}hy_i - \frac{1}{2}x_i h - \frac{1}{4}h^2 - \frac{1}{2}y_i h - \frac{1}{8}x_i h^2 - \frac{1}{8}y_i h^2 + \frac{2787}{763}x_i h + \frac{2787}{763}y_i h + \frac{5574}{6104}x_i h^2 + \frac{5574}{6104}y_i h^2 + \frac{2787}{24416}x_i h^3 + \frac{2787}{24416}y_i h^3 + \frac{2787}{3052}h^2 + \frac{2787}{12208}h^3$$

$$= x_i + y_i + \frac{10385}{3052}x_i h + \frac{10385}{3052}y_i h + \frac{4811}{6104}x_i h^2 + \frac{4811}{6104}y_i h^2 + \frac{2787}{24416}x_i h^3 + \frac{2787}{24416}y_i h^3 + \frac{1}{2}h + \frac{506}{763}h^2 + \frac{2787}{12208}h^3$$

$$k_5 = f\left(x_i + \frac{3}{4}h, y_i + \frac{3413}{2560}k_1 h + \frac{1}{2}k_2 h + \frac{1}{4}k_3 h + \frac{1}{8}k_4 h\right)$$

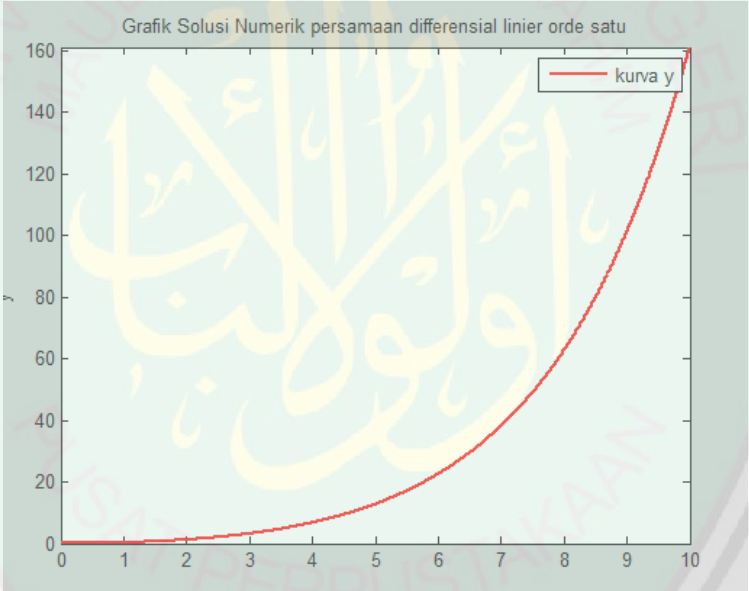
$$= x_i + \frac{3}{4}h + y_i + \frac{3413}{2560}(x_i + y_i)h + \frac{1}{2}\left(x_i + \frac{1}{2}h + y_i + \frac{1}{4}x_i h + \frac{1}{4}y_i h\right)h + \frac{1}{4}\left(x_i + y_i + \frac{2}{8}x_i h + \frac{2}{8}y_i h + \frac{1}{32}x_i h^2 + \frac{1}{32}y_i h^2 + \frac{1}{4}h + \frac{1}{16}h^2\right)h + \frac{1}{8}\left(x_i + y_i + \frac{10385}{3052}x_i h + \frac{10385}{3052}y_i h + \frac{4811}{6104}x_i h^2 + \frac{4811}{6104}y_i h^2 + \frac{2787}{24416}x_i h^3 + \frac{2787}{24416}y_i h^3 + \frac{1}{2}h + \frac{506}{763}h^2 + \frac{2787}{12208}h^3\right)h$$

$$= x_i + y_i + \frac{5653}{2560}x_i h + \frac{5653}{2560}y_i h + \frac{14963}{24416}x_i h^2 + \frac{14963}{24416}y_i h^2 + \frac{10385}{97664}x_i h^3 + \frac{10385}{97664}y_i h^3$$

$$+ \frac{2787}{195328}x_i h^4 + \frac{2787}{195328}y_i h^4 + \frac{3}{4}h + \frac{3}{8}h^2 + \frac{4811}{48832}h^3 + \frac{2787}{97664}h^4$$

Setelah menghasilkan nilai k, kemudian mensubtitusikan Nilai awal untuk y(1) = 0, nilai x berjalan dari 0 sampai 10 pada masing masing x dan y dan kemudian disubtitusikan pada persamaan y_{i+1} , untuk lebih mudahnya digunakan aplikasi Matlab untuk mengerjakan iterasi dari Runge Kutta.

