DISTANCE LAPLACIAN ENERGY DAN *DISTANCE SIGNLESS LAPLACIAN ENERGY* DARI KOMPLEMEN GRAF SUBGRUP DARI GRUP DIHEDRAL

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MALANG
2019

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menyatakan dengan sebenarnya bahwa skripsi yang saya tulis ini benar-benar merupakan hasil karya saya sendiri, bukan merupakan pengambilan data, tulisan, atau pikiran orang lain yang saya akui sebagai hasil tulisan atau pikiran saya sendiri, kecuali dengan mencantumkan sumber cuplikan pada daftar rujukan. Apabila di kemudian hari terbukti atau dapat dibuktikan skripsi ini hasil jiplakan, maka saya bersedia menerima sanksi atas perbuatan tersebut.

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Yang membuat pernyataan,

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"*Sebaik-baik manusia adalah yang paling bermanfaat bagi manusia*"
(HR. Ahmad, Ath-Thabrani, Ad-Daruqutni)

PERSEMBAHAN

Skripsi ini penulis persembahkan untuk:

Ayahanda Umar (Alm) dan ibunda Siti Asiyah.

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Malang, Mei 2019

Penulis

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Umar, Nabila. 2019. ***Distance Laplacian Energy* dan *Distance Signless Laplacian Energy* dari Komplemen Graf Subgrup dari Grup Dihedral**. Skripsi. Jurusan Matematika, Fakultas Sains dan Teknologi, Universitas Islam Negeri Maulana Malik Ibrahim Malang. Pembimbing: (I) Dr. Abdussakir, M.Pd (II) Ach. Nashichuddin, M.A.

Kata Kunci: *distance Laplacian energy*, *distance signless Laplacian energy*, graf subgrup, komplemen graf subgrup, grup dihedral

Misalkan G subgrup dan H subgrup normal dari G. Graf subgrup $\Gamma_H(G)$ adalah graf dengan himpunan titik semua unsur di G dan dua titik berbeda x dan y terhubung langsung jika dan hanya jika $xy \in H$. Komplemen graf subgrup $\overline{\Gamma_H(G)}$ adalah graf dengan himpunan titik semua unsur di G sedemikian hingga dua titik akan terhubung langsung jika dan hanya jika tidak terhubung langsung di G.

Penelitian ini bertujuan untuk menentukan *distance Laplacian energy (DLE)* dan *distance signless Laplacian energy (DSLE)* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral D_{2n}. Hasil penelitian ini adalah sebagai berikut:

1. *DLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral D_{2n} adalah
   a. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_{2n})$ untuk $n \geq 3$ dan *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah
   $$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$
   b. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^2\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah
   $$ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$
   c. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{2n})$ untuk $n \geq 6$ dengan n kelipatan tiga adalah
   $$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) = \frac{14n^2 - 10n}{3}$$
2. *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$ dan $\langle r^3\rangle$ dari grup dihedral D_{2n} adalah
   a. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_{2n})$ untuk $n \geq 3$ dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah
   $$ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$
   b. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah
   $$ED_L^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$

c. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{2n})$ untuk $n \geq 9$ dengan n kelipatan tiga adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}\right) = \frac{14n^2 - 10n}{3}$$

Penelitian selanjutnya diharapkan dapat menemukan teorema terkait energi dari komplemen graf subgrup dari grup dihedral untuk subgrup normal yang lainnya atau dari grup lainnya.

ABSTRACT

Umar, Nabila. 2019. **Distance Laplacian Energy and Distance Signless Laplacian Energy of Complement of subgroup Graph of Dihedral Group**. Thesis. Department of Mathematics, Faculty of Science and Technology, Maulana Malik Ibrahim State Islamic University of Malang. Advisors: (I) Dr. Abdussakir, M.Pd (II) Ach. Nashichuddin, M.A.

Kata Kunci: distance Laplacian energy, distance signless Laplacian energy, subgroup graph, complement of subgroup graph, dihedral group

Let G be a subgroup and H is a normal subgroup of G. Subgroup graph $\Gamma_H(G)$ is a graph with a set of point of all elements in G and two distinct vertices x and y are adjacent if and only if $xy \in H$. A complement of subgroup graph $\overline{\Gamma_H(G)}$ is a graph with a set of point of all elements in G such that two distinct vertices will be adjacent if and only if not adjacent in G.

The purpose of this research is determine the distance Laplacian energy (DLE) and distance signless Laplacian energy (DSLE) of complement of subgroup graph $\langle r\rangle, \langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, and $\langle r^3\rangle$ of dihedral group D_{2n}. The result of this study are as follows:

1. DLE of complement of subgroup graph $\langle r\rangle, \langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, and $\langle r^3\rangle$ of dihedral group D_{2n} is
   a. DLE of complement of subgroup graph $\Gamma_{\langle r\rangle}(D_{2n})$ for $n \geq 3$ and DLE of complement of subgroup graph $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ for $n \geq 4$ with n is even is
   $$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$
   b. DLE of complement of subgroup graph $\Gamma_{\langle r^2\rangle}(D_{2n})$ for $n \geq 4$ with n is even is
   $$ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$
   c. DLE of complement of subgroup graph $\Gamma_{\langle r^3\rangle}(D_{2n})$ for $n \geq 6$ with n is multiple of three is
   $$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) = \frac{14n^2 - 10n}{3}$$
2. DSLE of complement of subgroup graph $\langle r\rangle, \langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, and $\langle r^3\rangle$ of dihedral group D_{2n} is
   a. DSLE of complement of subgroup graph $\Gamma_{\langle r\rangle}(D_{2n})$ for $n \geq 3$ and DSLE of complement of subgroup graph $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ for $n \geq 4$ with n is even is
   $$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$
   b. DSLE of complement of subgroup graph $\Gamma_{\langle r^2\rangle}(D_{2n})$ for $n \geq 4$ with n is even is
   $$ED_L^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$

c. DSLE of complement of subgroup graph $\Gamma_{\langle r^3 \rangle}(D_{2n})$ for $n \geq 9$ with n is multiple of three is

$$ED_L^+\overline{\left(\Gamma_{\langle r^3 \rangle}(D_{2n})\right)} = \frac{14n^2 - 10n}{3}$$

For the further research the author suggest to determine the theorem related to energy obtained from the complement of subgroup graph for the other normal subgroup or the other group.

ملخص

عمر، نبلى. ٢٠١٩. *Distance Laplacian Energy* و *Signless Laplacian Energy Distance* **على مكملة مخطط زمرة جزئية من زمرة زوجية**. بحث الجامعي. شعبة الرياضيات، كلية العلوم والتكنولوجيا، جامعة مولانا مالك إبراهيم الحكومية الإسلامية في مالانج. المشرف : (١) الدكتور عبد الشاكر الماجستير(٢) احمد ناصيح الدين الماجستير.

الكلمات الرئيسية: *distance Laplacian energy*، *distance signless Laplacian energy*، مخطط زمرة جزئية، مكملة مخطط زمرة جزئية، زمرة زوجية.

على سبيل المثال G هو زمرة جزئية و H زمرة زوجية نورمل من G. مخطط زمرة جزئية $\Gamma_G(H)$ هو مخطط يحتوي على مجموعة من رؤوس لجميع العناصر ويتم توصيل نقطتين مختلفين x و y متصل مباشرة اذا وفقط اذا $xy \in H$. مضار من مخطط زمرة جزئية $\overline{\Gamma_G(H)}$ يحتوي على مجموعة من رؤوس لجميع العناصر G اذا توجيه اثنا شيئ مختلفين و متصل مباشرة اذا لا متصلهما في G.

الغرض لهذه الدارسة هو تحديد *distance Laplacian energy (DLE)* و *distance signless Laplacian energy (DSLE)* على مكملة مخطط زمرة جزئية $\langle r\rangle$، $\langle r^2\rangle$، $\langle r^2,s\rangle$، $\langle r^2,sr\rangle$، و $\langle r^3\rangle$ من زمرة زوجية D_{2n}. نتائج هذه الدراسية هي على النحو التالي:

1. DLE على مكملة مخطط زمرة جزئية $\langle r\rangle$، $\langle r^2\rangle$، $\langle r^2,s\rangle$، $\langle r^2,sr\rangle$، و $\langle r^3\rangle$ من زمرة زوجية D_{2n} هو

ا. DLE على مكملة مخطط زمرة جزئية $\Gamma_{\langle r\rangle}(D_{2n})$ ل $n \geq 3$ و DLE على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^2,s\rangle}(D_{2n})$، $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ ل $n \geq 4$ و n شفع هو

$$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

ب. DLE على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^2\rangle}(D_{2n})$ ل $n \geq 4$ و n شفع هو

$$ED_L\overline{\left(\Gamma_{\langle r^2\rangle}(D_{2n})\right)} = 5n^2 - 4n$$

ت. DLE على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^3\rangle}(D_{2n})$ ل $n \geq 6$ و n المضاعفات 3 هو

$$ED_L\overline{\left(\Gamma_{\langle r^3\rangle}(D_{2n})\right)} = \frac{14n^2 - 10n}{3}$$

2. $DSLE$ على مكملة مخطط زمرة جزئية $\langle r\rangle$، $\langle r^2\rangle$، $\langle r^2, s\rangle$، $\langle r^2, sr\rangle$، و $\langle r^3\rangle$ من زمرة زوجية D_{2n} هو

ا. $DSLE$ على مكملة مخطط زمرة جزئية $\Gamma_{\langle r\rangle}(D_{2n})$ ل $n \geq 3$ و $DSLE$ على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^2,s\rangle}(D_{2n})$، $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ ل $n \geq 4$ و n شفع هو

$$ED_L^+\overline{\left(\Gamma_{\langle r\rangle}(D_{2n})\right)} = ED_L^+\overline{\left(\Gamma_{\langle r^2,s\rangle}(D_{2n})\right)} = ED_L^+\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{2n})\right)} = 6n^2 - 4n$$

ب. $DSLE$ على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^2\rangle}(D_{2n})$ ل $n \geq 4$ و n شفع هو

$$ED_L^+\overline{\left(\Gamma_{\langle r^2\rangle}(D_{2n})\right)} = 5n^2 - 4n$$

ت. $DSLE$ على مكملة مخطط زمرة جزئية $\Gamma_{\langle r^3\rangle}(D_{2n})$ ل $n \geq 9$ و n المضاعفات 3 هو

$$ED_L^+\overline{\left(\Gamma_{\langle r^3\rangle}(D_{2n})\right)} = \frac{14n^2 - 10n}{3}$$

لمزيد من البحث من المرغوب فيه العثور على النظريات المتعلقة *energy* التي تم الحصول عليها على مكملة مخطط زمرة جزئية ل زمرة جزئية نورمل الاخرى او على زمرة الاخرى.

BAB I

PENDAHULUAN

1.1 Latar Belakang

Al-Quran adalah kalam Allah Swt yang di dalamnya terkandung hal-hal yang wajib dan sunnah bagi umat Islam. Banyak sekali anjuran di dalam al-Quran yang bisa dilakukan untuk menambah pahala kebaikan pada hari akhirat kelak. Salah satunya yaitu anjuran untuk saling tolong menolong antar sesama manusia. Anjuran untuk saling tolong menolong antar sesama manusia dalam al-Quran terdapat pada surat al-Maidah ayat 2 sebagai berikut

وَتَعَاوَنُوا۟ عَلَى ٱلْبِرِّ وَٱلتَّقْوَىٰ ۖ وَلَا تَعَاوَنُوا۟ عَلَى ٱلْإِثْمِ وَٱلْعُدْوَٰنِ ۚ وَٱتَّقُوا۟
ٱللَّهَ ۖ إِنَّ ٱللَّهَ شَدِيدُ ٱلْعِقَابِ ﴿٢﴾

"Dan tolong-menolonglah kamu dalam (mengerjakan) kebajikan dan takwa, dan jangan tolong menolong dalam berbuat dosa dan permusuhan. Bertakwalah kepada Allah, sungguh Allah sangat berat siksa-Nya"

Ayat di atas menjelaskan bahwasanya Allah Swt menganjurkan untuk saling tolong menolong sesama umat manusia dalam hal kebajikan dan takwa, baik pribadi maupun kelompok, baik perkara agama maupun dunia sesuai dengan kemampuan masing-masing. Sehingga sebagai seorang hamba yang beriman haruslah bisa melakukan anjuran dalam al-Quran tersebut (Ad-Dimasyqi, 2001:173).

Orang yang berilmu haruslah menolong sesama umat manusia dengan cara membagikan ilmu yang dimilikinya (Al-Qurṭubī, 2008:116). Termasuk para ahli matematika dalam menemukan suatu rumus atau pola pada kajian teori graf aljabar.

Teori graf merupakan suatu kajian yang digunakan untuk menyelesaikan permasalahan hubungan antara suatu titik dengan titik lainnya. Gabungan dari titik tersebut menggambarkan suatu rute, lintasan atau *circuit*. Graf G adalah pasangan $\left(V(G), E(G)\right)$ dengan $V(G)$ adalah himpunan tidak kosong dan berhingga dari objek-objek yang disebut *titik*, dan $E(G)$ adalah himpunan (mungkin kosong) pasangan tak berurutan dari titik-titik berbeda di $V(G)$ yang disebut sisi. Banyaknya unsur di $V(G)$ dinamakan order dari G dan dilambangkan dengan $p(G)$, dan banyaknya unsur di $E(G)$ disebut *ukuran* dari G dan dilambangkan dengan $q(G)$. Apabila permasalahan hanya pada graf G, *order* dan *ukuran* dapat dituliskan sebagai p dan q atau $G(p, q)$ (Abdussakir, dkk, 2009).

Misalkan terdapat graf G dengan $V(G)$ adalah himpunan titik dan $E(G)$ adalah himpunan sisi. Komplemen suatu graf G adalah graf dengan himpunan titik $V(G)$ sedemikian hingga dua titik akan terhubung langsung jika dan hanya jika dua titik tersebut tidak terhubung langsung di G. Komplemen suatu graf G dinotasikan dengan $\bar{G}$ (Abdussakir, dkk, 2009).

Misalkan G adalah graf terhubung berorder n dengan himpunan titik $V(G)$ dan himpunan sisi $E(G)$. Misalkan $u, v \in V(G)$ maka *distance* atau jarak antara u dan v adalah panjang lintasan terpendek dari u dan v (Xing & Zhou, 2013). Jarak antara u dan v dinotasikan dengan $d_G(u, v)$ atau d_{uv}. Matriks *distance* yang dinotasikan dengan $D(G)$ adalah suatu matriks berukuran $n \times n$ dengan unsur ke$-(i, j)$ adalah $d_{v_i v_j}$. Transmisi $T_{r_G}(v)$ dari titik v didefinisikan sebagai penjumlahan *distance* dari v ke titik lainnya di G (Xing dan Zhou, 2013). Matriks transmisi adalah suatu matriks diagonal dari transmisi titik pada G yang dinotasikan dengan $T_r(G)$. Jadi $T_r(G)$ adalah matriks dengan unsur ke$-ij$ adalah $T_{r_G}(v_i)$ dan 0 untuk lainnya (Alhevaz, dkk, 2018).

Terdapat konsep energi baru tentang energi Laplace dan energi signless Laplace yaitu *distance Laplacian energy (DLE)* dan *distance signless Laplacian energy (DSLE)*. Misalkan terdapat graf G berorder n, matriks *distance Laplacian* dari G didefinisikan sebagai $D_L(G) = T_r(G) - D(G)$ sedangkan matriks *distance signless laplacian* dari G didefinisikan sebagai $D_L{}^+(G) = T_r(G) + D(G)$ (Gutman, dkk, 2013). Polinomial karakteristik dari $D_L(G)$ dan $D_L^+(G)$ adalah $p_D(\lambda) = \det(D_L(G) - \lambda I)$ dan $p_D(\lambda^+) = \det(D_L^+(G) - \lambda^+ I)$. Akar-akar suatu persamaan karakteristik dinamakan sebagai nilai eigen (Kaladevi dan Abinayaa, 2017). Nilai eigen dari $D_L(G)$ dan $D_L^+(G)$ disebut sebagai nilai eigen dari *distance Laplacian* dan nilai eigen *distance signless Laplacian* (Atik, 2018). Nilai eigen dari $D_L(G)$ dan $D_L^+(G)$ dinotasikan dengan $\lambda_1, \lambda_2, \ldots, \lambda_n$ dan $\lambda_1^+, \lambda_2^+, \ldots, \lambda_n^+$ (Xing & Zhou, 2013). *DLE* dari suatu graf terhubung G didefinisikan sebagai $ED_L(G) = \sum_{i=1}^{n} |\lambda_i|$ dengan λ_i adalah nilai eigen dari $D_L(G)$, begitupula dengan *DSLE* yaitu $ED_L^+(G) = \sum_{i=1}^{n} |\lambda_i^+|$ dengan λ_i^+ adalah nilai eigen dari $D_L^+(G)$ (Alhevaz, dkk, 2018).

Salah satu graf yang baru diperkenalkan adalah graf subgrup. Misalkan G grup dan H adalah subgrup dari G. Misalkan $\Gamma_H(G)$ adalah graf berarah (*digraph*) dengan himpunan titik memuat semua unsur di G dan titik x terhubung langsung ke y jika dan hanya jika $x \neq y$ dan $xy \in H$. Jika $xy \in H$ dan $yx \in H$ untuk $x, y \in H$ dengan $x \neq y$ maka unsur x dan y dihubung langsungkan oleh sisi tak berarah. Oleh karena itu, akan diperoleh graf $\Gamma_H(G)$ yang disebut sebagai graf subgrup dari G (Anderson, dkk, 2012).

Apabila H merupakan subgrup normal dari G maka *graf* subgrup $\Gamma_H(G)$ akan jelas eksistensinya. Jika $xy \in H$ maka belum tentu $yx \in H$. Jika H subgrup normal di G, maka $xy \in H$ berakibat $yx = x^{-1}(xy)x \in H$. Oleh karena itu, ketika H subgrup normal maka graf subgrup $\Gamma_H(G)$ dan $\overline{\Gamma_H(G)}$ berbentuk graf (*graph*), bukan graf berarah (*digraph*) (Kakeri dan Erfanian, 2015).

Penelitian mengenai *distance* pada suatu graf sudah pernah dilakukan oleh Bo Zhou Rundan Xing pada tahun 2013 tentang *distance* dan *distance signless Laplacian energy* dari graf *spectral radii of bicyclic*. Penelitian lain oleh Jienshan dkk pada tahun 2013 juga menerangkan masalah *bounds on distance Laplacian energy* dari suatu graf G. Kemudian penelitian terbaru di antaranya penelitian Abdollah pada tahun 2018 tentang *distance signless Laplacian spectrum and energy of regular graph*, penelitian Fousul Atik dan Pratima Panigrahi pada tahun 2018 tentang *on distance and distance signless Laplacian eigenvalues of graphs and the smallest gersgorin disc*, kemudian penelitian Abdussakir dkk. pada tahun 2018 tentang *on distance spectrum and distance energy of complement of subgroup graphs of dihedral group*.

Berdasarkan paparan di atas, maka penelitian mengenai *distance Laplacian energy* dan *distance signless Laplacian energy* dari komplemen graf subgrup dari grup dihedral belum pernah dilakukan. Oleh karena itu, peneliti akan menelitinya sebagai topik baru pada teori graf aljabar.

1.2 Rumusan Masalah

Berdasarkan latar belakang di atas maka didapatkan rumusan masalah sebagai berikut

1. Bagaimana rumus *DLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral?
2. Bagaimana rumus *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral?

1.3 Tujuan Penelitian

Berdasarkan rumusan masalah diatas didapatkan tujuan penelitian sebagai berikut

1. Mengetahui rumus *DLE* dari komplemen graf subgrup $\langle r\rangle$, $\langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral
2. Mengetahui rumus *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle$, $\langle r^2, s\rangle$, $\langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral

1.4 Manfaat Penelitian

Penelitian ini diharapkan dapat memberi manfaat sebagai berikut

1. Memberikan informasi mengenai rumus *DLE* dari komplemen graf subgrup $\langle r\rangle$, $\langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral
2. Memberikan informasi mengenai rumus *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral.

1.5 Batasan Masalah

Penelitian ini akan memfokuskan pembahasannya pada *DLE* dan *DSLE*. Graf yang digunakan untuk menentukannya yaitu komplemen graf subgrup dari grup dihedral. Subgrup yang diambil yaitu subgrup normal $\langle r \rangle$ untuk $n \geq 3$, subgrup normal $\langle r^2 \rangle$, $\langle r^2, s \rangle$, $\langle r^2, sr \rangle$ untuk $n \geq 4$ dengan n genap dan subgrup normal $\langle r^3 \rangle$ untuk n kelipatan 3 dengan $n \geq 6$ dalam menentukan *DLE* dan untuk $n \geq 9$ berlaku dalam menentukan *DSLE*.

1.6 Metode Penelitian

Penelitian merupakan penelitian pustaka (*library research)*. Referensi yang dijadikan rujukan berasal dari beberapa jurnal, buku, dan artikel yang berkaitan dengan teori graf, aljabar linier, dan aljabar abstrak. Kajian pada teori graf dan jurnal terkait penelitian dikhususkan pada kajian mengenai graf. Kajian pada buku-buku aljabar linier berkaitan dengan topik matriks, khususnya mengenai nilai eigen dan energi dari matriks. Kajian pada buku aljabar abstrak berkaitan dengan topik grup dan subgrup normal.