Dibawah ini merupakan hasil simulasi grafik solusi numerik persamaan differensial linier orde satu menggunakan Runge Kutta orde lima.



Gambar 3.1 Grafik Solusi Numerik persamaan differensial linier orde satu

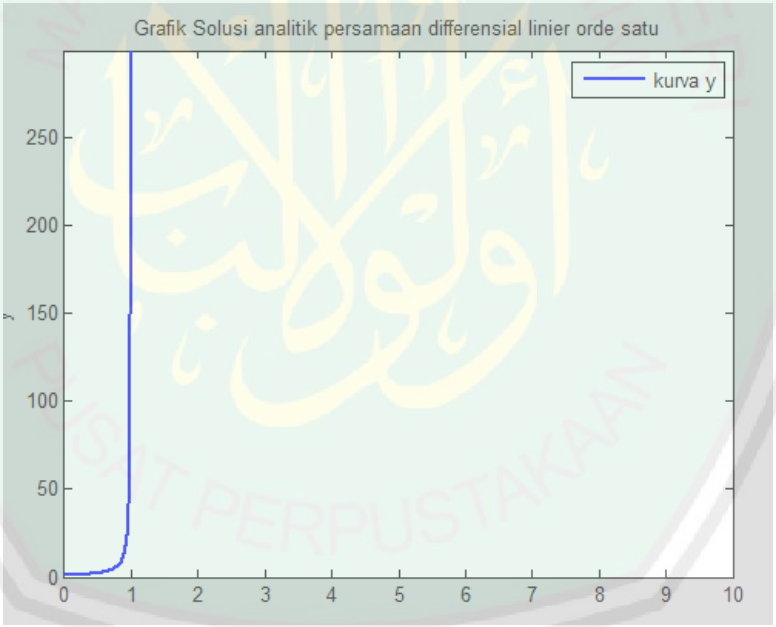
Pada gambar 3.1 dapat dilihat iterasi dilakukan dari $x = 0$ sampai $x = 10$. Dan terdapat sebanyak 2001 iterasi, dari $x = 0$ didapatkan nilai $y_i = 0$, sedangkan $x = 0.005$ didapatkan nilai $y_i = 0.000015727683649$, sedangkan $x = 0.01$ didapatkan nilai $y_i = 0.000042280748525$, sedangkan $x = 0.015$ didapatkan nilai $y_i = 0.000079682558910$, sedangkan $x = 0.020$ didapatkan nilai $y_i =$

0.000127956529513, sedangkan $x = 0.025$ didapatkan nilai $y_i = 0.000187126125580$ sedangkan $x = 0.030$ didapatkan nilai $y_i =$ 0.000257214863001, sedangkan $x = 10$ didapatkan y_i =1.610042024823768e+002.

Dari persamaan 3.5 dilakukan perhitungan secara analitik dan diperoleh hasil sebagai berikut:

$$y = \frac{\frac{1}{2}x^2+1}{1-x} \tag{3.6}$$

Dibawah ini merupakan hasil simulasi grafik solusi analitik persamaan differensial linier orde satu