Adapun langkah-langkah penelitian ini menggunakan pendekatan kualitatif. Yaitu melakukan pembuktian dari hal-hal khusus (induktif) hingga ditemukannya pola. Pola tersebut nantinya akan digeneralisasikan secara deduktif menjadi sebuah rumus. Langkah-langkah dalam penelitian ini yaitu:

1. Menentukan semua subgrup normal $\langle r \rangle$, $\langle r^2 \rangle$, $\langle r^2, s \rangle$, $\langle r^2, sr \rangle$, dan $\langle r^3 \rangle$ pada suatu grup D_{2n} untuk beberapa kasus n, yaitu $n = 3, 4, 5, 6, 7, 8$ untuk subgrup normal $\langle r \rangle$, $n = 4, 6, 8$ untuk subgrup normal $\langle r^2 \rangle$, $\langle r^2, s \rangle$, $\langle r^2, sr \rangle$, $n =$

$6, 9, 12$ untuk subgrup normal $\langle r^3 \rangle$ dalam menentukan *DLE* dan $n = 9, 12, 15$ untuk subgrup normal $\langle r^3 \rangle$ dalam menentukan *DSLE*

2. Menggambar graf subgrup dan komplemennya. Kemudian menyatakan keterhubungan titik ke dalam bentuk matriks distance, transmisi, *distance Laplacian*, dan *distance signless Laplacian*
3. Mencari nilai eigen dari matriks *distance Laplacian* dan *distance signless Laplacian*
4. Menentukan *DLE* dan *DSLE*
5. Membuat dugaan (konjektur) tentang energi berdasarkan pola yang ditemukan untuk masing-masing kasus
6. Merumuskan atau mengeneralisasikan konjektur tentang energi menjadi suatu teorema
7. Menghasilkan suatu teorema tentang energi dengan menyertakan bukti secara deduktif.

1.7 Sistematika Penulisan

Sistematika penulisan yang digunakan dalam penelitian ini terdiri dari empat bab, masing-masing dibagi ke dalam subbab yaitu sebagai berikut:

Bab I Pendahuluan

Pendahuluan tersusun atas latar belakang, rumusan masalah, tujuan penelitian, manfaat penelitian, batasan penelitian, metode penelitian, dan sistematika penelitian.

Bab II Kajian Pustaka

Kajian Pustaka tersusun atas teori-teori yang dapat digunakan dalam menjawab rumusan masalah sehingga membantu dalam pembahasan. Pada penelitian ini, teori yang digunakan yaitu: teori graf, matriks, energi, grup, dan subgrup. Juga disertai dengan tafsir ayat al-Quran mengenai pentingnya tolong menolong antar sesama manusia dalam kebaikan.

Bab III Pembahasan

Pembahasan tersusun atas bagaimana menentukan rumus umum *DLE* dan *DSLE* komplemen graf subgrup $\langle r \rangle$, $\langle r^2 \rangle$, $\langle r^2, s \rangle$, $\langle r^2, sr \rangle$, dan $\langle r^3 \rangle$ dari grup dihedral dan perhitungan energi dalam pandangan Islam.

Bab V Penutup

Penutup tersusun atas kesimpulan dari pembahasan dan saran-saran sesuai dengan hasil penelitian.

BAB II

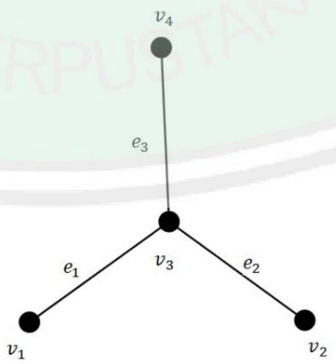
KAJIAN PUSTAKA

2.1 Graf

Graf G adalah pasangan $(V(G), E(G))$ dengan $V(G)$ adalah himpunan tidak kosong dan berhingga dari objek-objek yang disebut *titik*, dan $E(G)$ adalah himpunan (mungkin kosong) pasangan tak berurutan dari titik-titik berbeda di $V(G)$ yang disebut sisi. Banyaknya unsur di $V(G)$ dinamakan order dari G dan dilambangkan dengan $p(G)$, dan banyaknya unsur di $E(G)$ disebut *ukuran* dari G dan dilambangkan dengan $q(G)$. Apabila permasalahan hanya pada graf G, *order* dan *ukuran* dapat dituliskan sebagai p dan q atau $G(p, q)$ (Abdussakir, dkk, 2009).

Apabila $e = (u, v)$ merupakan suatu sisi pada graf G maka dikatakan bahwa titik u terhubung langsung (*adjacent)* dengan titik v. Titik u dengan sisi e juga titik v dengan sisi e dinamakan terkait langsung (*incident)* (Abdussakir, dkk, 2009).

Mengikuti definisi graf, maka diberikan contoh graf G di bawah ini:

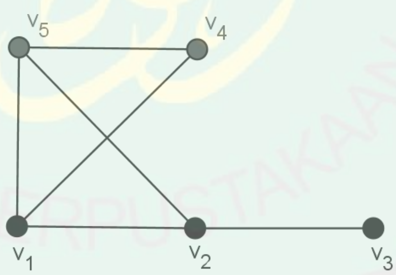


Gambar 2.1 Graf G berorder 4

Berdasarkan Gambar 2.1, graf G mempunyai 4 titik dan 3 sisi yang dinotasikan dengan $G(4,3)$ dengan $V(G) = \{v_1, v_2, v_3, v_4\}, E(G) = \{e_1, e_2, e_3\}$. Pada graf G terlihat bahwa semua sisi pada graf G saling terkait langsung *(incident)* di titik v_1. Hal tersebut menyebabkan semua titik pada graf G yaitu v_1 dengan v_2, v_1 dengan v_3, v_1 dengan v_4 saling terhubung langsung *(adjacent)*. Titik v_2 dengan v_3, v_3 dengan v_4, v_2 dengan v_4 saling terhubung. Titik v_1 dengan sisi e_2, v_3 dengan sisi e_2, v_4 dengan e_3 dinamakan terkait langsung.

2.2 Derajat Suatu Titik

Berdasarkan penjelasan sebelumnya bahwa titik pada suatu graf G dinotasikan dengan v, maka derajat pada titik tersebut merupakan banyaknya sisi di graf G yang terhubung langsung pada v. Notasi derajat suatu titik v adalah $deg_G(v)$. Ganjil atau genapnya derajat titik v tersebut menjadi acuan titik v dikatakan ganjil atau genap (Abdussakir, dkk, 2009).



Gambar 2.2 Graf $G1$ berorder 5

Berdasarkan Gambar 2.2 dapat dilihat bahwa derajat setiap titik adalah $deg_{G1}(v_1) = 3$, $deg_{G1}(v_2) = 3$, $deg_{G1}(v_3) = 1$, $deg_{G1}(v_4) = 2$, dan $deg_{G1}(v_5) = 3$. Kemudian diketahui titik v_1, v_2, v_5 berderajat ganjil, titik v_4 berderajat genap dan titik ujung yaitu v_3 berderajat 1.

2.3 Graf Terhubung

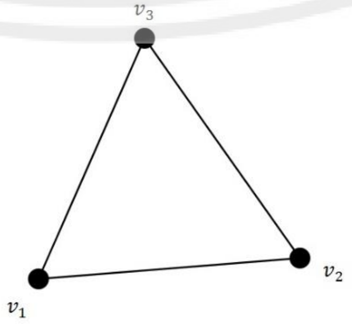
Asumsikan terdapat graf G yang mengandung suatu titik x dan y dengan tidak harus berbeda. Deretan berbatas dan berselang-seling pada graf G disebut jalan x ke y. Deretan tersebut dinotasikan

$$W: x = y_0, e_1, y_1, e_2, y_2, \ldots, e_n, y_n = y$$

Antara sisi dan titik yang diawali dengan titik kemudian diakhiri dengan titik. Diketahui bahwa $e_i = y_{i-1}y_i$ dan $i = 1, 2, 3, \ldots, n$ merupakan sisi di graf G. y_0 merupakan titik awal, y_n merupakan titik akhir, $y_1, y_2, \ldots, y_{n-1}$ merupakan titik internal, kemudian n menujukkan panjang W. W merupakan jalan terbuka bila $y_0 \neq y_n$ dan tertutup bila $y_0 = y_n$. Jalan trivial merupakan jalan yang tidak memiliki sisi. Karena satu sisi terhubung oleh dua titik, maka jalan x ke y dinotasikan $W: x = y_0, e_1, y_1, e_1, y_2, \ldots, e_n, y_n = y$ atau $W: x = y_0, y_1, \ldots, y_{n-1}, y_n = y$ (Abdussakir, dkk, 2009).

Misalkan u dan v titik berbeda pada graf G. Suatu graf G dikatakan terhubung *(connected)*, jika untuk setiap titik u dan v di G terdapat lintasan u-v di G. Sebaliknya, jika ada dua titik u dan v di G, tetapi tidak ada lintasan u-v di G, maka G dikatakan tak terhubung *(disconnected)* (Abdussakir, dkk, 2009).

Diberikan contoh graf terhubung berikut



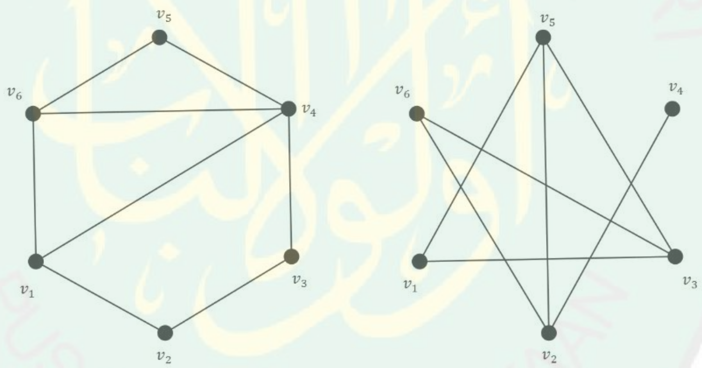
Gambar 2.3 Graf $G2$ berorder 3

Berdasarkan Gambar 2.3, graf $G2$ merupakan graf terhubung setiap dua titik yang berbeda di $G2$ dihubungkan oleh lintasan.

2.4 Komplemen Graf

Misalkan terdapat graf G dengan $V(G)$ adalah himpunan titik dan $E(G)$ adalah himpunan sisi. Komplemen suatu graf G adalah graf dengan himpunan titik $V(G)$ sedemikian hingga dua titik akan terhubung langsung jika dan hanya jika dua titik tersebut tidak terhubung langsung di G. Komplemen suatu graf G dinotasikan dengan $\bar{G}$. Sehingga diperoleh bahwa $V(\bar{G}) = V(G)$ dan $uv \in E(\bar{G})$ jika dan hanya jika $uv \notin E(G)$ (Abdussakir, dkk, 2009).

Diberikan contoh graf dan komplemennya berikut



Gambar 2.4 Graf $G3$ dan Komplemennya

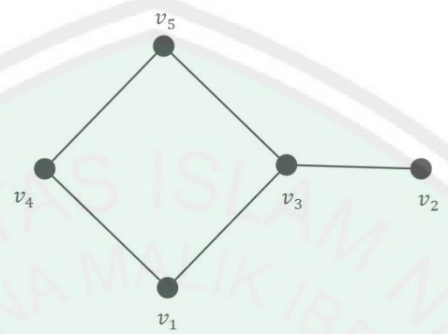
2.5 Graf dan Matriks

2.5.1 Matriks *Distance*

Misalkan G adalah graf terhubung dengan himpunan titik $V(G)$ dan himpunan sisi $E(G)$. Diketahui bahwa $u, v \in V(G)$ maka menurut Xing & Zhou (2013), *distance* atau jarak antara u dan v adalah panjang lintasan terpendek dari u dan v. *Distance* dinotasikan dengan $d_G(u, v)$ atau d_{uv}. Misalkan $V(G) =$

$\{v_1, v_2, \dots, v_n\}$. Menurut Alhevaz, dkk (2018), Matriks *distance* yang dinotasikan dengan $\boldsymbol{D}(G)$ adalah suatu matriks berukuran $n \times n$ dengan unsur ke$-(i,j)$ adalah $d_{v_i v_j}$.

Diberikan graf $G4$ berikut



Gambar 2.5 Graf $G4$ berorder 5

Berdasarkan Gambar 2.5, dapat diketahui bahwa $d_{v_1 v_2} = 2, d_{v_1 v_3} = 1$, $d_{v_1 v_4} = 1$, $d_{v_1 v_5} = 2$, $d_{v_2 v_3} = 1$, $d_{v_2 v_4} = 3$, $d_{v_2 v_5} = 2$, $d_{v_3 v_4} = 2$, $d_{v_3 v_5} = 1$, $d_{v_4 v_5} = 1$ dan jarak d_{ij} dengan $i = j = 0$, sehingga matriks *distance* dari graf $G4$ ditulis sebagai berikut

$$\boldsymbol{D}(G4) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 1 & 2 \\ 2 & 0 & 1 & 3 & 2 \\ 1 & 1 & 0 & 2 & 1 \\ 1 & 3 & 2 & 0 & 1 \\ 2 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

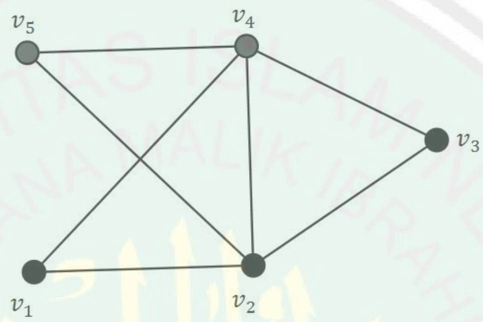
2.5.2 Matriks Transmisi

Misalkan terdapat graf G dengan $V(G) = \{v_1, v_2, \dots, v_n\}$, Transmisi $T_{r_G}(v)$ dari titik v, didefinisikan sebagai penjumlahan *distance* dari v ke titik lainnya di G (Xing dan Zhou, 2013), yaitu

$$T_{r_G}(v) = \sum_{u \in V(G)} d_G(u,v)$$

Matriks transmisi adalah suatu matriks diagonal dari transmisi titik pada G yang dinotasikan dengan $\boldsymbol{T_r}(G)$. Jadi $\boldsymbol{T_r}(G)$ adalah matriks dengan unsur ke$-ii$ adalah $T_{r_G}(v_i)$ dan 0 untuk lainnya (Alhevaz, dkk, 2018).

Diberikan graf $G5$ berikut.



Gambar 2.6 Graf $G5$ berorder 5

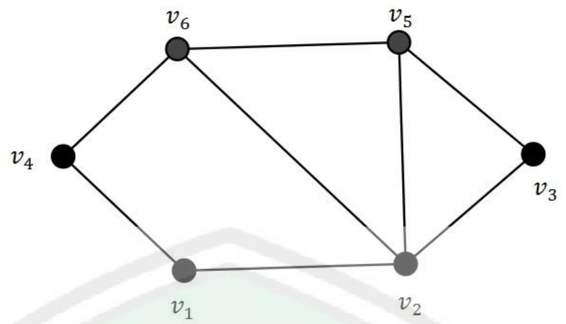
Berdasarkan Gambar 2.6, dapat diketahui bahwa $T_{r_{G5}}(v_1) = 7$, $T_{r_{G5}}(v_2) = 4$, $T_{r_{G5}}(v_3) = 6$, $T_{r_{G5}}(v_4) = 4$, dan $T_{r_{G5}}(v_5) = 6$ sehingga matriks transmisi ditulis sebagai berikut

$$\boldsymbol{T_r}(G4) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array}$$

2.2.3 Matriks *Distance Laplacian*

Matriks *distance Laplacian* dari graf G adalah matriks $\boldsymbol{D_L}(G) = \boldsymbol{T_r}(G) - \boldsymbol{D}(G)$ dengan $\boldsymbol{T_r}(G)$ adalah matriks transmisi dan $\boldsymbol{D}(G)$ adalah matriks *distance* dari G (Gutman, dkk, 2013).

Diberikan graf $G6$ berikut.



Gambar 2.7 Graf $G6$ berorder 6

Berdasarkan Gambar 2.7, diperoleh matriks *distance* dan transmisi sebagai berikut

$$\boldsymbol{D}(G6)=\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array}\\ \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 2\\ 1 & 0 & 1 & 2 & 1 & 1\\ 2 & 1 & 0 & 3 & 1 & 2\\ 1 & 2 & 3 & 0 & 2 & 1\\ 2 & 1 & 1 & 2 & 0 & 1\\ 2 & 1 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array} \quad \boldsymbol{T_r}(G6)=\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array}\\ \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0\\ 0 & 6 & 0 & 0 & 0 & 0\\ 0 & 0 & 8 & 0 & 0 & 0\\ 0 & 0 & 0 & 9 & 0 & 0\\ 0 & 0 & 0 & 0 & 7 & 0\\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \end{array}$$

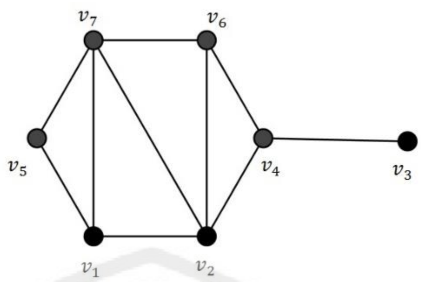
Sehingga berdasarkan definisi, matriks *distance Laplacian* $\boldsymbol{D_L}(G6)=\boldsymbol{T_r}(G6)-\boldsymbol{D}(G6)$ sebagai berikut

$$\boldsymbol{D_L}(G5)=\begin{array}{c} \\ v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array}\\ \begin{bmatrix} 8 & -1 & -2 & -1 & -2 & -2\\ -1 & 6 & -1 & -2 & -1 & -1\\ -2 & -1 & 8 & -3 & -1 & -1\\ -1 & -2 & -3 & 9 & -2 & -1\\ -2 & -1 & -1 & -2 & 7 & -1\\ -2 & -1 & -2 & -1 & -1 & 7 \end{bmatrix} \end{array}$$

2.2.4 Matriks *Distance Signless Laplacian*

Matriks *distance signless Laplacian* dari graf G adalah matriks $\boldsymbol{D_L}^{+}(G)=\boldsymbol{T_r}(G)+\boldsymbol{D}(G)$ dengan $\boldsymbol{T_r}(G)$ adalah matriks transmisi dan $\boldsymbol{D}(G)$ adalah matriks *distance* dari titik G (Alhevaz, dkk, 2018).

Diberikan graf $G6$ berikut



Gambar 2.8 Graf $G7$ berorder 7

Berdasarkan Gambar 2.8, diperoleh matriks *distance* dan transmisi sebagai berikut

$$\boldsymbol{D}(G6) = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{matrix} & \begin{bmatrix} 0 & 1 & 3 & 2 & 1 & 2 & 1 \\ 1 & 0 & 2 & 1 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 & 4 & 2 & 3 \\ 2 & 1 & 1 & 0 & 3 & 1 & 2 \\ 1 & 2 & 4 & 3 & 0 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 1 & 3 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array} \quad \boldsymbol{T_r}(G6) = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{matrix} & \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix} \end{array}$$

Sehingga berdasarkan definisi, matriks *distance signless Laplacian* $\boldsymbol{D}_L^+(G6) = \boldsymbol{T_r}(G6) - \boldsymbol{D}(G6)$ sebagai berikut

$$\boldsymbol{D_L}^+(G6) = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{matrix} & \begin{bmatrix} 10 & 1 & 3 & 2 & 1 & 2 & 1 \\ 1 & 8 & 2 & 1 & 2 & 1 & 1 \\ 3 & 2 & 15 & 1 & 4 & 2 & 3 \\ 2 & 1 & 1 & 10 & 3 & 1 & 2 \\ 1 & 2 & 4 & 3 & 13 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 9 & 1 \\ 1 & 1 & 3 & 2 & 1 & 1 & 9 \end{bmatrix} \end{array}$$

2.6 Energi

Misalkan $\boldsymbol{A}$ matriks $n \times n$, maka polinomial karakteristik $\boldsymbol{A}$ memiliki derajat n dan koefisien variabel λ^n adalah 1, sehingga polinomial karakteristik $p(\lambda)$ dari matriks $n \times n$ adalah

$$p(\lambda) = \det(\boldsymbol{A} - \lambda I) = \lambda^n + c\lambda^{n-1} - c_1\lambda^{n-2} + \cdots + c_n = 0$$

(Anton, dkk, 2011)

Misalkan graf G berorder n dan $\boldsymbol{A}$ adalah matriks keterhubungan dari graf G. Suatu vektor $\boldsymbol{x}$ bukan nol dikatakan vektor eigen dari $\boldsymbol{A}$ jika $\boldsymbol{Ax}$ adalah suatu kelipatan skalar dari $\boldsymbol{x}$, yaitu $\boldsymbol{Ax} = \lambda\boldsymbol{x}$, untuk sebarang skalar λ. Notasi λ disebut nilai eigen dari $\boldsymbol{A}$ sedangkan $\boldsymbol{x}$ sebagai vektor eigen dari $\boldsymbol{A}$ yang bersesuaian dengan λ. Persamaan $\boldsymbol{Ax} = \lambda\boldsymbol{x}$ ditulis kembali dalam bentuk $(\boldsymbol{A} - \lambda I)\boldsymbol{x} = 0$ dengan I adalah matriks identitas berordo $n \times n$ untuk menentukan nilai eigen dari matriks $\boldsymbol{A}$. Persamaan tersebut mempunyai solusi bukan nol jika dan hanya jika $det(\boldsymbol{A} - \lambda I) = 0$. Persamaan $det(\boldsymbol{A} - \lambda I) = 0$ akan menghasilkan persamaan polinomial dalam variabel λ dan disebut persamaan karakteristik dari matriks $\boldsymbol{A}$. Nilai-nilai eigen dari matriks $\boldsymbol{A}$ adalah skalar-skalar λ yang memenuhi persamaan karakteristik (Anton, dkk, 2011).

Energi suatu graf G adalah penjumlahan nilai eigen mutlak dari nilai eigen matriks dari graf G. Misalkan graf G berorder n. Misalkan $\beta_1, \beta_2, \dots, \beta_n$ dengan $\beta_1 \geq \beta_2 \geq \cdots \geq \lambda_n$ adalah nilai eigen dari suatu matriks, maka energi dari G adalah $E(G) = \sum_{i=1}^{n} |\beta_i(G)|$ (Ilic', 2010). Misalkan $\lambda_1, \lambda_2, \dots, \lambda_n$ adalah nilai eigen berbeda dan $m(\lambda_1), m(\lambda_2), \dots, m(\lambda_p)$ adalah banyaknya basis ruang vektor eigen dari λ_i, maka energi graf G adalah $E(G) = \sum_{i=1}^{n} m(\lambda_i(G))|\lambda_i(G)|$ sehingga dapat diketahui $E(G) = \sum_{i=1}^{n} |\beta_i(G)| = \sum_{i=1}^{n} m(\lambda_i(G))|\lambda_i(G)|$ (Kaladevi dan Abinayaa, 2017).

2.7 Grup dan Subgrup

2.7.1 Grup

Grup merupakan himpunan tak kosong G yang di dalamnya terdapat operasi biner "$*$". Sedemikian hingga menurut (Herstein, 1975), struktur aljabar $(G,*)$ disebut grup jika dan hanya jika:

1. Tertutup di G (untuk setiap $a, b \in G$ berakibat $a + b \in G$ atau $a * b \in G$).
2. Bersifat asosiatif (untuk setiap $a, b, c \in G$ berakibat $a * b * c = a * b * c$).
3. Memiliki elemen identitas yaitu e (terdapat suatu elemen $e \in G$ sedemikian sehingga $a * e = e * a = a$ untuk setiap $a \in G$).
4. Memiliki unsur invers dari a di G (untuk setiap $a \in G$, terdapat suatu elemen $a^{-1} \in G$ sedemikian sehingga $a * a^{-1} = a^{-1} * a = e$.

Grup $(G,*)$ dikatakan abelian (komutatif) jika untuk setiap $a, b \in G$ berlaku $a * b = b * a$ (Arifin, 2000).

Misalkan $(G,*)$ adalah sebarang grup. Misalkan g adalah sebarang elemen dari G. Order dari suatu elemen g pada suatu grup G adalah bilangan bulat positif n sedemikian hingga $g^n = e$ (e adalah elemen identitas di G). Order dari suatu elemen g dilambangkan dengan $|g| = n$ (Gallian, 2010).

2.7.2 Grup Dihedral

Grup dihedral dengan notasi D_{2n} merupakan suatu grup yang terdiri dari himpunan dari simetri-simetri (rotasi dan refleksi) pada segi-n beraturan (*polygon*) dengan $n \in Z^+$ dan $n \geq 3$. Rotasi pada segi n beraturan sebanyak n rotasi dan refleksi pada segi n beraturan sebanyak n refleksi (Dummit dan Foote, 1991).

Grup dihedral ini akan sangat diaplikasikan sehingga dibutuhkan beberapa notasi atau hitungan untuk membantu terlaksananya pengamatan-pengamatan pada D_{2n}, diantaranya:

a. $1, r, r^2, \ldots, r^{n-1}$ adalah unsur-unsur yang berbeda dengan $r^n = 1$ sehingga $|r| = n$ (order r adalah n)

b. $|s| = 2$ atau order s adalah 2 sehingga $s \circ s = 1$

c. $s \neq r^i$ untuk semua i

d. $sr^i \neq sr^j$untuk semua $0 \leq i, j \leq n-1$dengan $i \neq j$ akibatnya setiap elemen dari $D_{2n} = \{1, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$ dapat ditulis secara tunggal dalam bentuk $s^k r^i$ untuk suatu $k = 0$ atau 1 dan $0 \leq i \leq n-1$

e. $sr = r^{-1}s$, ini menunjukkan bahwa r dan s tidak komutatif sehingga D_{2n} adalah non abelian

f. $sr^i = r^{-i}s$, untuk setiap $0 \leq i \leq n$ (Dummit dan Foote, 1991).

Diberikan contoh, grup dihedral D_6 yang memuat semua himpunan simetri (rotasi dan refleksi) pada bangun segitiga sehingga $D_6 = \{1, r, r^2, s, sr, sr^2\}$.

2.7.3 Subgrup dan Subgrup Normal

Misalkan $(G,*)$ merupakan suatu grup, terdapat $S \subseteq G$ dan S bukan merupakan himpunan kosong. Kemudian S disebut sebagai subgrup dari G jika dan hanya jika untuk setiap $a, b \in S$ maka berlaku $a * b \in S$.

Dari definisi subgrup di atas menghasilkan suatu lemma (syarat cukup dan perlu), yaitu:

Misalkan $(G,*)$ merupakan suatu grup, terdapat $S \subseteq G$ dan S bukan merupakan himpunan kosong. Kemudian S disebut sebagai subgrup dari G jika dan hanya jika untuk setiap $a, b \in S$ maka berlaku $a * b^{-1} \in G$ (Gallian, 2010).

Subgrup S dari G dinotasikan sebagai $S \leq G$. Jika di dalam subgrup tersebut berlaku gsg^{-1} pada setiap $s \in S$, $g \in G$ maka S akan dikatakan sebagai subgrup normal dari G (Gallian, 2010). Semua subgrup dikatakan normal apabila grupnya abelian karena $s = es = gg^{-1}s = gsg^{-1} \in S$ untuk setiap $s \in S$, $g \in G$ (Gallian, 2010).

Misalkan grup D_{2n}=$\{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ adalah grup dihedral berorder $2n$. Subgrup normal dari D_{2n} dengan n ganjil adalah $\langle r \rangle, \langle r^d \rangle$ ketika d membagi n dan D_{2n} sendiri. Subgrup normal dari D_{2n} dengan n genap adalah $\langle r \rangle, \langle r^d \rangle$ ketika d membagi n, $\langle r^2, s \rangle, \langle r^2, sr \rangle$, dan D_{2n} sendiri (Abdussakir, 2017).

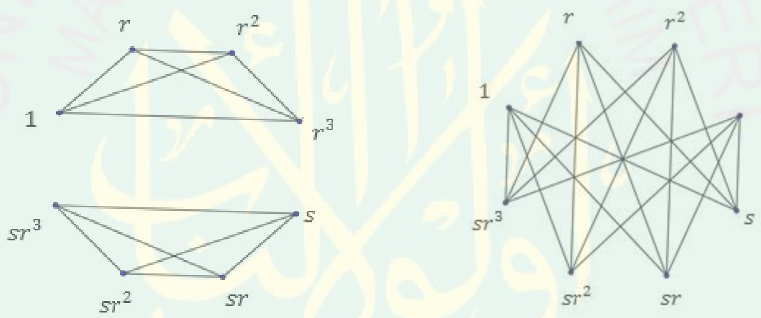
Diberikan contoh, misalkan grup dihedral $D_6 = \{1, r, r^2, s, sr, sr^2\}$ untuk $n = 3$ dengan n ganjil. Berdasarkan definisi subgrup maka diperoleh subgrup dari D_6 adalah $\{1\}, \{1, s\}, \{1, sr\}, \{1, sr^2\}, \{1, r, r^2\}$, dan $\{1, r, r^2, s, sr, sr^2\}$. Sedangkan subgrup normal dari D_6 adalah $\{1, r, r^2\}$ dan D_6 sendiri.

2.8 Graf Subgrup dan Komplemennya

Misalkan H merupakan subgrup dari G. Misalkan $\Gamma_H(G)$ adalah graf berarah (*digraph*) dengan himpunan titik memuat semua unsur di G dan titik x terhubung langsung ke y jika dan hanya jika $x \neq y$ dan $xy \in H$. Jika $xy \in H$ dan $yx \in H$ untuk $x, y \in H$ dengan $x \neq y$ maka unsur x dan y dihubung langsungkan oleh sisi tak berarah. Oleh karena itu, akan diperoleh graf $\Gamma_H(G)$ yang disebut sebagai graf subgrup dari G (Anderson, dkk, 2012).

Apabila H merupakan subgrup normal dari G maka graf subgrup $\Gamma_H(G)$ akan jelas eksistensinya. Jika $xy \in H$ maka belum tentu $yx \in H$. Jika H subgrup normal di G, maka $xy \in H$ berakibat $yx = x^{-1}(xy)x \in H$. Oleh karena itu, ketika H subgrup normal maka graf subgrup $\Gamma_H(G)$ dan $\overline{\Gamma_H(G)}$ berbentuk graf (*graph*), bukan graf berarah (*digraph*) (Kakeri dan Erfanian, 2015).

Misalkan terdapat grup dihedral D_8 yang dibangun oleh subgrup normal $\langle r \rangle = \{1, r, r^2, r^3\}$, sehingga diperoleh graf subgrup dan komplemennya sebagai berikut



Gambar 2.9 Graf $\Gamma_{\langle r \rangle}(D_6)$ dan $\overline{\Gamma_{\langle r \rangle}(D_6)}$

2.9 Anjuran Tolong Menolong dalam Al-Quran

Al-Quran adalah kalam Allah Swt yang didalamnya terkandung hal yang wajib dan sunnah. Banyak sekali anjuran di dalam al-Quran yang bisa dilakukan untuk menambah pahala kebaikan pada hari akhirat kelak, salah satunya yaitu anjuran untuk saling tolong menolong antar sesama manusia. Anjuran untuk saling tolong menolong antar sesama manusia dalam al-Quran terdapat pada surat al-Maidah ayat 2 sebagai berikut

وَتَعَاوَنُواْ عَلَى ٱلۡبِرِّ وَٱلتَّقۡوَىٰۖ وَلَا تَعَاوَنُواْ عَلَى ٱلۡإِثۡمِ وَٱلۡعُدۡوَٰنِۚ وَٱتَّقُواْ ٱللَّهَۖ إِنَّ ٱللَّهَ شَدِيدُ ٱلۡعِقَابِ ۝٢

"Dan tolong-menolonglah kamu dalam (mengerjakan) kebajikan dan takwa, dan jangan tolong menolong dalam berbuat dosa dan permusuhan. Bertakwalah kepada Allah, sungguh Allah sangat berat siksa-Nya"

Allah Swt memerintahkan kepada hamba-Nya yang beriman untuk saling tolong menolong dalam berbuat kebaikan, yaitu kebajikan dan meninggalkan hal-hal yang munkar; hal ini dinamakan ketakwaan (Ad-Dimasyqi, 2001:173). Menurut Az-Zuhaili (2012), kebajikan dan takwa adalah dua kata yang memiliki kesamaan arti, disebut berulang dengan kata yang berbeda sebagai penegasan dan penekanan, sebab setiap kebajikan adalah takwa dan setiap takwa adalah kebajikan. Kemudian Allah Swt melarang mereka bantu-membantu dalam kebatilan serta tolong menolong dalam perbuatan dosa dan hal-hal yang diharamkan (Ad-Dimasyqi, 2001:173).

Penjelasan tafsir surat al-Maidah ayat 2 menurut beberapa ulama' sebagai berikut

وَتَعَاوَنُواْ عَلَى ٱلۡبِرِّ وَٱلتَّقۡوَىٰۖ

"Dan tolong-menolonglah kamu dalam (mengerjakan) kebajikan dan takwa".

Al Mawardi berkata, "Allah menganjurkan untuk saling tolong menolong dalam kebajikan, dan Allah pun menyertakan ketakwaan kepada-Nya terhadap anjuran itu. Sebab dalam ketakwaan terdapat keridhaan Allah, sedangkan dalam kebajikan terdapat keridhaan manusia. Sementara orang yang menyatukan antara keridhaan Allah dan keridhaan manusia, maka sesungguhnya sempurnalah kebahagiaanya dan luaslah nikmatnya" (Al-Qurṭubī, 2008:115).

Ibnu Khuwaizimandad berkata dalam *Ahkam*-nya, "Tolong menolong dalam mengerjakan kebajikan dan takwa dapat dilakukan dengan berbagai cara. Adalah suatu hal yang wajib bagi seorang alim untuk menolong manusia dengan ilmunya, sehingga dia mau mengajari mereka, sedangkan orang yang kaya wajib menolong mereka dengan hartanya. Adapun seorang pemberani, (dia wajib memberikan pertolongan) di jalan Allah dengan keberanianya" (Al-Qurṭubī, 2008:116).

Selanjutnya Allah memberikan larangan, Allah berfirman,

وَلَا تَعَاوَنُوا۟ عَلَى ٱلْإِثْمِ وَٱلْعُدْوَٰنِ ۚ

"Dan jangan tolong menolong dalam berbuat dosa dan permusuhan".

Maksud ayat tersebut adalah ketetapan yang diperuntukkan bagi dosa dan *udwan*, yaitu menzhalimi manusia (Al-Qurṭubī, 2008:116). Dalam (Maraghi, dkk, 1987:85), *udwan* diartikan sebagai melampaui batas-batas syari'at dan adat dalam soal mu'amalat, dan tidak berlaku adil pada keduanya.

Setelah itu Allah Swt memerintahkan agar bertakwa dan mengeluarkan ancaman. Allah Swt berfirman

وَٱتَّقُوا۟ ٱللَّهَ ۖ إِنَّ ٱللَّهَ شَدِيدُ ٱلْعِقَابِ ﴿٢﴾

"Bertakwalah kepada Allah, sungguh Allah sangat berat siksa-Nya"

Bertakwa dalam arti mengikuti sunnah-sunnah Allah yang terdapat dalam al-Quran maupun dalam sistem yang berlaku pada makhluk-Nya. Sehingga, orang yang bertakwa tidak akan terkena hukuman melainkan bila tidak sesuai dengan petunjuk-Nya dan wajib diketahui bagi orang yang bertakwa bahwa siksa Allah sangatlah pedih. Karena tidak ada kasihan dan damai lagi bila hukuman Allah telah tiba. Allah memang takkan memerintahkan sesuatu kecuali yang beguna, dan tidak mencegah sesuatu kecuali yang berbahaya (Maraghi, dkk, 1987:87).

Berdasarkan penjelasan beberapa tafsir surat al-Maidah ayat 2 tersebut bahwa perilaku saling tolong menolong antar sesama manusia dalam hal kebajikan sangat dianjurkan di dalam al-Quran. Bahkan kebajikan sekecil apapun sesuai kemampuan masing-masing manusia. Inilah gambaran bahwa orang yang berilmu haruslah menolong sesama umat manusia dengan cara membagikan ilmu yang dimilikinya, termasuk ahli matematika dalam menemukan suatu pola atau rumus umum.

BAB III

PEMBAHASAN

3.1 *DLE* dan *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r \rangle}(D_{2n})}$ dari Grup Dihedral

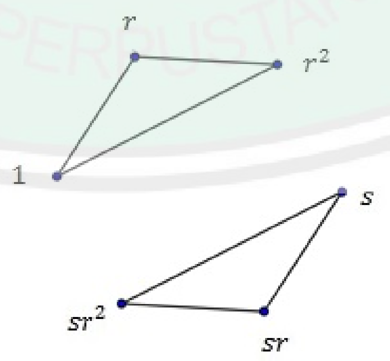
3.1.1 Grup Dihedral D_6

Diketahui anggota grup D_6 adalah $\{1, r, r^2, s, sr, sr^2\}$. Subgrup yang dibangun oleh $\langle r \rangle$ pada grup dihedral D_6 adalah $\{1, r, r^2\}$. Operasi unsur-unsur di D_6 yang hasilnya berada di $\langle r \rangle$ seperti pada tabel berikut

Tabel 3.1 Tabel Cayley Grup Dihedral D_6

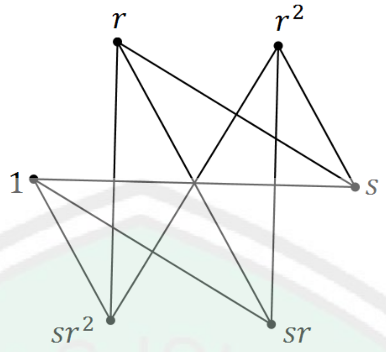
$\circ$	1	r	r^2	s	sr	sr^2
1	1	r	r^2	s	sr	sr^2
r	r	r^2	1	sr^2	s	sr
r^2	r^2	1	r	sr	sr^2	s
s	s	sr	sr	1	r^2	r
sr	sr	s	sr^2	r^2	1	r^2
sr^2	sr^2	sr	s	r	r^2	1

Berdasarkan Tabel 3.1 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r \rangle}(D_6)$ sebagai berikut



Gambar 3.1 Graf $\Gamma_{\langle r \rangle}(D_6)$

Dari Gambar 3.1 diperoleh komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_6)$ sebagai berikut



Gambar 3.2 Graf $\overline{\Gamma_{\langle r\rangle}(D_6)}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ s \\ sr \\ sr^2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & r & r^2 & s & sr & sr^2 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 1 & 1 & 1 \\ 2 & 0 & 2 & 1 & 1 & 1 \\ 2 & 2 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 1 & 2 & 0 & 2 \\ 1 & 1 & 1 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ s \\ sr \\ sr^2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & r & r^2 & s & sr & sr^2 \end{array} \\ \begin{bmatrix} 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \end{array}$$

Matriks distance Laplacian diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ s \\ sr \\ sr^2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & r & r^2 & s & sr & sr^2 \end{array} \\ \begin{bmatrix} 7 & -2 & -2 & -1 & -1 & -1 \\ -2 & 7 & -2 & -1 & -1 & -1 \\ -2 & -2 & 7 & -1 & -1 & -1 \\ -1 & -1 & -1 & 7 & -2 & -2 \\ -1 & -1 & -1 & -2 & 7 & -2 \\ -1 & -1 & -1 & -2 & -2 & 7 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* terlebih dahulu akan di cari polinomial karakteristik dari matriks tersebut dengan cara:

$$\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right)-\lambda\boldsymbol{I}\right)=$$

$$\det\begin{pmatrix} 7-\lambda & -2 & -2 & -1 & -1 & -1 \\ -2 & 7-\lambda & -2 & -1 & -1 & -1 \\ -2 & -2 & 7-\lambda & -1 & -1 & -1 \\ -1 & -1 & -1 & 7-\lambda & -2 & -2 \\ -1 & -1 & -1 & -2 & 7-\lambda & -2 \\ -1 & -1 & -1 & -2 & -2 & 7-\lambda \end{pmatrix}$$

Matriks tersebut direduksi menggunakan eliminasi baris elementer pada aplikasi Maple 18 dan diperoleh matriks segitiga atas sebagai berikut

$$\begin{bmatrix} 7-\lambda & -2 & -2 & -1 & -1 & -1 \\ 0 & -\frac{(\lambda-5)(\lambda-9)}{-7+\lambda} & -\frac{2(\lambda-9)}{-7+\lambda} & -\frac{\lambda-9}{-7+\lambda} & -\frac{\lambda-9}{-7+\lambda} & -\frac{\lambda-9}{-7+\lambda} \\ 0 & 0 & -\frac{(\lambda-3)(\lambda-9)}{\lambda-5} & -\frac{\lambda-9}{\lambda-5} & -\frac{\lambda-9}{\lambda-5} & -\frac{\lambda-9}{\lambda-5} \\ 0 & 0 & 0 & -\frac{\lambda^2-10\lambda+18}{\lambda-3} & -\frac{2\lambda-9}{\lambda-3} & -\frac{2\lambda-9}{\lambda-3} \\ 0 & 0 & 0 & 0 & -\frac{(\lambda-9)(\lambda^2-8\lambda+9)}{\lambda^2-10\lambda+18} & -\frac{(\lambda-9)(2\lambda-9)}{\lambda^2-10\lambda+18} \\ 0 & 0 & 0 & 0 & 0 & -\frac{(\lambda-6)(\lambda-9)\lambda}{\lambda^2-8\lambda+9} \end{bmatrix}$$