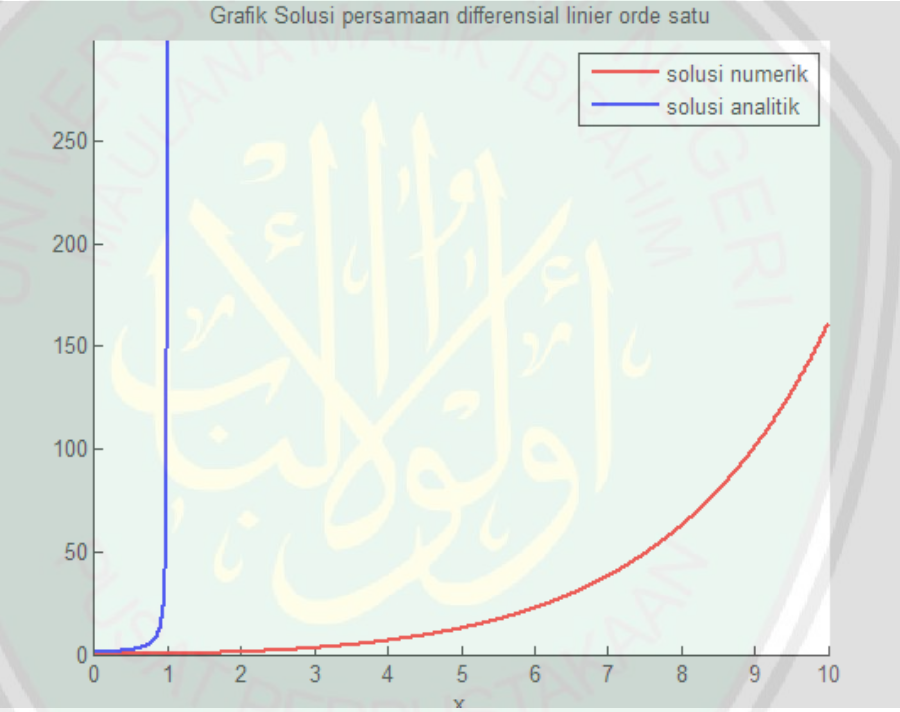


Gambar 3.2 Grafik Solusi analitik persamaan differensial linier orde satu

Pada gambar 3.2 dapat dilihat iterasi dilakukan dari $x = 0$ sampai $x = 10$. Dari $x = 0$ didapatkan nilai $y_i = 1$, sedangkan $x = 0.005$ didapatkan nilai $y_i =$ 1.005037688442211, sedangkan $x = 0.01$ didapatkan nilai y_i =1.010151515151515, sedangkan $x = 0.015$ didapatkan nilai

y_i =1.015342639593909, sedangkan $x = 0.020$ didapatkan nilai y_i = 1.020612244897959, sedangkan $x = 0.025$ didapatkan nilai y_i = 1.025961538461538 sedangkan $x = 0.030$ didapatkan nilai y_i =1.031391752577320, sedangkan $x = 10$ didapatkan nilai y_i =-5.666666666666667.

Untuk lebih jelasnya dapat dilihat di grafik dibawah ini untuk solusi numerik dan solusi analitik



Gambar 3.3 Grafik Solusi persamaan differensial linier orde satu

3.3 Analisis Galat Metode Runge Kutta Orde lima

Dengan melihat gambar 3.1 dan 3.2 dapat dilakukan perbandingan nilai antara solusi numerik dan solusi analitik. Hal ini dilakukan untuk mendapatkan nilai error, nilai error didapatkan dengan cara $\varepsilon = |a - \grave{a}|$ dimana a adalah nilai sejati

dan $\grave{a}$ adalah nilai hampiran. Berikut ini adalah nilai error dari persamaan differensial linier orde satu .

Tabel 3.1 perbandingan nilai eksak dengan nilai hampiran persamaan differensial orde satu

i	x_i	solusi hampiran RK-5	solusi analitik	error
0	0	0	1.0000000000000000	1
1	0.005	0.000015727683649	1.005037688442211	1.005021960
2	0.01	0.000042280748525	1.010151515151515	1.010109234
3	0.015	0.000079682558910	1.015342639593909	1.015262957
4	0.02	0.000127956529513	1.020612244897959	1.020484288
5	0.025	0.000187126125580	1.025961538461538	1.025774412
6	0.03	0.000257214863001	1.031391752577320	1.031134538
⋮	⋮	⋮	⋮	⋮
2001	10	1.610042024823768e+002	-5.666666666666667	1.666708691490435e+002

Dari perhitungan error di atas, dapat dilihat bahwa error yang didapat cukup besar dan menjauhi nol, maka masih ada kesalahan dalam menurunkan metode runge kutta orde lima.

BAB IV

PENUTUP

4.1 Kesimpulan

Berdasarkan hasil pembahasan yang dipaparkan pada bab tiga maka dapat disimpulkan sebagai berikut.

1. Metode Runge Kutta orde lima memiliki sembarang untuk masing masing parameternya. Dari hasil penelitian ini didapatkan hasil metode Runge Kutta orde lima $y_{i+1} = y_i + \left(-\frac{943}{480}k_1 + \frac{6}{5}k_2 + \frac{37}{40}k_3 + \frac{763}{480}k_4 - \frac{4}{3}k_5\right)h$ dimana $k_1 = f(x_i, y_i)$, $k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{4}k_1h)$, $k_3 = f(x_i + \frac{1}{4}h, y_i + \frac{1}{8}k_1h + \frac{1}{8}k_2h)$, $k_4 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{4}k_1h - \frac{1}{2}k_2h + \frac{2787}{763}k_3h)$, dan $k_5 = f(x_i + \frac{3}{4}h, y_i + \frac{3413}{2560}k_1h + \frac{1}{2}k_2h + \frac{1}{4}k_3h + \frac{1}{8}k_4h)$.
2. Solusi numerik model persamaan differensial linier orde satu menggunakan metode Runge Kutta orde lima diperoleh nilai $y_0 = 0$, $y_1 = 0.000015727683649, y_2 = 0.000042280748525$, $y_3 = 0.000079682558910$, $y_4 = 0.000127956529513$, $y_5 = 0.000187126125580$, $y_6 = 0.000257214863001$, y_{2001} =1.610042024823768e+002.
3. Penyelesaian Galat dari metode Runge Kutta orde lima didapatkan nilai $\varepsilon_1 = 1$, $\varepsilon_2 = 1.005021960$, $\varepsilon_3 = 1.010109234$, $\varepsilon_4 = 1.015262957$, $\varepsilon_{2001} = 1.666708691490435e + 002$.