Sehingga polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ diperoleh dari perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut adalah

$$\begin{aligned} p(\lambda) &= (7-\lambda)\left(-\frac{(\lambda-5)(\lambda-9)}{-7+\lambda}\right)\left(-\frac{(\lambda-3)(\lambda-9)}{\lambda-5}\right)\left(-\frac{\lambda^2-10\lambda+18}{\lambda-3}\right) \\ &\quad \left(-\frac{(\lambda-9)(\lambda^2-8\lambda+9)}{\lambda^2-10\lambda+18}\right)\left(-\frac{(\lambda-6)(\lambda-9)\lambda}{\lambda^2-8\lambda+9}\right) \\ &= (\lambda-9)^4(\lambda-6)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=9, \lambda_2=6$, dan $\lambda_3=0$ dengan multiplisitas masing-masing yaitu $m(\lambda_1)=4, m(\lambda_2)=1$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_6)}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$

$$= (4 \cdot |9|) + (1 \cdot |6|) + (1 \cdot |0|)$$

$$= 42$$

Sedangkan matriks *distance signless Laplacian* diperoleh dari perhitungan $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ s \\ sr \\ sr^2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & r & r^2 & s & sr & sr^2 \end{array} \\ \begin{bmatrix} 7 & 2 & 2 & 1 & 1 & 1 \\ 2 & 7 & 2 & 1 & 1 & 1 \\ 2 & 2 & 7 & 1 & 1 & 1 \\ 1 & 1 & 1 & 7 & 2 & 2 \\ 1 & 1 & 1 & 2 & 7 & 2 \\ 1 & 1 & 1 & 2 & 2 & 7 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance signless Laplacian* terlebih dahulu akan di cari polinomial karakteristik dari matriks tersebut dengan cara:

$$\det\left(\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right) - \lambda^+ \boldsymbol{I}\right) =$$

$$\det\begin{pmatrix} 7-\lambda^+ & 2 & 2 & 1 & 1 & 1 \\ 2 & 7-\lambda^+ & 2 & 1 & 1 & 1 \\ 2 & 2 & 7-\lambda^+ & 1 & 1 & 1 \\ 1 & 1 & 1 & 7-\lambda^+ & 2 & 2 \\ 1 & 1 & 1 & 2 & 7-\lambda^+ & 2 \\ 1 & 1 & 1 & 2 & 2 & 7-\lambda^+ \end{pmatrix}$$

Matriks tersebut direduksi menggunakan eliminasi baris elementer pada aplikasi Maple 18 dan diperoleh matriks segitiga atas sebagai berikut

$$\begin{bmatrix} 7-\lambda^+ & 2 & 2 & 1 & 1 & 1 \\ 0 & -\frac{(\lambda^+-5)(\lambda^+-9)}{-7+\lambda^+} & \frac{2(\lambda^+-5)}{-7+\lambda^+} & \frac{\lambda^+-5}{-7+\lambda^+} & \frac{\lambda^+-5}{-7+\lambda^+} & \frac{\lambda^+-5}{-7+\lambda^+} \\ 0 & 0 & -\frac{(\lambda^+-5)(\lambda^+-11)}{\lambda^+-9} & \frac{\lambda^+-5}{\lambda^+-9} & \frac{\lambda^+-5}{\lambda^+-9} & \frac{\lambda^{\wedge}+-5}{\lambda^+-9} \\ 0 & 0 & 0 & -\frac{\lambda^{+2}-18\lambda^++74}{\lambda^+-11} & \frac{2\lambda^+-19}{\lambda^+-11} & \frac{2\lambda^+-19}{\lambda^+-11} \\ 0 & 0 & 0 & 0 & -\frac{(\lambda^+-5)(\lambda^{+2}-20\lambda^++93)}{\lambda^{+2}-18\lambda^++74} & \frac{(\lambda^+-5)(2\lambda^+-19)}{\lambda^{+2}-18\lambda^++74} \\ 0 & 0 & 0 & 0 & 0 & -\frac{(\lambda^+-5)(\lambda^+-8)(\lambda^+-14)}{\lambda^{+2}-20\lambda^++93} \end{bmatrix}$$

Sehingga polinomial karakteristik dari $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r \rangle}(D_6)}\right)$ diperoleh dari perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut adalah

$$\begin{aligned} p(\lambda^{+}) &= (7-\lambda^{+})\left(-\frac{(\lambda^{+}-5)(\lambda^{+}-9)}{-7+\lambda^{+}}\right)\left(-\frac{(\lambda^{+}-5)(\lambda^{+}-11)}{\lambda^{+}-9}\right)\left(-\frac{{\lambda^{+}}^{2}-18\lambda^{+}+74}{\lambda^{+}-11}\right) \\ &\quad \left(-\frac{(\lambda^{+}-5)\left({\lambda^{+}}^{2}-20\lambda^{+}+93\right)}{{\lambda^{+}}^{2}-18\lambda^{+}+74}\right)\left(-\frac{(\lambda^{+}-5)(\lambda^{+}-8)(\lambda^{+}-14)}{{\lambda^{+}}^{2}-20\lambda^{+}+93}\right) \\ &= (\lambda^{+}-5)^{4}(\lambda^{+}-8)(\lambda^{+}-14) \end{aligned}$$

Dengan menetapkan $p(\lambda^{+}) = 0$, diperoleh nilai eigen $\lambda_1^{+} = 5, \lambda_2^{+} = 8,$ dan $\lambda_3^{+} = 14$ dengan multiplisitas masing-masing yaitu $m(\lambda_1^{+}) = 4, m(\lambda_2^{+}) = 1,$ dan $m(\lambda_3^{+}) = 1$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_6)}$ adalah

$$\begin{aligned} ED_L^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_6)}\right) &= \sum_{i=1}^{3} m(\lambda_i^{+})\left|{\lambda_i}^{+}\right| \\ &= (4 \cdot |5|) + (1 \cdot |8|) + (1 \cdot |14|) \\ &= 42. \end{aligned}$$

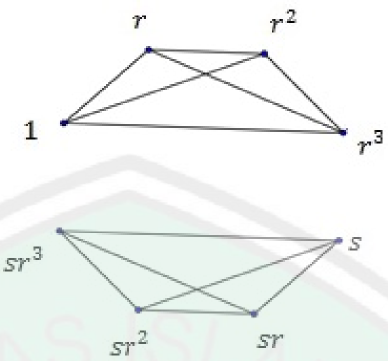
3.1.2 Grup Dihedral D_8

Diketahui anggota grup D_8 adalah $\{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$. Subgrup yang dibangun oleh $\langle r\rangle$ pada grup dihedral D_8 adalah $\{1, r, r^2, r^3\}$. Operasi unsur-unsur di D_8 yang hasilnya berada di $\langle r\rangle$ seperti pada tabel berikut

Tabel 3.2 Tabel Cayley Grup Dihedral D_8

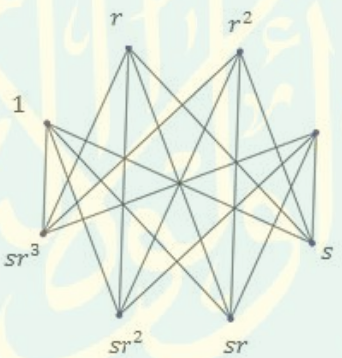
$\circ$	1	r	r^2	r^3	s	sr	sr^2	sr^3
1	1	r	r^2	r^3	s	sr	sr^2	sr^3
r	r	r^2	r^3	1	sr^3	s	sr	sr^2
r^2	r^2	r^3	1	r	sr^2	sr^3	s	sr
r^3	r^3	1	r	r^2	sr	sr^2	sr^3	s
s	s	sr	sr^2	sr^3	1	r	r^2	r^3
sr	sr	sr^2	sr^3	s	r^3	1	r	r^2
sr^2	sr^2	sr^3	s	sr	r^2	r^3	1	r
sr^3	sr^3	s	sr	sr^2	r	r^2	r^3	1

Berdasarkan Tabel 3.2 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r\rangle}(D_8)$ sebagai berikut



Gambar 3.3 Graf $\Gamma_{\langle r\rangle}(D_8)$

Dari Gambar 3.3 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_8)$ sebagai berikut



Gambar 3.4 Graf $\overline{\Gamma_{\langle r\rangle}(D_8)}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 2 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \end{array}$$

Matriks distance Laplacian diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 10 & -2 & -2 & -2 & -1 & -1 & -1 & -1 \\ -2 & 10 & -2 & -2 & -1 & -1 & -1 & -1 \\ -2 & -2 & 10 & -2 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & 10 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 10 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -2 & 10 & -2 & -2 \\ -1 & -1 & -1 & -1 & -2 & -2 & 10 & -2 \\ -1 & -1 & -1 & -1 & -2 & -2 & -2 & 10 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh polinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda) &= (10-\lambda)\left(-\frac{(\lambda-8)(\lambda-12)}{-10+\lambda}\right)\left(-\frac{(\lambda-6)(\lambda-12)}{\lambda-8}\right)\left(-\frac{(\lambda-4)(\lambda-12)}{\lambda-6}\right)\left(-\frac{\lambda^2-14\lambda+36}{\lambda-4}\right) \\ &\quad \left(-\frac{(\lambda-12)(\lambda^2-12\lambda+24)}{\lambda^2-14\lambda+36}\right)\left(-\frac{(\lambda-12)(\lambda^2-10\lambda+12)}{\lambda^2-12\lambda+24}\right)\left(-\frac{(\lambda-8)(\lambda-12)\lambda}{\lambda^2-10\lambda+12}\right) \\ &= (\lambda-12)^6(\lambda-8)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 12, \lambda_2 = 8$, dan $\lambda_3 = -0$ dengan multiplisitas masing-masing $m(\lambda_1) = 6, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_8)}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$

$= (6 \cdot |12|) + (1 \cdot |8|) + (1 \cdot |-0|)$

$= 80$

Sedangkan matriks *distance signless Laplacian* diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 10 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\ 2 & 10 & 2 & 2 & 1 & 1 & 1 & 1 \\ 2 & 2 & 10 & 2 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 10 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 10 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 10 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 10 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 10 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda^+) &= (10-\lambda^+)\left(-\frac{(\lambda^+-8)(\lambda^+-12)}{-10+\lambda^+}\right)\left(-\frac{(\lambda^+-8)(\lambda^+-14)}{\lambda^+-12}\right)\left(-\frac{(\lambda^+-8)(\lambda^+-16)}{\lambda^+-14}\right) \\ &\quad \left(-\frac{{\lambda^+}^2-26\lambda^++156}{\lambda^+-16}\right)\left(-\frac{(\lambda^+-8)\left({\lambda^+}^2-28\lambda^++184\right)}{{\lambda^+}^2-26\lambda^++156}\right)\left(-\frac{(\lambda^+-8)\left({\lambda^+}^2-30\lambda^++212\right)}{{\lambda^+}^2-28\lambda^++184}\right) \\ &\quad \left(-\frac{(\lambda^+-20)(\lambda^+-12)(\lambda^+-8)}{{\lambda^+}^2-10\lambda^++12}\right) \\ &= (\lambda^+-20)(\lambda^+-12)(\lambda^+-8)^6 \end{aligned}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 20, \lambda_2^+ = 12$, dan $\boldsymbol{\lambda_3^+} = 8$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 6$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_8)}$ adalah

$$\begin{aligned} ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\ &= (1 \cdot |20|) + (1 \cdot |12|) + (6 \cdot |8|) \\ &= 80. \end{aligned}$$

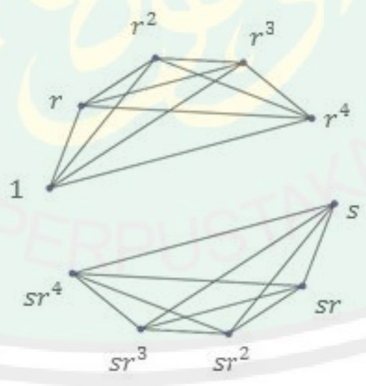
3.1.3 Grup Dihedral D_{10}

Diketahui $D_{10} = \{1, r, r^2, r^3, r^4, s, sr, sr^2, sr^3, sr^4\}$. Subgrup yang dibangun oleh $\langle r \rangle$ pada grup dihedral D_{10} adalah $\{1, r, r^2, r^3, r^4\}$. Operasi unsur-unsur di D_{10} yang hasilnya berada di $\langle r \rangle$ seperti pada tabel berikut

Tabel 3.3 Tabel Cayley Grup Dihedral D_{10}

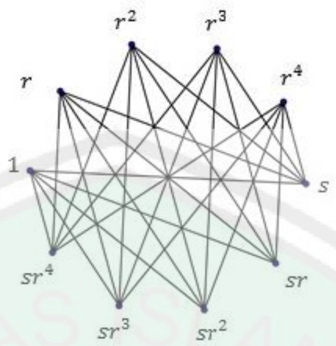
$\circ$	1	r	r^2	r^3	r^4	s	sr	sr^2	sr^3	sr^4
1	1	r	r^2	r^3	r^4	s	sr	sr^2	sr^3	sr^4
r	r	r^2	r^3	r^4	1	sr^4	s	sr	sr^2	sr^3
r^2	r^2	r^3	r^4	1	r	sr^3	sr^4	s	sr	sr^2
r^3	r^3	r^4	1	r	r^2	sr^2	sr^3	sr^4	s	sr
r^4	r^4	1	r	r^2	r^3	sr	sr^2	sr^3	sr^4	s
s	s	sr^4	sr^3	sr^2	sr	1	r	r^2	r^3	r^4
sr	sr	s	sr^4	sr^3	sr^2	r^4	1	r	r^2	r^3
sr^2	sr^2	sr	s	sr^4	sr^3	r^3	r^4	1	r	r^2
sr^3	sr^3	sr^2	sr	s	sr^4	r^2	r^3	r^4	1	r
sr^4	sr^4	sr^3	sr^2	sr	s	r	r^2	r^3	r^4	1

Berdasarkan Tabel 3.3 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r \rangle}(D_{10})$ sebagai berikut



Gambar 3.5 Graf $\Gamma_{\langle r \rangle}(D_{10})$

Dari Gambar 3.5 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{10})}$ sebagai berikut



Gambar 3.6 Graf $\overline{\Gamma_{\langle r\rangle}(D_{10})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & r & r^2 & r^3 & r^4 & s & sr & sr^2 & sr^3 & sr^4 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & r & r^2 & r^3 & r^4 & s & sr & sr^2 & sr^3 & sr^4 \end{array} \\ \begin{bmatrix} 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 \end{bmatrix} \end{array}$$

Matriks distance Laplacian yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right)$ adalah

$$D_L\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = \begin{array}{c c} & \begin{array}{cccccccccc} 1 & r & r^2 & r^3 & r^4 & s & sr & sr^2 & sr^3 & sr \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \end{array} & \begin{bmatrix} 13 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 \\ -2 & 13 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & 13 & -2 & -2 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & 13 & -2 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & -2 & 13 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 13 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -2 & 13 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -2 & -2 & 13 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & 13 & -2 \\ -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & 13 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda) &= (13-\lambda)\left(-\frac{(\lambda-11)(\lambda-15)}{-13+\lambda}\right)\left(-\frac{(\lambda-9)(\lambda-15)}{\lambda-11}\right)\left(-\frac{(\lambda-7)(\lambda-15)}{\lambda-9}\right) \\ &\quad \left(-\frac{(\lambda-5)(\lambda-15)}{\lambda-7}\right)\left(-\frac{\lambda^2-18\lambda+60}{\lambda-5}\right)\left(-\frac{(\lambda-15)(\lambda^2-16\lambda+45)}{\lambda^2-18\lambda+60}\right)\left(-\frac{(\lambda-15)(\lambda^2-14\lambda+30)}{\lambda^2-16\lambda+45}\right) \\ &\quad \left(-\frac{(\lambda-15)(\lambda^2-12\lambda+15)}{\lambda^2-14\lambda+30}\right)\left(-\frac{(\lambda-10)(\lambda-15)\lambda}{\lambda^2-12\lambda+15}\right) \\ &= (\lambda-15)^8(\lambda-10)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1 = 15, \lambda_2 = 10$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 8, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{10})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (8\cdot|15|) + (1\cdot|10|) + (1\cdot|0|) \\ &= 130 \end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $D_L{}^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = T_r\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) + D\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right)$ adalah

$$D_L{}^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & r & r^2 & r^3 & r^4 & s & sr & sr^2 & sr^3 & sr^4 \end{array} \\ \begin{bmatrix} 13 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 13 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 13 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 13 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 13 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 13 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 13 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 13 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 13 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 13 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda^+) &= (13-\lambda^+)\left(-\frac{(\lambda^+-11)(\lambda^+-15)}{-13+\lambda^+}\right)\left(-\frac{(\lambda^+-11)(\lambda^+-17)}{\lambda^+-15}\right)\left(-\frac{(\lambda^+-11)(\lambda^+-19)}{\lambda^+-17}\right) \\ &\quad \left(-\frac{(\lambda^+-11)(\lambda^+-21)}{\lambda^+-19}\right)\left(-\frac{{\lambda^+}^2-34\lambda^++268}{\lambda^+-21}\right)\left(-\frac{(\lambda^+-11)\left({\lambda^+}^2-36\lambda^++305\right)}{\lambda^2-34\lambda+268}\right) \\ &\quad \left(-\frac{(\lambda^+-11)\left({\lambda^+}^2-38\lambda^++342\right)}{{\lambda^+}^2-36\lambda^++305}\right)\left(-\frac{(\lambda^+-11)\left({\lambda^+}^2-40\lambda^++379\right)}{{\lambda^+}^2-38\lambda^++342}\right) \\ &\quad \left(-\frac{(\lambda^+-11)(\lambda^+-26)(\lambda^+-16)}{{\lambda^+}^2-40\lambda^++379}\right) \\ &= (\lambda^+-26)(\lambda^+-16)(\lambda^+-11)^8 \end{aligned}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 26, \lambda_2^+ = 16$, dan $\lambda_3^+ = 11$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 8$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{10})}$ adalah

$$\begin{aligned} ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{10})}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\ &= (1\cdot|26|) + (1\cdot|16|) + (8\cdot|11|) \\ &= 130. \end{aligned}$$

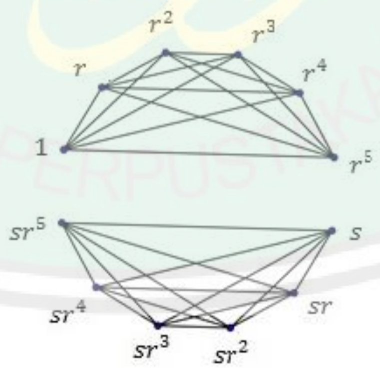
3.1.4 Grup Dihedral D_{12}

Diketahui $D_{12} = \{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$. Subgrup yang dibangun oleh $\langle r \rangle$ pada grup dihedral D_{12} adalah $\{1, r, r^2, r^3, r^4, r^5\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r \rangle$ seperti pada tabel berikut

Tabel 3.4 Tabel Cayley Grup Dihedral D_{12}

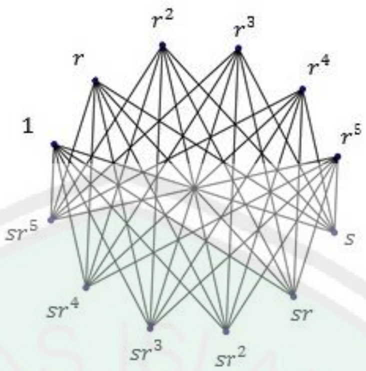
$\circ$	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
1	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
r	r	r^2	r^3	r^4	r^5	1	sr^5	s	sr	sr^2	sr^3	sr^4
r^2	r^2	r^3	r^4	r^5	1	r	sr^4	sr^5	s	sr	sr^2	sr^3
r^3	r^3	r^4	r^5	1	r	r^2	sr^3	sr^4	sr^5	s	sr	sr^2
r^4	r^4	r^5	1	r	r^2	r^3	sr^2	sr^3	sr^4	sr^5	s	sr
r^5	r^5	1	r	r^2	r^3	r^4	sr	sr^2	sr^3	sr^4	sr^5	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	1	r	r^2	r^3	r^4	r^5
sr	sr	sr^2	sr^3	sr^4	sr^5	s	r^5	1	r	r^2	r^3	r^4
sr^2	sr^2	sr^3	sr^4	sr^5	s	sr	r^4	r^5	1	r	r^2	r^3
sr^3	sr^3	sr^4	sr^5	s	sr	sr^2	r^3	r^4	r^5	1	r	r^2
sr^4	sr^4	sr^5	s	sr	sr^2	sr^3	r^2	r^3	r^4	r^5	1	r
sr^5	sr^5	s	sr	sr^2	sr^3	sr^4	r	r^2	r^3	r^4	r^5	1

Berdasarkan Tabel 3.3 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r \rangle}(D_{12})$ sebagai berikut



Gambar 3.7 Graf $\Gamma_{\langle r \rangle}(D_{12})$

Dari Gambar 3.7 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{12})}$ sebagai berikut



Gambar 3.8 Graf $\overline{\Gamma_{\langle r\rangle}(D_{12})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)$ adalah

$$D_L\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)=\begin{array}{c}\\ 1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5\end{array}\begin{array}{c}\begin{array}{cccccccccccc}1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5\end{array}\\ \begin{bmatrix}16 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & 16 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & -2 & 16 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & -2 & -2 & 16 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & -2 & -2 & -2 & 16 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & -2 & -2 & -2 & -2 & 16 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & 16 & -2 & -2 & -2 & -2 & -2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & 16 & -2 & -2 & -2 & -2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & 16 & -2 & -2 & -2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & 16 & -2 & -2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & 16 & -2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & 16\end{bmatrix}\end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned}p(\lambda) &= (16-\lambda)\left(-\frac{(\lambda-14)(\lambda-18)}{-16+\lambda}\right)\left(-\frac{(\lambda-12)(\lambda-18)}{\lambda-14}\right)\left(-\frac{(\lambda-10)(\lambda-18)}{\lambda-12}\right)\\ &\quad\left(-\frac{(\lambda-8)(\lambda-18)}{\lambda-10}\right)\left(-\frac{(\lambda-6)(\lambda-18)}{\lambda-8}\right)\left(-\frac{\lambda^2-22\lambda+90}{\lambda-6}\right)\left(-\frac{(\lambda-18)(\lambda^2-20\lambda+72)}{\lambda^2-22\lambda+90}\right)\\ &\quad\left(-\frac{(\lambda-18)(\lambda^2-18\lambda+54)}{\lambda^2-20\lambda+72}\right)\left(-\frac{(\lambda-18)(\lambda^2-16\lambda+36)}{\lambda^2-18\lambda+54}\right)\left(-\frac{\lambda^3-32\lambda^2+272\lambda-342}{\lambda^2-16\lambda+36}\right)\\ &\quad\left(-\frac{(\lambda-8)(\lambda^3-30\lambda^2+218\lambda-18)}{\lambda^3-32\lambda^2+272\lambda-342}\right)\\ &= (\lambda-18)^{10}(\lambda-12)\lambda\end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=18, \lambda_2=12$, dan $\lambda_3=0$ dengan multiplisitas masing-masing $m(\lambda_1)=10, m(\lambda_2)=1$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{12})}$ adalah

$$\begin{aligned}ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|\\ &= (10\cdot|18|)+(1\cdot|12|)+(1\cdot|0|)\\ &= 192\end{aligned}$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)$ adalah

$$
\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right)=
\begin{array}{c}
\\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\
\begin{bmatrix}
16 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 16 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 16 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 16 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 16 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 & 2 & 16 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 16 & 2 & 2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 2 & 16 & 2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 16 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 16 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 16 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 16
\end{bmatrix}
\end{array}
$$

Dengan menggunakan cara yang sama pada grup dihedral D_6, diperoleh poinomial karakteristik sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (16-\lambda^+)\left(-\frac{(\lambda^+-14)(\lambda^+-18)}{-16+\lambda^+}\right)\left(-\frac{(\lambda^+-14)(\lambda^+-20)}{\lambda^+-18}\right)\left(-\frac{(\lambda^+-14)(\lambda^+-22)}{\lambda^+-20}\right) \\
&\quad \left(-\frac{(\lambda^+-14)(\lambda^+-24)}{\lambda^+-22}\right)\left(-\frac{(\lambda^+-14)(\lambda^+-26)}{\lambda^+-24}\right)\left(-\frac{{\lambda^+}^2-42\lambda^++410}{\lambda^+-26}\right) \\
&\quad \left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-44\lambda^++456\right)}{{\lambda^+}^2-42\lambda^++410}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-46\lambda^++502\right)}{{\lambda^+}^2-44\lambda^++456}\right) \\
&\quad \left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-48\lambda^++548\right)}{{\lambda^+}^2-46\lambda^++502}\right)\left(-\frac{{\lambda^+}^3-64{\lambda^+}^2+1296\lambda^+-8362}{{\lambda^+}^2-48\lambda^++548}\right) \\
&\quad \left(-\frac{(\lambda^+-14)\left({\lambda^+}^3-66{\lambda^+}^2+1370\lambda^+-9006\right)}{{\lambda^+}^3-64{\lambda^+}^2+1296\lambda^+-8362}\right) \\
&= (\lambda^+-32)(\lambda^+-20)(\lambda^+-14)^{10}
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+)=0$, diperoleh nilai eigen $\lambda_1^+=32, \lambda_2^+=20$, dan $\lambda_3^+=14$ dengan multiplisitas masing-masing $m(\lambda_1^+)=1, m(\lambda_2^+)=1$, dan $m(\lambda_3^+)=10$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{12})}$ adalah

$$
\begin{aligned}
ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{12})}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\
&= (1\cdot|32|)+(1\cdot|20|)+(10\cdot|14|) \\
&= 192.
\end{aligned}
$$

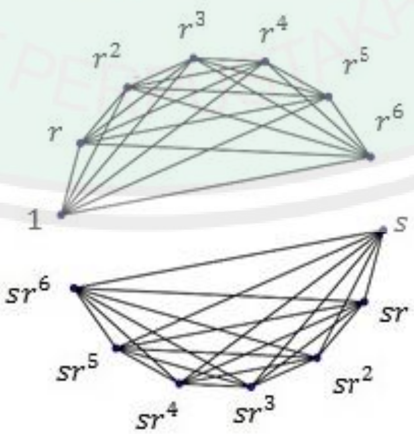
3.1.5 Grup Dihedral D_{14}

Diketahui $D_{14} = \{1, r, r^2, r^3, \dots, r^6, s, sr, sr^2, sr^3, \dots, sr^6\}$. Subgrup yang dibangun oleh $\langle r \rangle$ pada grup dihedral D_{14} adalah $\{1, r, r^2, r^3, r^4, r^5, r^6\}$. Operasi unsur-unsur di D_{14} yang hasilnya berada di $\langle r \rangle$ seperti pada tabel berikut

Tabel 3.5 Tabel Cayley Grup Dihedral D_{14}

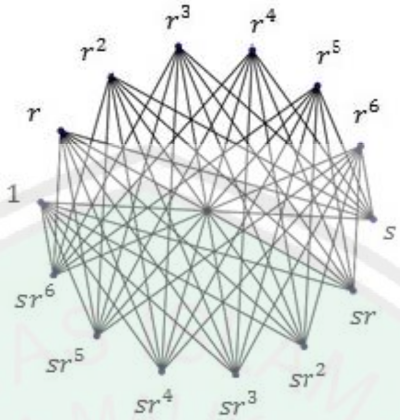
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
1	1	r	r^2	r^3	r^4	r^5	r^6	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r	r	r^2	r^3	r^4	r^5	r^6	1	sr^6	s	sr	sr^2	sr^3	sr^4	sr^5
r^2	r^2	r^3	r^4	r^5	r^6	1	r	sr^5	sr^6	s	sr	sr^2	sr^3	sr^4
r^3	r^3	r^4	r^5	r^6	1	r	r^2	sr^4	sr^5	sr^6	s	sr	sr^2	sr^3
r^4	r^4	r^5	r^6	1	r	r^2	r^3	sr^3	sr^4	sr^5	sr^6	s	sr	sr^2
r^5	r^5	r^6	1	r	r^2	r^3	r^4	sr^2	sr^3	sr^4	sr^5	sr^6	s	sr
r^6	r^6	1	r	r^2	r^3	r^4	r^5	sr	sr^2	sr^3	sr^4	sr^5	sr^6	s
s	s	sr^6	sr^5	sr^4	sr^3	sr^2	sr	1	r	r^2	r^3	r^4	r^5	r^6
sr	sr	s	sr^6	sr^5	sr^4	sr^3	sr^2	r^6	1	r	r^2	r^3	r^4	r^5
sr^2	sr^2	sr	s	sr^6	sr^5	sr^4	sr^3	r^5	r^6	1	r	r^2	r^3	r^4
sr^3	sr^3	sr^2	sr	s	sr^6	sr^5	sr^4	r^4	r^5	r^6	1	r	r^2	r^3
sr^4	sr^4	sr^3	sr^2	sr	s	sr^6	sr^5	r^3	r^4	r^5	r^6	1	r	r^2
sr^5	sr^5	sr^4	sr^3	sr^2	sr	s	sr^6	r^2	r^3	r^4	r^5	r^6	1	r
sr^6	sr^6	sr^5	sr^4	sr^3	sr^2	sr	s	r	r^2	r^3	r^4	r^5	r^6	1

Berdasarkan Tabel 3.5 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r \rangle}(D_{14})$ sebagai berikut



Gambar 3. 9 Graf $\Gamma_{\langle r \rangle}(D_{14})$

Dari Gambar 3.9 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{14})}$ sebagai berikut



Gambar 3.10 Graf $\overline{\Gamma_{\langle r\rangle}(D_{14})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 \end{array} \\ \begin{bmatrix} 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \end{array} & \begin{bmatrix} 19 & -2 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & 19 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & 19 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & 19 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & -2 & 19 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & -2 & -2 & 19 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -2 & -2 & -2 & -2 & -2 & 19 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 19 & -2 & -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & 19 & -2 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & 19 & -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & 19 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & 19 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & 19 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & -2 & 19 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda) &= (19-\lambda)\left(-\frac{(\lambda-17)(\lambda-21)}{-19+\lambda}\right)\left(-\frac{(\lambda-15)(\lambda-21)}{\lambda-17}\right)\left(-\frac{(\lambda-13)(\lambda-21)}{\lambda-15}\right) \\ &\quad \left(-\frac{(\lambda-11)(\lambda-21)}{\lambda-13}\right)\left(-\frac{(\lambda-9)(\lambda-21)}{\lambda-11}\right)\left(-\frac{(\lambda-7)(\lambda-21)}{\lambda-9}\right)\left(-\frac{\lambda^2-26\lambda+126}{\lambda-7}\right) \\ &\quad \left(-\frac{(\lambda-21)(\lambda^2-24\lambda+105)}{\lambda^2-26\lambda+126}\right)\left(-\frac{(\lambda-21)(\lambda^2-22\lambda+84)}{\lambda^2-24\lambda+105}\right)\left(-\frac{(\lambda-21)(\lambda^2-20\lambda+63)}{\lambda^2-22\lambda+84}\right) \\ &\quad \left(-\frac{(\lambda-21)(\lambda^2-18\lambda+42)}{\lambda^2-20\lambda+63}\right)\left(-\frac{(\lambda-21)(\lambda^2-16\lambda+21)}{\lambda^2-18\lambda+42}\right)\left(-\frac{(\lambda-14)(\lambda-21)\lambda}{\lambda^2-16\lambda+21}\right) \\ &= (\lambda-21)^{12}(\lambda-14)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1 = 21, \lambda_2 = 14$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 12, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{14})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (12\cdot|21|) + (1\cdot|14|) + (1\cdot|0|) \end{aligned}$$

$= 266$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r\rangle}(D_{14})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r\rangle}(D_{14})\right)} + \boldsymbol{D}\overline{\left(\Gamma_{\langle r\rangle}(D_{14})\right)}$ adalah

$$\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r\rangle}(D_{14})\right)} = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 \end{array} \\ \begin{bmatrix} 19 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 19 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 19 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 19 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 19 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 19 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 19 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 19 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 19 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 19 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 19 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 19 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 19 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 19 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda^+) &= (19-\lambda^+)\left(-\frac{(\lambda^+-17)(\lambda^+-21)}{-19+\lambda^+}\right)\left(-\frac{(\lambda^+-17)(\lambda^+-23)}{\lambda^+-21}\right)\left(-\frac{(\lambda^+-17)(\lambda^+-25)}{\lambda^+-23}\right) \\ &\quad \left(-\frac{(\lambda^+-17)(\lambda^+-27)}{\lambda^+-25}\right)\left(-\frac{(\lambda^+-17)(\lambda^+-29)}{\lambda^+-27}\right)\left(-\frac{(\lambda^+-17)(\lambda^+-31)}{\lambda^+-29}\right) \\ &\quad \left(-\frac{{\lambda^+}^2-50\lambda^++582}{\lambda^+-31}\right)\left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-52\lambda^++637\right)}{{\lambda^+}^2-50\lambda^++582}\right)\left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-54\lambda^++692\right)}{{\lambda^+}^2-52\lambda^++637}\right) \\ &\quad \left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-56\lambda^++747\right)}{{\lambda^+}^2-54\lambda^++692}\right)\left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-58\lambda^++802\right)}{{\lambda^+}^2-56\lambda^++747}\right) \\ &\quad \left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-60\lambda^++857\right)}{{\lambda^+}^2-58\lambda^++802}\right)\left(-\frac{(\lambda^+-17)(\lambda^+-38)(\lambda^+-24)}{{\lambda^+}^2-60\lambda^++857}\right) \\ &= (\lambda^+-38)(\lambda^+-24)(\lambda^+-17)^{12} \end{aligned}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 38, \lambda_2^+ = 24$, dan $\lambda_3^+ = 17$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 12$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{14})}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{14})}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1 \cdot |38|) + (1 \cdot |24|) + (10 \cdot |17|)$$

$$= 266.$$

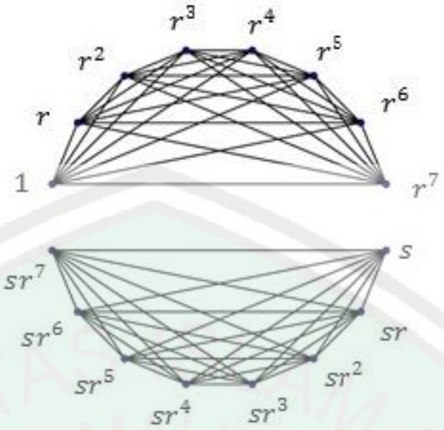
3.1.6 Grup Dihedral D_{16}

Diketahui $D_{16} = \{1, r, r^2, r^3, \dots, r^7, s, sr, sr^2, sr^3, \dots, sr^7\}$. Subgrup yang dibangun oleh $\langle r\rangle$ pada grup dihedral D_{16} adalah $\{1, r, r^2, r^3, r^4, r^5, r^6, r^7\}$. Operasi unsur-unsur di D_{16} yang hasilnya berada di $\langle r\rangle$ seperti pada tabel berikut

Tabel 3.6 Tabel Cayley Grup Dihedral D_{16}

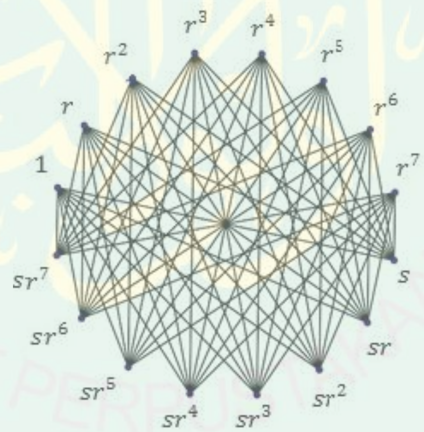
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r	r	r^2	r^3	r^4	r^5	r^6	r^7	1	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^2	r^2	r^3	r^4	r^5	r^6	r^7	1	r	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5
r^3	r^3	r^4	r^5	r^6	r^7	1	r	r^2	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4
r^4	r^4	r^5	r^6	r^7	1	r	r^2	r^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3
r^5	r^5	r^6	r^7	1	r	r^2	r^3	r^4	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2
r^6	r^6	r^7	1	r	r^2	r^3	r^4	r^5	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr
r^7	r^7	1	r	r^2	r^3	r^4	r^5	r^6	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	r^7	1	r	r^2	r^3	r^4	r^5	r^6
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	r^6	r^7	1	r	r^2	r^3	r^4	r^5
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	r^5	r^6	r^7	1	r	r^2	r^3	r^4
sr^4	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	r^4	r^5	r^6	r^7	1	r	r^2	r^3
sr^5	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	r^3	r^4	r^5	r^6	r^7	1	r	r^2
sr^6	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	r^2	r^3	r^4	r^5	r^6	r^7	1	r
sr^7	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r	r^2	r^3	r^4	r^5	r^6	r^7	1

Berdasarkan Tabel 3.6 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r\rangle}(D_{16})$ sebagai berikut



Gambar 3.11 Graf $\Gamma_{\langle r\rangle}(D_{16})$

Dari Gambar 3.11 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{16})}$ sebagai berikut



Gambar 3.12 Graf $\overline{\Gamma_{\langle r\rangle}(D_{16})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \begin{bmatrix} 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 \end{bmatrix} \end{array}$$

Dmengan demikian matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right)$ adalah

$$\boldsymbol{D}_L\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \left[\begin{array}{cccccccccccccccc}
22 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & 22 & -2 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & 22 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & -2 & 22 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & -2 & -2 & 22 & -2 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & -2 & -2 & -2 & 22 & -2 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & -2 & -2 & -2 & -2 & 22 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -2 & -2 & -2 & -2 & -2 & -2 & 22 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 22 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & 22 & -2 & -2 & -2 & -2 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & 22 & -2 & -2 & -2 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & 22 & -2 & -2 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & 22 & -2 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & 22 & -2 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & -2 & 22 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & 22
\end{array}\right] \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda) &= (22-\lambda)\left(-\frac{(\lambda-20)(\lambda-24)}{-22+\lambda}\right)\left(-\frac{(\lambda-18)(\lambda-24)}{\lambda-20}\right)\left(-\frac{(\lambda-16)(\lambda-24)}{\lambda-18}\right) \\ &\quad \left(-\frac{(\lambda-14)(\lambda-24)}{\lambda-16}\right)\left(-\frac{(\lambda-12)(\lambda-24)}{\lambda-14}\right)\left(-\frac{(\lambda-10)(\lambda-24)}{\lambda-12}\right)\left(-\frac{(\lambda-8)(\lambda-24)}{\lambda-10}\right) \\ &\quad \left(-\frac{\lambda^2-30\lambda+168}{\lambda-8}\right)\left(-\frac{(\lambda-24)(\lambda^2-28\lambda+144)}{\lambda^2-30\lambda+168}\right)\left(-\frac{(\lambda-20)(\lambda-6)(\lambda-24)}{\lambda^2-28\lambda+144}\right) \\ &\quad \left(-\frac{(\lambda-24)(\lambda^2-24\lambda+96)}{(\lambda-6)(\lambda-20)}\right)\left(-\frac{(\lambda-24)(\lambda-4)(\lambda-18)}{\lambda^2-24\lambda+96}\right)\left(-\frac{(\lambda-24)(\lambda^2-20\lambda+48)}{(\lambda-4)(\lambda-18)}\right) \\ &\quad \left(-\frac{(\lambda-24)(\lambda^2-18\lambda+24)}{\lambda^2-20\lambda+48}\right)\left(-\frac{(\lambda-16)(\lambda-24)\lambda}{\lambda^2-18\lambda+24}\right) \\ &= (\lambda-24)^{14}(\lambda-16)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1 = 24, \lambda_2 = 16$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 14, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{16})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (14\cdot|24|)+(1\cdot|16|)+(1\cdot|0|) \\ &= 352 \end{aligned}$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r\rangle}(D_{16})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r\rangle}(D_{16})\right)} + \boldsymbol{D}\overline{\left(\Gamma_{\langle r\rangle}(D_{16})\right)}$ adalah

$$\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r\rangle}(D_{16})\right)} = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{bmatrix} 22 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 22 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 22 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 22 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 22 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 22 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 22 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 22 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 22 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 22 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 22 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 22 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 22 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 22 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 22 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 22 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_6), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda^+) &= (22-\lambda^+)\left(-\frac{(\lambda^+-20)(\lambda^+-24)}{-22+\lambda^+}\right)\left(-\frac{(\lambda^+-20)(\lambda^+-26)}{\lambda^+-24}\right)\left(-\frac{(\lambda^+-20)(\lambda^+-28)}{\lambda^+-26}\right) \\ &\quad \left(-\frac{(\lambda^+-20)(\lambda^+-30)}{\lambda^+-28}\right)\left(-\frac{(\lambda^+-20)(\lambda^+-32)}{\lambda^+-30}\right)\left(-\frac{(\lambda^+-20)(\lambda^+-34)}{\lambda^+-32}\right) \\ &\quad \left(-\frac{(\lambda^+-20)(\lambda^+-36)}{\lambda^+-34}\right)\left(-\frac{{\lambda^+}^2-58\lambda^++784}{\lambda^+-36}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-60\lambda^++848\right)}{{\lambda^+}^2-58\lambda^++784}\right) \\ &\quad \left(-\frac{(\lambda^+-20)(\lambda^+-38)(\lambda^+-24)}{{\lambda^+}^2-60\lambda^++848}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-64\lambda^++976\right)}{(\lambda^+-24)(\lambda^+-38)}\right) \\ &\quad \left(-\frac{(\lambda^+-26)(\lambda^+-40)(\lambda^+-20)}{{\lambda^+}^2-64\lambda^++976}\right)\left(-\frac{\lambda^+\left({\lambda^+}^2-68\lambda^++1104\right)}{(\lambda^+-40)(\lambda^+-20)}\right) \\ &\quad \left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-70\lambda^++1168\right)}{{\lambda^+}^2-68\lambda^++1104}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^3-72{\lambda^+}^2+1236\lambda^+-128\right)}{{\lambda^+}^2-70\lambda^++1168}\right) \\ &= (\lambda^+-44)(\lambda^+-28)(\lambda^+-20)^{14} \end{aligned}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 44, \lambda_2^+ = 28$, dan $\lambda_3^+ = 20$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 14$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{16})}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{16})}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1\cdot|44|) + (1\cdot|28|) + (14\cdot|20|)$$

$$= 352.$$

Dari *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ untuk $n \geq 3$ yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.7 Tabel pola *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$

D_{2n}	n	*DLE*	*DSLE*
D_6	3	$42 = 6\cdot 7$ $= 2\cdot 3(3\cdot 3-2)$	42
D_8	4	$80 = 8\cdot 10$ $= 2\cdot 4(3\cdot 4-2)$	80
D_{10}	5	$130 = 10\cdot 13$ $= 2\cdot 5(3\cdot 5-2)$	130
D_{12}	6	$192 = 12\cdot 16$ $= 2\cdot 6(\cdot 6-2)$	192
D_{14}	7	$266 = 14\cdot 19$ $= 2\cdot 7(3\cdot 7-2)$	266
D_{16}	8	$352 = 16\cdot 22$ $= 2\cdot 8(3\cdot 8-2)$	352
$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$2n(3n-2) = 6n^2-4n$	$6n^2-4n$