4.2 Saran

Bagi penelitian selanjutnya disarankan untuk menganalisis kembali penurunan metode Runge Kutta orde lima atau metode Runge Kutta yang berorde lebih tinggi.

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LAMPIRAN-LAMPIRAN

1. Program MATLAB untuk Simulasi Persamaan Differensial Linier Orde Satu

```
clc; clear all; close all;clf;
format long
disp('Metode Runge-Kutta pada persamaan differensial linier
orde satu ')

%nilai parameter%
h=0.005;
x=0:h:10;
%persamaan differensial%
f=@(x,y) x+y;

%nilai awal%
y(1)=0;
n=length(x)-1;

%skema RK5
for i=1:n

    k1=x(i)+y(i);
    k2=x(i)+((1/2)*h)+y(i)+((1/4)*k1*h);
    k3=x(i)+((1/4)*h)+y(i)+((1/8)*k1*h)+((1/8)*k2*h);
    k4=x(i)+((1/2)*h)+y(i)+((1/4)*k1*h)-
((1/2)*k2*h)+((2787/763)*k3*h);

k5=x(i)+((3/4)*h)+y(i)+((3413/2560)*k1*h)+((1/2)*k2*h)+((1/4)*
k3*h)+((1/8)*k4*h);

    y(i+1)=y(i)+((-
943/480)*k1+(6/5)*k2+(37/40)*k3+(763/480)*k4-(4/3)*k5)*h;

end;

figure(1)
plot(x,y,'-r','LineWidth',2)
hold on
pause(0.0000000001)
ylabel('y')
legend('kurva y')
title('Grafik Solusi Numerik persamaan differensial linier
orde satu')

disp('          iterasi          y  ')
disp([x'  y' ])

x=0:h:10;

for i=1:length (x)
    y_analitik(i)=(0.5*(x(i)^2)+1)/(1-x(i));
end
```

```
figure(2)
plot(x,y_analitik,'-b','LineWidth',2)
hold on
pause(0.0000000001)
ylabel('y')
legend('kurva y')
title('Grafik Solusi analitik persamaan differensial linier
orde satu')

disp('     y_analitik  ')
disp([y_analitik' ])

figure(3)
hold on
plot(x,y,'-r',x,y_analitik,'-b','LineWidth',2)
pause(0.0000000001)
xlabel('x')
legend('solusi analitik','solusi numerik')
title('Grafik Solusi persamaan differensial linier orde satu')
hold off


for i=1:n
err(i)=abs(y_analitik(i)-y(i+1));
end

disp('          error  ')
disp([err' ])

disp('                iterasi              y           y_analitik
error')
disp([{x}'  {y}' {y_analitik}'   {err}' ])
```

RIWAYAT HIDUP

Isvina Unaizahroya, lahir di Jombang pada tanggal 28 februari 1997 dan biasa dipanggil Isvina. Penulis tinggal di Desa Ploso Genuk, Kecamatan Perak Kabupaten Jombang, Jawa Timur. Dia merupakan anak kelima dari lima bersaudara pasangan Bapak Ali Masyhar dan Ibu Siti Musmaidah.

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No	Tanggal	Hal	Tanda Tangan	
1	13 Mei 2019	Konsultasi Bab I dan Bab II	1.	
2	5 Agustus 2019	Revisi Bab I dan Bab II		2.
3	10 September 2019	Konsultasi Agama Bab I dan Bab II	3.	
4	13 September 2019	ACC Agama Bab I dan Bab II		4.
5	6 November 2019	Konsultasi Bab III	5.	
6	24 Januari 2020	Revisi Bab III		6.
7	14 Februari 2020	ACC untuk diseminarkan	7.	
8	20 Maret 2020	Revisi Bab III (Bimbingan Online)		8.
9	28 April 2020	Revisi Agama (Bimbingan Online)	9.	
10	29 April 2020	ACC Untuk disidangkan (online)		10.

Malang, 23 September 2020
Mengetahui,
Ketua Jurusan Matematika

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