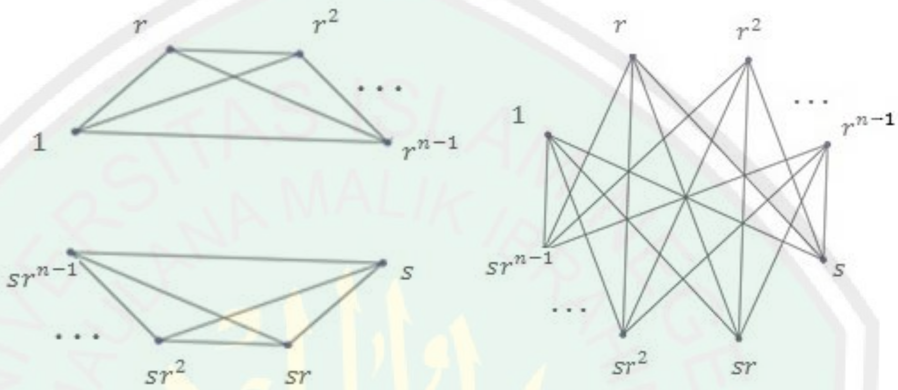
Teorema 1

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r\rangle = \{1, r, r^2, \dots, r^{n-1}\}$ subgrup normal dari D_{2n} untuk $n \geq 3$. *DLE* dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

Bukti:

Misalkan grup dihedral $D_{2n}=\{1,r,r^2,\dots,r^{n-1},s,sr,sr^2,\dots,sr^{n-1}\}$ untuk $n\geq 3$. Ambil $\langle r\rangle=\{1,r,r^2,\dots,r^{n-1}\}$ subgrup normal dari D_{2n}. Sesuai definisi graf subgrup dan komplemennya, maka diperoleh graf $\Gamma_{\langle r\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ yaitu:



Gambar 3.13 Graf $\Gamma_{\langle r\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$

Sehingga diperoleh matriks *distance* dari $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ \vdots\\ r^{n-2}\\ r^{n-1}\\ s\\ sr\\ sr^2\\ sr^3\\ \vdots\\ sr^{n-2}\\ sr^{n-1}\end{array}\begin{array}{c} \begin{array}{ccccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1}\end{array}\\ \begin{bmatrix} 0 & 2 & 2 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ 2 & 0 & 2 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ 2 & 2 & 0 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ 2 & 2 & 2 & 0 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots\\ 2 & 2 & 2 & 2 & \dots & 0 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ 2 & 2 & 2 & 2 & \dots & 2 & 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 0 & 2 & 2 & 2 & \dots & 2 & 2\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 0 & 2 & 2 & \dots & 2 & 2\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 0 & 2 & \dots & 2 & 2\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 0 & \dots & 2 & 2\\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 2 & \cdots & 0 & 2\\ 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 2 & \dots & 2 & 0 \end{bmatrix}\end{array}$$

dan matriks transmisi $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ adalah:

$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) =$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
3n-2 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 3n-2 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 3n-2 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 3n-2 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 3n-2 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 3n-2 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 3n-2 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 3n-2 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 3n-2 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 3n-2 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 3n-2 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 3n-2
\end{bmatrix}
\end{array}$$

Matriks *distance Laplacian Laplacian* yang diperoleh dari perhitungan

$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$ adalah

$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) =$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
3n-2 & -2 & -2 & -2 & \dots & -2 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & 3n-2 & -2 & -2 & \dots & -2 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & -2 & 3n-2 & -2 & \dots & -2 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & -2 & -2 & 3n-2 & \dots & -2 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2 & -2 & -2 & -2 & \dots & 3n-2 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & -2 & -2 & -2 & \dots & -2 & 3n-2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & 3n-2 & -2 & -2 & -2 & \dots & -2 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & 3n-2 & -2 & -2 & \dots & -2 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -2 & 3n-2 & -2 & \dots & -2 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -2 & -2 & 3n-2 & \dots & -2 & -2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -2 & -2 & -2 & \cdots & 3n-2 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -2 & -2 & -2 & \dots & -2 & 3n-2
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}\right)$.

Dengan eliminasi baris elementer pada $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$\begin{array}{cc}
 & \begin{array}{ccccc} 1 & r & r^2 & \dots & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1} \end{array} &
\begin{bmatrix}
(3n-2)-\lambda & \dots & \dots & \dots & \dots \\
0 & -\frac{(\lambda-(3n-4))(\lambda-3n)}{-(3n-2)+\lambda} & \dots & \dots & \dots \\
0 & 0 & -\frac{(\lambda-(3n-6))(\lambda-3n)}{\lambda-(3n-4)} & \dots & \dots \\
\vdots & \vdots & \vdots & & \dots \\
0 & 0 & 0 & \dots & -\frac{(\lambda-2n)(\lambda-3n)\lambda}{f(\lambda)}
\end{bmatrix}
\end{array}$$

Maka $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)-\lambda\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda)=\lambda(\lambda-2n)(\lambda-3n)^{2n-2}$$

Dengan menetapkan $p(\lambda)=0$ maka diperoleh $\lambda_1=0, \lambda_2=2n,$ dan $\lambda_3=3n$ dengan multiplisitas masing-masing $m(\lambda_1)=1, m(\lambda_2)=1,$ dan $m(\lambda_3)=2n-2$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) &= m(\lambda_1)|\lambda_1|+m(\lambda_2)|\lambda_2|+m(\lambda_3)|\lambda_3| \\ &= 1\cdot|0|+1\cdot|2n|+2n-2\cdot|3n| \\ &= 2n+6n^2-6n \\ &= 6n^2-4n \end{aligned}$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$ adalah

$$\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)=$$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
3n-2 & 2 & 2 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 3n-2 & 2 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 2 & 3n-2 & 2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 2 & 2 & 3n-2 & \dots & 2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
2 & 2 & 2 & 2 & \dots & 3n-2 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 2 & 2 & 2 & \dots & 2 & 3n-2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 3n-2 & 2 & 2 & 2 & \dots & 2 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 3n-2 & 2 & 2 & \dots & 2 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 3n-2 & 2 & \dots & 2 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 3n-2 & \dots & 2 & 2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 2 & \cdots & 3n-2 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 2 & 2 & 2 & \dots & 2 & 3n-2
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)-\lambda^+\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)-\lambda^+\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$
\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & r & r^2 & \cdots & sr^{n-1} \end{array} \\
\begin{bmatrix}
(3n-2)-\lambda & \cdots & \cdots & \cdots & \cdots \\
0 & \frac{(\lambda-(3n-4))(\lambda-3n)}{-(3n-2)+\lambda} & \cdots & \cdots & \cdots \\
0 & 0 & -\frac{(\lambda-(4n-7))(\lambda-(3n+2))}{\lambda-3n} & \cdots & \cdots \\
\vdots & \vdots & \vdots & & \cdots \\
0 & 0 & 0 & \cdots & -\frac{(\lambda-(3n-4))(\lambda-(4n-4))(\lambda-(6n-4))}{f(\lambda)}
\end{bmatrix}
\end{array}
$$

Maka $\det\left(\boldsymbol{D}_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)-\lambda^+\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D}_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right)$, yaitu:

$$
\begin{aligned}
p(\lambda^+) &= (\lambda^+-4n+4)(\lambda^+-6n+4)(\lambda^+-3n+4)^{2n-2} \\
&= (\lambda^+-(4n-4))(\lambda^+-(6n-4))(\lambda^+-(3n-4))^{2n-2}
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+)=0$, diperoleh $\lambda_1^+=4n-4, \lambda_2^+=6n-4$, dan $\lambda_3^+=3n-4$ dengan multiplisitas masing-masing $m(\lambda_1^+)=1, m(\lambda_2^+)=1$, dan $m(\lambda_3^+)=2n-2$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r\rangle}(D_{2n})}$ adalah

$$
\begin{aligned}
ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) &= m(\lambda_1^+)|\lambda_1^+|+m(\lambda_2^+)|\lambda_2^+|+m(\lambda_3^+)|\lambda_3^+| \\
&= 1\cdot|4n-4|+1\cdot|6n-4|+2n-2\cdot|3n-4| \\
&= 4n-4+6n-4+6n^2-8n-6n+8 \\
&= 6n^2-4n.
\end{aligned}
$$

3.2 *DLE* dan *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$ dari Grup Dihedral

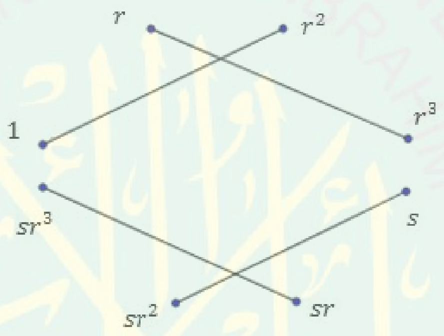
3.2.1 Grup dihedral D_8

Diketahui anggota grup D_8 adalah $\{1,r,r^2,r^3,s,sr,sr^2,sr^3\}$. Subgrup yang dibangun oleh $\langle r^2\rangle$ pada grup dihedral D_8 adalah $\{1,r^2\}$. Operasi unsur-unsur di D_8 yang hasilnya berada di $\langle r^2\rangle$ seperti pada tabel berikut

Tabel 3.8 Tabel Cayley Grup Dihedral D_8

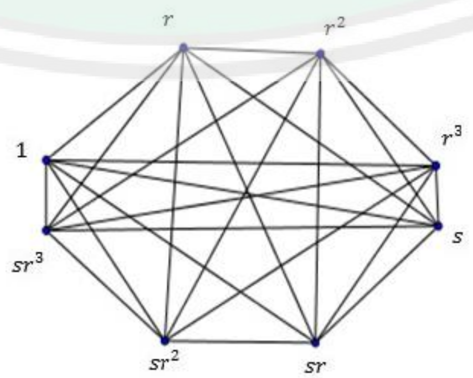
$\circ$	1	r	r^2	r^3	s	sr	sr^2	sr^3
1	1	r	r^2	r^3	s	sr	sr^2	sr^3
r	r	r^2	r^3	1	sr^3	s	sr	sr^2
r^2	r^2	r^3	1	r	sr^2	sr^3	s	sr
r^3	r^3	1	r	r^2	sr	sr^2	sr^3	s
s	s	sr	sr^2	sr^3	1	r	r^2	r^3
sr	sr	sr^2	sr^3	s	r^3	1	r	r^2
sr^2	sr^2	sr^3	s	sr	r^2	r^3	1	r
sr^3	sr^3	s	sr	sr^2	r	r^2	r^3	1

Berdasarkan Tabel 3.8 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2 \rangle}(D_8)$ sebagai berikut



Gambar 3.14 Graf $\Gamma_{\langle r^2 \rangle}(D_8)$

Dari Gambar 3.14 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^2 \rangle}(D_8)$ sebagai berikut



Gambar 3.15 Graf $\overline{\Gamma_{\langle r^2 \rangle}(D_8)}$

Matriks *distance* $\boldsymbol{D}(\overline{\Gamma_{\langle r^2 \rangle}(D_8)})$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right)$ sebagai berikut

$$\boldsymbol{D}(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}) = \begin{array}{c c} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \begin{array}{c c} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix} \end{array}$$

Matriks distance Laplacian yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \begin{array}{c c} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 8 & -1 & -2 & -1 & -1 & -1 & -1 & -1 \\ -1 & 8 & -1 & -2 & -1 & -1 & -1 & -1 \\ -2 & -1 & 8 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & 8 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 8 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & 8 & -1 & -2 \\ -1 & -1 & -1 & -1 & -2 & -1 & 8 & -1 \\ -1 & -1 & -1 & -1 & -1 & -2 & -1 & 8 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* terlebih dahulu akan di cari polinomial karakteristik dari matriks tersebut dengan cara:

$$\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r\rangle}(D_8)}\right)-\lambda\boldsymbol{I}\right)=$$

$$\det\begin{pmatrix}8-\lambda & -1 & -2 & -1 & -1 & -1 & -1 & -1\\ -1 & 8-\lambda & -1 & -2 & -1 & -1 & -1 & -1\\ -2 & -1 & 8-\lambda & -1 & -1 & -1 & -1 & -1\\ -1 & -2 & -1 & 8-\lambda & -1 & -1 & -1 & -1\\ -1 & -1 & -1 & -1 & 8-\lambda & -1 & -2 & -1\\ -1 & -1 & -1 & -1 & -1 & 8-\lambda & -1 & -2\\ -1 & -1 & -1 & -1 & -2 & -1 & 8-\lambda & -1\\ -1 & -1 & -1 & -1 & -1 & -2 & -1 & 8-\lambda\end{pmatrix}$$

Matriks tersebut direduksi menggunakan eliminasi baris elementer pada aplikasi Maple 18 dan diperoleh matriks segitiga atas sebagai berikut

$$\begin{bmatrix}8-\lambda & -1 & -2 & -1 & -1 & -1 & -1 & -1\\ 0 & -\frac{(\lambda-7)(\lambda-9)}{\lambda-8} & -\frac{\lambda-10}{\lambda-8} & -\frac{2(\lambda-17)}{\lambda-8} & -\frac{\lambda-9}{\lambda-8} & -\frac{\lambda-9}{\lambda-8} & -\frac{\lambda-9}{\lambda-8} & -\frac{\lambda-9}{\lambda-8}\\ 0 & 0 & -\frac{(\lambda-10)(\lambda^2-14\lambda+46)}{(\lambda-7)(\lambda-9)} & -\frac{(\lambda-10)^2}{(\lambda-7)(\lambda-9)} & -\frac{\lambda-10}{\lambda-7} & -\frac{\lambda-10}{\lambda-7} & -\frac{\lambda-10}{\lambda-7} & -\frac{\lambda-10}{\lambda-7}\\ 0 & 0 & 0 & -\frac{(\lambda-10)(\lambda-8)(\lambda-4)}{\lambda^2-14\lambda+46} & -\frac{(\lambda-10)(\lambda-8)}{\lambda^2-14\lambda+46} & -\frac{(\lambda-10)(\lambda-8)}{\lambda^2-14\lambda+46} & -\frac{(\lambda-10)(\lambda-8)}{\lambda^2-14\lambda+46} & -\frac{(\lambda-10)(\lambda-8)}{\lambda^2-14\lambda+46}\\ 0 & 0 & 0 & 0 & -\frac{\lambda^2-12\lambda+28}{\lambda-4} & -\frac{\lambda-8}{\lambda-4} & -\frac{2(\lambda-6)}{\lambda-4} & -\frac{\lambda-8}{\lambda-4}\\ 0 & 0 & 0 & 0 & 0 & -\frac{(\lambda-9)(\lambda^2-11\lambda+20)}{\lambda^2-12\lambda+28} & -\frac{(\lambda-10)(\lambda-8)}{\lambda^2-12\lambda+28} & -\frac{2\lambda^2-29\lambda+100}{\lambda^2-12\lambda+28}\\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(\lambda-8)(\lambda-10)(\lambda^2-10\lambda+10)}{(\lambda^2-11\lambda+20)} & -\frac{(\lambda-10)^2(\lambda-8)}{(\lambda^2-11\lambda+20)}\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{(\lambda-8)(\lambda-10)\lambda}{\lambda^2-10\lambda+12}\end{bmatrix}$$

Sehingga polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_8)}\right)$ diperoleh dari perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut adalah

$$\begin{aligned}p(\lambda) &= (8-\lambda)\left(-\frac{(\lambda-7)(\lambda-9)}{\lambda-8}\right)\left(-\frac{(\lambda-10)(\lambda^2-14\lambda+46)}{(\lambda-7)(\lambda-9)}\right)\left(-\frac{(\lambda-10)(\lambda-8)(\lambda-4)}{\lambda^2-14\lambda+46}\right)\\ &\quad\left(-\frac{\lambda^2-12\lambda+28}{\lambda-4}\right)\left(-\frac{(\lambda-9)(\lambda^2-11\lambda+20)}{\lambda^2-12\lambda+28}\right)\left(-\frac{(\lambda-8)(\lambda-10)(\lambda^2-10\lambda+10)}{(\lambda^2-11\lambda+20)}\right)\\ &\quad\left(-\frac{(\lambda-8)(\lambda-10)\lambda}{\lambda^2-10\lambda+12}\right)\\ &= (\lambda-10)^4(\lambda-8)^3\lambda\end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=10, \lambda_2=8$, dan $\lambda_3=0$ dengan multiplisitas masing-masing $m(\lambda_1)=4, m(\lambda_2)=3$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_8)}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_8)}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$

$$= (4 \cdot |10|) + (3 \cdot |8|) + (1 \cdot |0|)$$

$$= 64$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+ \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \boldsymbol{T_r} \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) + \boldsymbol{D} \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right)$adalah

$$\boldsymbol{D_L}^+ \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array} \begin{array}{c} \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{bmatrix} 8 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 8 & 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 8 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 8 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 8 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 8 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 8 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 8 \end{bmatrix} \end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+} \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right)$ sebagai berikut

$$p(\lambda^+) = (8-\lambda^+)\left(-\frac{(\lambda^+-7)(\lambda^+-9)}{\lambda-8}\right)\left(-\frac{(\lambda^+-6)\left({\lambda^+}^2-18\lambda^++78\right)}{(\lambda-7)(\lambda-9)}\right)$$

$$\left(-\frac{(\lambda^+-6)(\lambda^+-12)(\lambda^+-8)}{{\lambda^+}^2-18\lambda^++78}\right)\left(-\frac{{\lambda^+}^2-20\lambda^++92}{\lambda^+-12}\right)\left(-\frac{(\lambda^+-7)\left({\lambda^+}^2-21\lambda^++100\right)}{{\lambda^+}^2-20\lambda^++92}\right)$$

$$\left(-\frac{(\lambda^+-6)(\lambda^+-8)\left({\lambda^+}^2+22\lambda^+-106\right)}{(\lambda^+-7)({\lambda^+}^2-21\lambda^++100)}\right)\left(-\frac{(\lambda^+-16)(\lambda^+-6)(\lambda^+-8)}{{\lambda^+}^2+22\lambda^+-10}\right)$$

$$= (\lambda^+ - 16)(\lambda^+ - 8)^3(\lambda^+ - 6)^4$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 16, \lambda_2^+ = 8$, dan $\boldsymbol{\lambda_3^+} = 6$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 3$, dan $m(\lambda_3^+) = 4$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2 \rangle}(D_8)}$ adalah

$$ED_L^+ \left(\overline{\Gamma_{\langle r^2 \rangle}(D_8)}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1 \cdot |16|) + (3 \cdot |8|) + (4 \cdot |6|)$$

$= 64.$

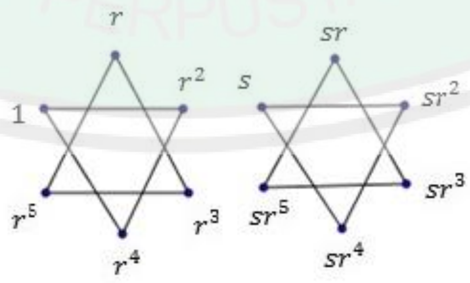
3.2.2 Grup dihedral D_{12}

Diketahui $D_{12} = \{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$. Subgrup yang dibangun oleh $\langle r^2 \rangle$ pada grup dihedral D_{12}adalah $\{1, r^2, r^4\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r^2 \rangle$ seperti pada tabel berikut

Tabel 3.9 Tabel Cayley Grup Dihedral D_{12}

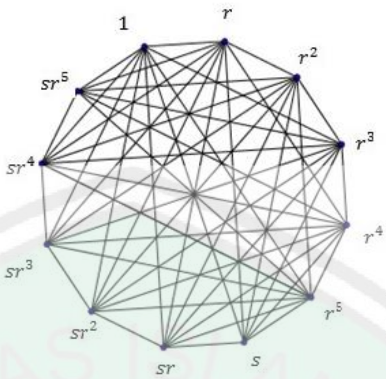
$\circ$	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
1	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
r	r	r^2	r^3	r^4	r^5	1	sr^5	s	sr	sr^2	sr^3	sr^4
r^2	r^2	r^3	r^4	r^5	1	r	sr^4	sr^5	s	sr	sr^2	sr^3
r^3	r^3	r^4	r^5	1	r	r^2	sr^3	sr^4	sr^5	s	sr	sr^2
r^4	r^4	r^5	1	r	r^2	r^3	sr^2	sr^3	sr^4	sr^5	s	sr
r^5	r^5	1	r	r^2	r^3	r^4	sr	sr^2	sr^3	sr^4	sr^5	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	1	r	r^2	r^3	r^4	r^5
sr	sr	sr^2	sr^3	sr^4	sr^5	s	r^5	1	r	r^2	r^3	r^4
sr^2	sr^2	sr^3	sr^4	sr^5	s	sr	r^4	r^5	1	r	r^2	r^3
sr^3	sr^3	sr^4	sr^5	s	sr	sr^2	r^3	r^4	r^5	1	r	r^2
sr^4	sr^4	sr^5	s	sr	sr^2	sr^3	r^2	r^3	r^4	r^5	1	r
sr^5	sr^5	s	sr	sr^2	sr^3	sr^4	r	r^2	r^3	r^4	r^5	1

Berdasarkan Tabel 3.9 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2 \rangle}(D_{12})$ sebagai berikut



Gambar 3.16 Graf $\Gamma_{\langle r^2 \rangle}(D_{12})$

Dari Gambar 3.16 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}$ sebagai berikut



Gambar 3.17 Graf $\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}\right)$ dan transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 \end{bmatrix} \end{array}$$

Matriks distance Laplacian yang diperoleh dari perhitungan $\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2\rangle}(D_{12})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r^2\rangle}(D_{12})\right)} - \boldsymbol{D}\overline{\left(\Gamma_{\langle r^2\rangle}(D_{12})\right)}$ adalah

$$\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2\rangle}(D_{12})\right)} = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 13 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 13 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -1 & 13 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & 13 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -1 & -2 & -1 & 13 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & -2 & -1 & 13 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 13 & -1 & -2 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 13 & -1 & -2 & -1 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & 13 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & 13 & -1 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & 13 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & 13 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_8), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda) &= (13-\lambda)\left(-\frac{(\lambda-12)(\lambda-14)}{\lambda-13}\right)\left(-\frac{(\lambda-15)(\lambda^2-24\lambda+141)}{(\lambda-12)(\lambda-14)}\right)\left(-\frac{(\lambda-15)(\lambda-13)(\lambda-9)}{\lambda^2-24\lambda+141}\right) \\ &\quad \left(-\frac{(\lambda-15)(\lambda^2-20\lambda+93)}{(\lambda-13)(\lambda-9}\right)\left(-\frac{(\lambda-15)(\lambda-6)(\lambda-12)}{\lambda^2-20\lambda+93}\right)\left(-\frac{\lambda^2-19\lambda+72}{\lambda-6}\right) \\ &\quad \left(-\frac{(\lambda-14)(\lambda^2-18\lambda+60)}{\lambda^2-19\lambda+72}\right)\left(-\frac{(\lambda-15)(\lambda^3-30\lambda^2+267\lambda-600)}{(\lambda-14)(\lambda^2-18\lambda+60)}\right) \\ &\quad \left(-\frac{(\lambda-13)(\lambda-15)(\lambda^2-15\lambda+30)}{\lambda^3-30\lambda^2+267\lambda-600}\right)\left(-\frac{(\lambda-12)(\lambda-15)(\lambda^2-14\lambda+15)}{(\lambda-13)(\lambda^2-15\lambda+30)}\right) \\ &\quad \left(-\frac{(\lambda-12)(\lambda-15)\lambda}{\lambda^2-14\lambda+15}\right) \\ &= (\lambda-15)^8(\lambda-12)^3\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 15, \lambda_2 = 12,$ dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 8, m(\lambda_2) = 3,$ dan $m(\lambda_3) = 1$.

Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{12})}$ adalah

$$\begin{aligned} ED_L\overline{\left(\Gamma_{\langle r^2\rangle}(D_{12})\right)} &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (8\cdot|15|) + (3\cdot|12|) + (1\cdot|0|) \end{aligned}$$

$$= 156$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{12})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{12})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{12})}\right)$ adalah

$$\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 13 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 13 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 13 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 13 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 13 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 13 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 13 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 13 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 13 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 13 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 13 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 13 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_8), diperoleh poinomial karakteristik sebagai berikut

$$\begin{aligned} p(\lambda^{+}) &= (13-\lambda^{+})\left(-\frac{(\lambda^{+}-12)(\lambda^{+}-14)}{\lambda^{+}-13}\right)\left(-\frac{(\lambda^{+}-11)\left({\lambda^{+}}^{2}-28\lambda^{+}+193\right)}{(\lambda^{+}-12)(\lambda^{+}-14)}\right) \\ &\left(-\frac{(\lambda^{+}-11)(\lambda^{+}-17)(\lambda^{+}-13)}{{\lambda^{+}}^{2}-28\lambda^{+}+193}\right)\left(-\frac{(\lambda^{+}-11)\left({\lambda^{+}}^{2}-32\lambda^{+}+249\right)}{(\lambda^{+}-13)(\lambda^{+}-17)}\right) \\ &\left(-\frac{(\lambda^{+}-20)(\lambda^{+}-11)(\lambda^{+}-14)}{{\lambda^{+}}^{2}-32\lambda^{+}+249}\right)\left(-\frac{{\lambda^{+}}^{2}-33\lambda^{+}+254}{\lambda^{+}-20}\right)\left(-\frac{(\lambda^{+}-12)\left({\lambda^{+}}^{2}-34\lambda^{+}+268\right)}{{\lambda^{+}}^{2}-33\lambda^{+}+254}\right) \\ &\left(-\frac{(\lambda^{+}-11)\left({\lambda^{+}}^{3}-48{\lambda^{+}}^{2}+735\lambda^{+}-3638\right)}{(\lambda^{+}-12)\left({\lambda^{+}}^{2}-34\lambda^{+}+268\right)}\right)\left(-\frac{(\lambda^{+}-11)(\lambda^{+}-13)\left({\lambda^{+}}^{2}-37\lambda^{+}+316\right)}{{\lambda^{+}}^{3}-48{\lambda^{+}}^{2}+735\lambda^{+}-3638}\right) \\ &\left(-\frac{(\lambda^{+}-11)(\lambda^{+}-14)\left({\lambda^{+}}^{2}-38\lambda^{+}+327\right)}{(\lambda^{+}-13)\left({\lambda^{+}}^{2}-37\lambda^{+}+316\right)}\right)\left(-\frac{(\lambda^{+}-11)(\lambda^{+}-14)(\lambda^{+}-26)}{{\lambda^{+}}^{2}-38\lambda^{+}+327}\right) \\ &= (\lambda^{+}-26)(\lambda^{+}-14)^{3}(\lambda^{+}-11)^{8} \end{aligned}$$

Dengan menetapkan $p(\lambda^{+}) = 0$, diperoleh nilai eigen $\lambda_1^{+} = 26, \lambda_2^{+} = 14$, dan $\lambda_3^{+} = 11$ dengan multiplisitas masing-masing $m(\lambda_1^{+}) = 1, m(\lambda_2^{+}) = 3$, dan $m(\lambda_3^{+}) = 8$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{12})}$ adalah

$$ED_L^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{12})}\right) = \sum_{i=1}^{3} m(\lambda_i^{+})|\lambda_i^{+}|$$

$$= (1\cdot|26|) + (3\cdot|14|) + (8\cdot|11|)$$

$$= 156.$$

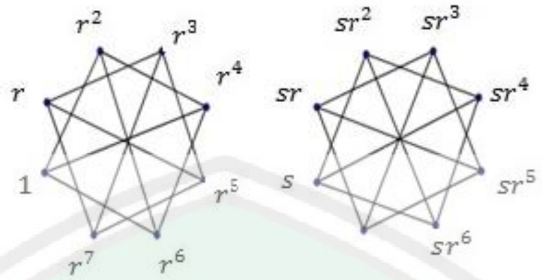
3.2.3 Grup dihedral D_{16}

Diketahui $D_{16} = \{1, r, r^2, r^3, \dots, r^7, s, sr, sr^2, sr^3, \dots, sr^7\}$. Subgrup yang dibangun oleh $\langle r^2\rangle$ pada grup dihedral D_{16} adalah $\{1, r^2, r^4, r^6\}$. Operasi unsur-unsur di D_{16} yang hasilnya berada di $\langle r^2\rangle$ seperti pada tabel berikut

Tabel 3.10 Tabel Cayley Grup Dihedral D_{16}

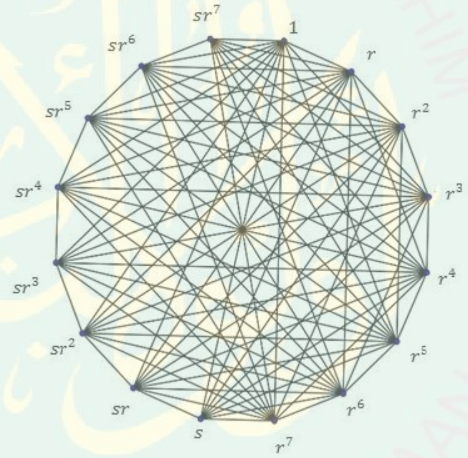
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r	r	r^2	r^3	r^4	r^5	r^6	r^7	1	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^2	r^2	r^3	r^4	r^5	r^6	r^7	1	r	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5
r^3	r^3	r^4	r^5	r^6	r^7	1	r	r^2	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4
r^4	r^4	r^5	r^6	r^7	1	r	r^2	r^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3
r^5	r^5	r^6	r^7	1	r	r^2	r^3	r^4	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2
r^6	r^6	r^7	1	r	r^2	r^3	r^4	r^5	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr
r^7	r^7	1	r	r^2	r^3	r^4	r^5	r^6	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	r^7	1	r	r^2	r^3	r^4	r^5	r^6
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	r^6	r^7	1	r	r^2	r^3	r^4	r^5
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	r^5	r^6	r^7	1	r	r^2	r^3	r^4
sr^4	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	r^4	r^5	r^6	r^7	1	r	r^2	r^3
sr^5	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	r^3	r^4	r^5	r^6	r^7	1	r	r^2
sr^6	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	r^2	r^3	r^4	r^5	r^6	r^7	1	r
sr^7	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r	r^2	r^3	r^4	r^5	r^6	r^7	1

Berdasarkan Tabel 3.10 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2\rangle}(D_{16})$ sebagai berikut



Gambar 3.18 Graf $\Gamma_{\langle r^2\rangle}(D_{16})$

Dari Gambar 3.18 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{16})}$ sebagai berikut



Gambar 3.19 Graf $\overline{\Gamma_{\langle r^2\rangle}(D_{16})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right)$ dan matriks transmisi $\boldsymbol{T}_r\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \begin{bmatrix} 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \end{array} & \begin{bmatrix} 18 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 18 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -1 & 18 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & 18 & -1 & -2 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -1 & -2 & -1 & 18 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & -2 & -1 & 18 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -2 & -1 & -2 & -1 & -2 & -1 & 18 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & 18 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 18 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 18 & -1 & -2 & -1 & -2 & -1 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & 18 & -1 & -2 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & 18 & -1 & -2 & -1 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & 18 & -1 & -2 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & 18 & -1 & -2 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 18 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 18 \end{bmatrix} \end{array}$$

Dengan menggunakan cara yang sama pada grup dihedral (D_8), diperoleh poinomial karakteristik sebagai berikut

$$
\begin{aligned}
p(\lambda) &= (18-\lambda)\left(-\frac{(\lambda-17)(\lambda-19)}{\lambda-18}\right)\left(-\frac{(\lambda-20)(\lambda^2-34\lambda+286)}{(\lambda-17)(\lambda-19)}\right)\left(-\frac{(\lambda-20)(\lambda-18)(\lambda-14)}{\lambda^2-34\lambda+286}\right) \\
&\left(-\frac{(\lambda-20)(\lambda^2-30\lambda+218)}{(\lambda-18)(\lambda-14)}\right)\left(-\frac{(\lambda-20)(\lambda-11)(\lambda-17)}{\lambda^2-30\lambda+218}\right)\left(-\frac{(\lambda-20)(\lambda^2-26\lambda+156)}{(\lambda-11)(\lambda-17)}\right) \\
&\left(-\frac{(\lambda-20)(\lambda-16)(\lambda-8)}{\lambda^2-26\lambda+156}\right)\left(-\frac{\lambda^2-26\lambda+136}{\lambda-8}\right)\left(-\frac{(\lambda-19)(\lambda^2-25\lambda+120)}{\lambda^2-26\lambda+136}\right) \\
&\left(-\frac{(\lambda-20)(\lambda^3-42\lambda^2+534\lambda-1840)}{(\lambda-19)(\lambda^2-25\lambda+120)}\right)\left(-\frac{(\lambda-18)(\lambda-20)(\lambda^2-22\lambda+80)}{\lambda^3-42\lambda^2+534\lambda-1840}\right) \\
&\left(-\frac{(\lambda-20)(\lambda^3-38\lambda^2+418\lambda-1040)}{(\lambda-18)(\lambda^2-22\lambda+80)}\right)\left(-\frac{(\lambda-17)(\lambda-20)(\lambda^2-19\lambda+40)}{\lambda^3-38\lambda^2+418\lambda-1040}\right) \\
&\left(-\frac{(\lambda-16)(\lambda-20)(\lambda^2-18\lambda+20)}{(\lambda-17)(\lambda^2-19\lambda+40)}\right)\left(-\frac{(\lambda-16)(\lambda-20)\lambda}{\lambda^2-18\lambda+20}\right) \\
&= (\lambda-20)^{12}(\lambda-16)^3\lambda
\end{aligned}
$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 20, \lambda_2 = 16$, dan $\lambda_3 = -0$ dengan multiplisitas masing-masing $m(\lambda_1) = 12, m(\lambda_2) = 3$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{16})}$ adalah

$$
\begin{aligned}
ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\
&= (12\cdot|20|) + (3\cdot|16|) + (1\cdot|-0|) \\
&= 288
\end{aligned}
$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right)$ adalah

$$
\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) =
\begin{array}{c}
\\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array} \\
\begin{bmatrix}
18 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 18 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 18 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 18 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 2 & 1 & 18 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 2 & 1 & 18 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 18 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 18 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 18 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 18 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 18 & 1 & 2 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 18 & 1 & 2 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 18 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 18 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 18 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 18
\end{bmatrix}
\end{array}
$$

Dengan menggunakan cara yang sama pada grup dihedral (D_8), diperoleh poinomial karakteristik sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (18-\lambda^+)\left(-\frac{(\lambda^+-17)(\lambda^+-19)}{\lambda^+-18}\right)\left(-\frac{(\lambda^+-16)\left({\lambda^+}^2-38\lambda^++358\right)}{(\lambda^+-17)(\lambda^+-19)}\right) \\
&\quad \left(-\frac{(\lambda^+-16)(\lambda^+-22)(\lambda^+-18)}{{\lambda^+}^2-38\lambda^++358}\right)\left(-\frac{(\lambda^+-16)\left({\lambda^+}^2-42\lambda^++434\right)}{(\lambda^+-18)(\lambda^+-22)}\right) \\
&\quad \left(-\frac{(\lambda^+-25)(\lambda^+-16)(\lambda^+-19)}{{\lambda^+}^2-42\lambda^++434}\right)\left(-\frac{(\lambda^+-16)\left({\lambda^+}^2-46\lambda^++516\right)}{(\lambda^+-19)(\lambda^+-25)}\right) \\
&\quad \left(-\frac{(\lambda^+-20)(\lambda^+-16)(\lambda^+-28)}{{\lambda^+}^2-46\lambda^++516}\right)\left(-\frac{{\lambda^+}^2-46\lambda^++496}{\lambda^+-28}\right) \\
&\quad \left(-\frac{(\lambda^+-17)\left({\lambda^+}^2-47\lambda^++516\right)}{{\lambda^+}^2-46\lambda^++496}\right)\left(-\frac{(\lambda^+-16)\left({\lambda^+}^3-66{\lambda^+}^2+1398\lambda^+-9608\right)}{(\lambda^+-17)\left({\lambda^+}^2-47\lambda^++516\right)}\right) \\
&\quad \left(-\frac{(\lambda^+-16)(\lambda^+-18)\left({\lambda^+}^2-50\lambda^++584\right)}{{\lambda^+}^3-66{\lambda^+}^2+1398\lambda^+-9608}\right)\left(-\frac{(\lambda^+-16)\left({\lambda^+}^3-70{\lambda^+}^2+1570\lambda^+-11416\right)}{(\lambda^+-18)\left({\lambda^+}^2-50\lambda^++584\right)}\right) \\
&\quad \left(-\frac{(\lambda^+-16)(\lambda^+-19)\left({\lambda^+}^2-53\lambda^++652\right)}{{\lambda^+}^3-70{\lambda^+}^2+1570\lambda^+-11416}\right)\left(-\frac{(\lambda^+-16)(\lambda^+-20)\left({\lambda^+}^2-54\lambda^++668\right)}{(\lambda^+-19)\left({\lambda^+}^2-53\lambda^++652\right)}\right) \\
&\quad \left(-\frac{(\lambda^+-36)(\lambda^+-16)(\lambda^+-20)}{{\lambda^+}^2-54\lambda^++668}\right) \\
&= (\lambda^+-36)(\lambda^+-20)^3(\lambda^+-16)^{12}
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 36, \lambda_2^+ = 20$, dan $\lambda_3^+ = 16$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 3$, dan $m(\lambda_3^+) = 12$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{16})}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{16})}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1 \cdot |36|) + (3 \cdot |20|) + (12 \cdot |16|)$$

$$= 288.$$

Dari *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$ di grup dihedral yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.11 Tabel pola *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$

D_{2n}	n	*DLE*	*DSLE*
D_8	4	$64 = 4 \cdot 16$ $= 4(5 \cdot 4 - 4)$	64
D_{12}	6	$156 = 6 \cdot 26$ $= 6(5 \cdot 6 - 6)$	156
D_{16}	8	$288 = 8 \cdot 36$ $= 8(5 \cdot 8 - 8)$	288
$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$n(5n - 4) = 5n^2 - 4n$	$5n^2 - 4n$

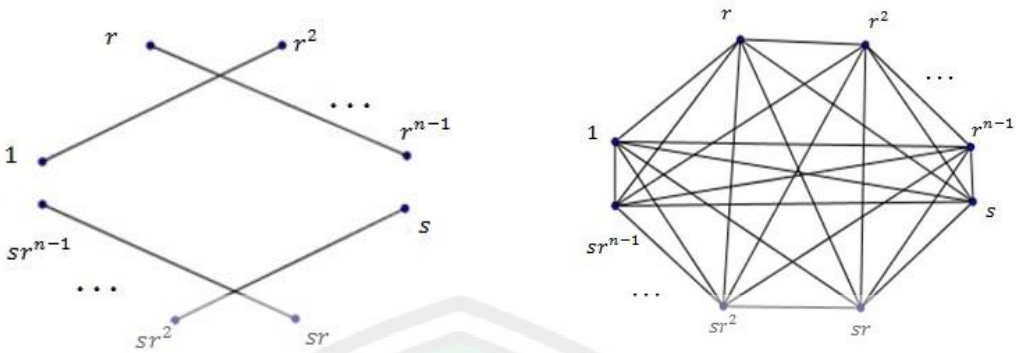
Teorema 2

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r^2\rangle = \{1, r^2, r^4, \dots, r^{n-2}\}$ subgrup normal dari grup dihedral D_{2n}untuk $n \geq 4$ dan n bilangan genap. *DLE* dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$

Bukti:

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ untuk n genap dengan $n \geq 4$. Ambil $\langle r^2\rangle = \{1, r^2, r^4, \dots, r^{n-2}\}$ subgrup normal dari grup dihedral D_{2n}. Sesuai definisi graf subgrup dan komplemennya, maka diperoleh graf $\Gamma_{\langle r^2\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$ yaitu:



Gambar 3.20 Graf $\Gamma_{\langle r^2 \rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}$

Sehingga diperoleh matriks *distance* dari $\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}$ sebagai berikut

$$
\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) =
\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc}
1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1}
\end{array} \\
\begin{bmatrix}
0 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 0 & 1 & 2 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 1 & 0 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 1 & 0 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
2 & 1 & 2 & 1 & \dots & 0 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 1 & 2 & \dots & 1 & 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 0 & 1 & 2 & 1 & \dots & 2 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 0 & 1 & 2 & \dots & 1 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 0 & 1 & \dots & 2 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 0 & \dots & 1 & 2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 2 & 1 & \cdots & 0 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 0
\end{bmatrix}
\end{array}
$$

dan matriks transmisi $\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}$ adalah:

$$
T_r\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) =
$$

$$
\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc}
1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1}
\end{array} \\
\begin{bmatrix}
5n-12 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 5n-12 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 5n-12 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 5n-12 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 5n-12 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 5n-12 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 5n-12 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 5n-12 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 5n-12 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 5n-12 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 5n-12 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 5n-12
\end{bmatrix}
\end{array}
$$

Matriks *distance Laplacian* dari yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) =$$

$$\begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{bmatrix}
5n-12 & -1 & -2 & -1 & \dots & -2 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & 5n-12 & -1 & -2 & \dots & -1 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & -1 & 5n-12 & -1 & \dots & -2 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -2 & -1 & 5n-12 & \dots & -1 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2 & -1 & -2 & -1 & \dots & 5n-12 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & 5n-12 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & 5n-12 & -1 & -2 & -1 & \dots & -2 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & 5n-12 & -1 & -2 & \dots & -1 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -1 & 5n-12 & -1 & \dots & -2 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & -2 & -2 & 5n-12 & \dots & -1 & -2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -1 & -2 & -1 & \cdots & 5n-12 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & -2 & -1 & -2 & \dots & -1 & 5n-12
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) - \lambda\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) - \lambda\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$\begin{array}{c} \\ 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1} \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & r & r^2 & \dots & sr^{n-1} \end{array} \\
\begin{bmatrix}
\left(\frac{5n}{2}-2\right)-\lambda & \dots & \dots & \dots & \dots \\
0 & -\frac{\left(\lambda-\left(\frac{5n}{2}-3\right)\right)\left(\lambda-\left(\frac{5n}{2}-1\right)\right)}{\lambda-\left(\frac{5n}{2}-2\right)} & \dots & \dots & \dots \\
0 & 0 & -\frac{\left(\lambda-\frac{5}{2}n\right)\left(\lambda^2-(5n-6)\lambda+\left(\frac{50}{8}n^2-15n+6\right)\right)}{\left(\lambda-\left(\frac{5n}{2}-3\right)\right)\left(\lambda-\left(\frac{5n}{2}-1\right)\right)} & \dots & \dots \\
\vdots & \vdots & \vdots & & \dots \\
0 & 0 & 0 & \dots & -\frac{(\lambda-2n)\left(\lambda-\frac{5n}{2}\right)\lambda}{\lambda^2-(2n-2)\lambda-\frac{1}{4}n^2+4n}
\end{bmatrix}
\end{array}$$

Maka $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) - \lambda\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda) = \lambda(\lambda-2n)^3\left(\lambda-\frac{5}{2}n\right)^{2n-4}$$

Dengan menetapkan $p(\lambda) = 0$ maka diperoleh $\lambda_1 = 0, \lambda_2 = 2n,$ dan $\lambda_3 = \frac{5}{2}n$ dengan multiplisitas masing-masing $m(\lambda_1) = 1, m(\lambda_2) = 3,$ dan $m(\lambda_3) = 2n - 4.$ Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) &= m(\lambda_1)|\lambda_1| + m(\lambda_2)|\lambda_2| + m(\lambda_3)|\lambda_3| \\ &= (1 \cdot |0|) + (3 \cdot |2n|) + \left((2n-4) \cdot \left|\frac{5}{2}n\right|\right) \\ &= 6n + 5n^2 - 10n \\ &= 5n^2 - 4n \end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right)$ adalah

$\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) =$

$$\begin{array}{cc}
 & \begin{matrix} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{matrix} \\
\begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{matrix} &
\begin{bmatrix}
5n-12 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 5n-12 & 1 & 2 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 1 & 5n-12 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 1 & 5n-12 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
2 & 1 & 2 & 1 & \dots & 5n-12 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 1 & 2 & \dots & 1 & 5n-12 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 5n-12 & 1 & 2 & 1 & \dots & 2 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 5n-12 & 1 & 2 & \dots & 1 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 5n-12 & 1 & \dots & 2 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 5n-12 & \dots & 1 & 2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 2 & 1 & \cdots & 5n-12 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 5n-12
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2n})}\right) - \lambda^+ \boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L^+}\overline{\left(\Gamma_{\langle r^2 \rangle}(D_{2n})\right)} - \lambda^+ \boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$\begin{array}{cc}
 & \begin{matrix} 1 & r & r^2 & \dots & sr^{n-1} \end{matrix} \\
\begin{matrix} 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1} \end{matrix} &
\begin{bmatrix}
\left(\frac{5n}{2} - 2\right) - \lambda & \dots & \dots & \dots & \dots \\
0 & -\frac{\left(\lambda - \left(\frac{5n}{2} - 3\right)\right)\left(\lambda - \left(\frac{5n}{2} - 1\right)\right)}{\lambda - \left(\frac{5n}{2} - 2\right)} & \dots & \dots & \dots \\
0 & 0 & -\frac{\left(\lambda - \left(\frac{5}{2}n - 4\right)\right)\left(\lambda^2 - (5n-2)\lambda + \left(\frac{50}{8}n^2 - 5n - 2\right)\right)}{\left(\lambda - \left(\frac{5n}{2} - 3\right)\right)\left(\lambda - \left(\frac{5n}{2} - 1\right)\right)} & \dots & \dots \\
\vdots & \vdots & \vdots & & \dots \\
0 & 0 & 0 & \dots & -\frac{(\lambda - (5n-4))\left(\lambda - \left(\frac{5n}{2} - 4\right)\right)(\lambda - (3n-4))}{\lambda^2 - (8n-10)\lambda + 15n^2 - \frac{79}{2}n + 24}
\end{bmatrix}
\end{array}$$

Maka $\det\left(\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right)-\lambda^{+}\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda^{+})=\left(\lambda^{+}-(5n-4)\right)\left(\lambda^{+}-(3n-4)\right)^{3}\left(\lambda^{+}-\left(\frac{5}{2}n-4\right)\right)^{2n-4}$$

Dengan menetapkan $p(\lambda^{+})=0$ maka diperoleh $\lambda_1^{+}=5n-4, \lambda_2^{+}=3n-4$, dan $\lambda_3^{+}=\frac{5}{2}n-4$ dengan multipisitas masing-masing $m(\lambda_1^{+})=1, m(\lambda_2^{+})=3$, dan $m(\lambda_3^{+})=2n-4$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$ adalah

$$\begin{aligned}ED_L^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) &= m(\lambda_1^{+})|\lambda_1^{+}|+m(\lambda_2^{+})|\lambda_2^{+}|+m(\lambda_3^{+})|\lambda_3^{+}|\\ &=(1\cdot|5n-4|)+(3\cdot|3n-4|)+\left(2n-4\cdot\left|\frac{5}{2}n-4\right|\right)\\ &=5n-4+9n-12+5n^2-8n-10n+16\\ &=5n^2-4n.\end{aligned}$$

3.3 *DLE* dan *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ dari Grup Dihedral

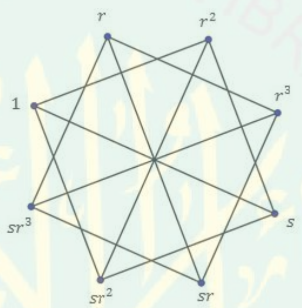
3.3.1 Grup dihedral D_8

Diketahui $D_8=\{1,r,r^2,r^3,s,sr,sr^2,sr^3\}$. Subgrup yang dibangun oleh $\langle r^2,s\rangle$ pada grup dihedral D_8 adalah $\{1,r^2,s,sr^2\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r^2,s\rangle$ seperti pada tabel berikut

Tabel 3.12 Tabel Cayley Grup Dihedral D_8

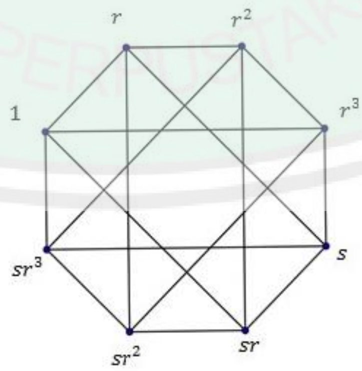
$\circ$	1	r	r^2	r^3	s	sr	sr^2	sr^3
1	1	r	r^2	r^3	s	sr	sr^2	sr^3
r	r	r^2	r^3	1	sr^3	s	sr	sr^2
r^2	r^2	r^3	1	r	sr^2	sr^3	s	sr
r^3	r^3	1	r	r^2	sr	sr^2	sr^3	s
s	s	sr	sr^2	sr^3	1	r	r^2	r^3
sr	sr	sr^2	sr^3	s	r^3	1	r	r^2
sr^2	sr^2	sr^3	s	sr	r^2	r^3	1	r
sr^3	sr^3	s	sr	sr^2	r	r^2	r^3	1

Berdasarkan Tabel 3.12 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_8)$ sebagai berikut



Gambar 3.21 $\Gamma_{\langle r^2,s\rangle}(D_8)$

Dari Gambar 3.21 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_8)$ sebagai berikut



Gambar 3.22 Graf $\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \begin{array}{cc} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \begin{array}{cc} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ sebagai berikut

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \begin{array}{cc} & \begin{matrix} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{matrix} \\ \begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{matrix} & \begin{bmatrix} 10 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & 10 & -1 & -2 & -1 & -2 & -1 & -2 \\ -2 & -1 & 10 & -1 & -2 & -1 & -2 & -1 \\ -1 & -2 & -1 & 10 & -1 & -2 & -1 & -2 \\ -2 & -1 & -2 & -1 & 10 & -1 & -2 & -1 \\ -1 & -2 & -1 & -2 & -1 & 10 & -1 & -2 \\ -2 & -1 & -2 & -1 & -2 & -1 & 10 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & 10 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) - \lambda \boldsymbol{I}\right)$.

Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) - \lambda \boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (10-\lambda)\left(-\frac{(\lambda-9)(\lambda-11)}{-10+\lambda}\right)\left(-\frac{(\lambda-12)(\lambda^2-18\lambda+78)}{(\lambda-9)(\lambda-11)}\right)\left(-\frac{(\lambda-10)(\lambda-6)(\lambda-12)}{\lambda^2-18\lambda+78}\right) \\ &\quad \left(-\frac{(\lambda-12)(\lambda^2-14\lambda+42)}{(\lambda-10)(\lambda-6)}\right)\left(-\frac{(\lambda-12)(\lambda-3)(\lambda-9)}{\lambda^2-14\lambda+42}\right)\left(-\frac{(\lambda-12)(\lambda^2-10\lambda+12)}{(\lambda-3)(\lambda-9)}\right) \\ &\quad \left(-\frac{(\lambda-8)(\lambda-12)\lambda}{\lambda^2-10\lambda+12}\right) \\ &= \lambda(\lambda-8)(\lambda-12)^6 \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1 = 12, \lambda_2 = 8$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 6, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (6\cdot|12|) + (1\cdot|8|) + (1\cdot|0|) \\ &= 80 \end{aligned}$$

Sedangkan matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 10 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 10 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 10 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 10 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 10 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 10 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 10 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 10 \end{bmatrix} \end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right)$ sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (10-\lambda^+)\left(-\frac{(\lambda^+-9)(\lambda^+-11)}{-10+\lambda^2}\right)\left(-\frac{(\lambda^+-8)\left({\lambda^+}^2-22\lambda^++118\right)}{(\lambda^+-9)(\lambda^+-11)}\right) \\
&\quad \left(-\frac{(\lambda^+-10)(\lambda^+-8)(\lambda^+-14)}{{\lambda^+}^2-22\lambda^++118}\right)\left(-\frac{(\lambda^+-8)({\lambda^+}^2-26\lambda^++162)}{(\lambda^+-10)(\lambda^+-14)}\right) \\
&\quad \left(-\frac{(\lambda^+-11)(\lambda^+-17)(\lambda^+-8)}{\lambda^2-26\lambda+162}\right)\left(-\frac{(\lambda^+-8)(\lambda^2-30\lambda^++212)}{(\lambda^+-11)(\lambda^+-17)}\right) \\
&\quad \left(-\frac{(\lambda^+-20)(\lambda^+-12)(\lambda^+-8)}{\lambda^2-30\lambda^++212}\right) \\
&= (\lambda^+-20)(\lambda^+-12)(\lambda^+-8)^6
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 20, \lambda_2^+ = 12$, dan $\lambda_3^+ = 8$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 6$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}$ adalah

$$
\begin{aligned}
ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_8)}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\
&= (1\cdot|20|) + (1\cdot|12|) + (6\cdot|8|) \\
&= 80.
\end{aligned}
$$

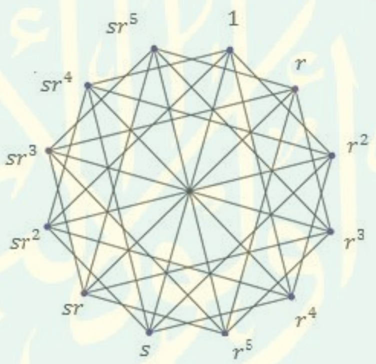
3.3.2 Grup dihedral D_{12}

Diketahui $D_{12} = \{1, r, r^2, r^3, r^4, r^5, s, sr, sr^2, sr^3, sr^4, sr^5\}$. Subgrup yang dibangun oleh $\langle r^2, s\rangle$ pada grup dihedral D_{12} adalah $\{1, r^2, r^4, s, sr^2, sr^4\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r^2, s\rangle$ seperti pada tabel berikut

Tabel 3.13 Tabel Cayley Grup Dihedral D_{12}

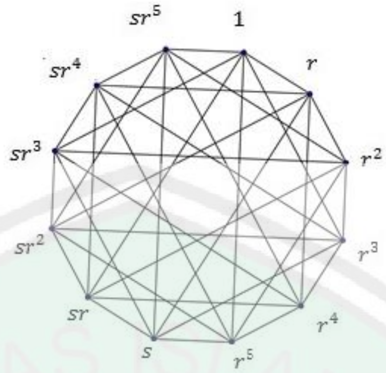
$\circ$	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
1	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
r	r	r^2	r^3	r^4	r^5	1	sr^5	s	sr	sr^2	sr^3	sr^4
r^2	r^2	r^3	r^4	r^5	1	r	sr^4	sr^5	s	sr	sr^2	sr^3
r^3	r^3	r^4	r^5	1	r	r^2	sr^3	sr^4	sr^5	s	sr	sr^2
r^4	r^4	r^5	1	r	r^2	r^3	sr^2	sr^3	sr^4	sr^5	s	sr
r^5	r^5	1	r	r^2	r^3	r^4	sr	sr^2	sr^3	sr^4	sr^5	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	1	r	r^2	r^3	r^4	r^5
sr	sr	sr^2	sr^3	sr^4	sr^5	s	r^5	1	r	r^2	r^3	r^4
sr^2	sr^2	sr^3	sr^4	sr^5	s	sr	r^4	r^5	1	r	r^2	r^3
sr^3	sr^3	sr^4	sr^5	s	sr	sr^2	r^3	r^4	r^5	1	r	r^2
sr^4	sr^4	sr^5	s	sr	sr^2	sr^3	r^2	r^3	r^4	r^5	1	r
sr^5	sr^5	s	sr	sr^2	sr^3	sr^4	r	r^2	r^3	r^4	r^5	1

Berdasarkan Tabel 3.13 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{12})$ sebagai berikut



Gambar 3.23 Graf $\Gamma_{\langle r^2,s\rangle}(D_{12})$

Dari Gambar 3.23 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}$ sebagai berikut



Gambar 3.24 Graf $\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} & \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} & \begin{bmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} & \begin{bmatrix} 16 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & 16 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\ -2 & -1 & 16 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & -2 & -1 & 16 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\ -2 & -1 & -2 & -1 & 16 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & -2 & -1 & -2 & -1 & 16 & -1 & -2 & -1 & -2 & -1 & -2 \\ -2 & -1 & -2 & -1 & -2 & -1 & 16 & -1 & -2 & -1 & -2 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & 16 & -1 & -2 & -1 & -2 \\ -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 16 & -1 & -2 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 16 & -1 & -2 \\ -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 16 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 16 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) - \lambda \boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) - \lambda \boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (16-\lambda)\left(-\frac{(\lambda-15)(\lambda-17)}{-16+\lambda}\right)\left(-\frac{(\lambda-18)(\lambda^2-30\lambda+222)}{(\lambda-15)(\lambda-17)}\right)\left(-\frac{(\lambda-16)(\lambda-12)(\lambda-18)}{\lambda^2-30\lambda+222}\right) \\ &\quad \left(-\frac{(\lambda-18)(\lambda^2-26\lambda+162)}{(\lambda-12)(\lambda-16)}\right)\left(-\frac{(\lambda-15)(\lambda-18)(\lambda-9)}{\lambda^2-26\lambda+162}\right)\left(-\frac{(\lambda-18)(\lambda^2-22\lambda+108)}{(\lambda-15)(\lambda-9)}\right) \\ &\quad \left(-\frac{(\lambda-6)(\lambda-18)(\lambda-14)}{\lambda^2-22\lambda+108}\right)\left(-\frac{(\lambda-18)(\lambda^2-18\lambda+60)}{(\lambda-6)(\lambda-14)}\right)\left(-\frac{(\lambda-13)(\lambda-3)(\lambda-18)}{\lambda^2-18\lambda+60}\right) \\ &\quad \left(-\frac{(\lambda-18)(\lambda^2-14\lambda+18)}{(\lambda-13)(\lambda-3)}\right)\left(-\frac{(\lambda-12)(\lambda-18)\lambda}{\lambda^2-14\lambda+18}\right) \\ &= (\lambda-12)(\lambda-18)^{10}\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 18, \lambda_2 = 12$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 10, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$

$$= (10\cdot|18|) + (1\cdot|12|) + (1\cdot|0|)$$

$$= 192$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 16 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 16 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 16 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 16 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 16 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 16 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 16 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 16 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 16 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 16 & 1 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 16 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 16 \end{bmatrix} \end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ sebagai berikut

$$p(\lambda^+) = (16-\lambda^+)\left(-\frac{(\lambda^+-15)(\lambda^+-17)}{-16+\lambda}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-34\lambda^++286\right)}{(\lambda^+-15)(\lambda^+-17)}\right)$$

$$\left(-\frac{(\lambda^+-20)(\lambda^+-16)(\lambda^+-14)}{{\lambda^+}^2-34\lambda^++286}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-38\lambda^++354\right)}{(\lambda^+-20)(\lambda^+-16)}\right)$$

$$\left(-\frac{(\lambda^+-17)(\lambda^+-23)(\lambda^+-14)}{{\lambda^+}^2-38\lambda^++354}\right)\left(-\frac{(\lambda^+-14)({\lambda^+}^2-42\lambda^++428)}{(\lambda^+-17)(\lambda^+-23)}\right)$$

$$\left(-\frac{(\lambda^+-26)(\lambda^+-18)(\lambda^+-14)}{{\lambda^+}^2-42\lambda^++428}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-46\lambda^++508\right)}{(\lambda^+-26)(\lambda^+-18)}\right)$$

$$\left(-\frac{(\lambda^+-14)(\lambda^+-29)(\lambda^+-19)}{{\lambda^+}^2-46\lambda^++508}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-50\lambda^++594\right)}{(\lambda^+-29)(\lambda^+-19)}\right)$$

$$\left(-\frac{(\lambda^+-20)(\lambda^+-32)(\lambda^+-14)}{{\lambda^+}^2-50\lambda^++594}\right)$$

$$= (\lambda^+-32)(\lambda^+-20)(\lambda^+-14)^2$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 32, \lambda_2^+ = 20$, dan $\lambda_3^+ = 14$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 10$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}$ adalah

$$\begin{aligned} ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\ &= (1\cdot|32|) + (1\cdot|20|) + (10\cdot|14|) \\ &= 192. \end{aligned}$$

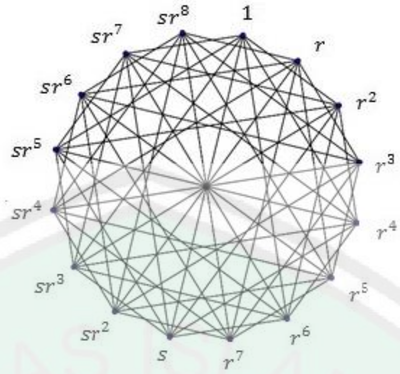
3.3.3 Grup dihedral D_{16}

Diketahui $D_{16} = \{1, r, r^2, r^3, \dots, r^7, s, sr, sr^2, sr^3, \dots, sr^7\}$. Subgrup yang dibangun oleh $\langle r^2, s\rangle$ pada grup dihedral D_{16} adalah$\{1, r^2, r^4, r^6, s, sr^2, sr^4, sr^6\}$. Operasi unsur-unsur di D_{16} yang hasilnya berada di $\langle r^2, s\rangle$ seperti pada tabel berikut

Tabel 3.14 Tabel Cayley Grup Dihedral D_{16}

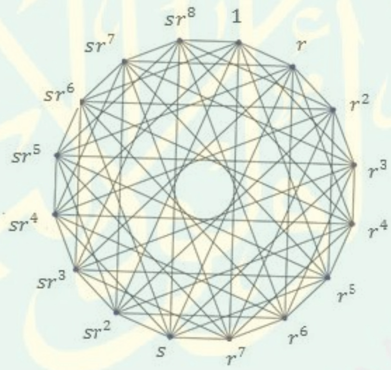
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r	r	r^2	r^3	r^4	r^5	r^6	r^7	1	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^2	r^2	r^3	r^4	r^5	r^6	r^7	1	r	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5
r^3	r^3	r^4	r^5	r^6	r^7	1	r	r^2	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4
r^4	r^4	r^5	r^6	r^7	1	r	r^2	r^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3
r^5	r^5	r^6	r^7	1	r	r^2	r^3	r^4	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2
r^6	r^6	r^7	1	r	r^2	r^3	r^4	r^5	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr
r^7	r^7	1	r	r^2	r^3	r^4	r^5	r^6	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	r^7	1	r	r^2	r^3	r^4	r^5	r^6
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	r^6	r^7	1	r	r^2	r^3	r^4	r^5
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	r^5	r^6	r^7	1	r	r^2	r^3	r^4
sr^4	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	r^4	r^5	r^6	r^7	1	r	r^2	r^3
sr^5	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	r^3	r^4	r^5	r^6	r^7	1	r	r^2
sr^6	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	r^2	r^3	r^4	r^5	r^6	r^7	1	r
sr^7	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r	r^2	r^3	r^4	r^5	r^6	r^7	1

Berdasarkan Tabel 3.14 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2,s\rangle}\,(D_{16})$ sebagai berikut



Gambar 3.25 Graf $\Gamma_{\langle r^2,s\rangle}(D_{16})$

Dari Gambar 3.25 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}\,(D_{16})}$ sebagai berikut



Gambar 3.26 Graf $\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc|}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^2 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^3 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^4 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^5 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^6 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^7 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
s & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
sr & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 \\
sr^2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\
sr^3 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\
sr^4 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\
sr^5 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \\
sr^6 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\
sr^7 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0
\end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc|}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
1 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^2 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^3 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^4 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^5 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^6 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
sr & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\
sr^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 \\
sr^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 \\
sr^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 \\
sr^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 \\
sr^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 \\
sr^7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22
\end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc|}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^2 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^3 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^4 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^5 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^6 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^7 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
s & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
sr & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 \\
sr^2 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 \\
sr^3 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 \\
sr^4 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 \\
sr^5 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 \\
sr^6 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 \\
sr^7 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22
\end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) - \lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) - \lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (22-\lambda)\left(-\frac{(\lambda-21)(\lambda-23)}{-22+\lambda}\right)\left(-\frac{(\lambda-24)(\lambda^2-42\lambda+438)}{(\lambda-21)(\lambda-23)}\right)\left(-\frac{(\lambda-22)(\lambda-18)(\lambda-24)}{\lambda^2-42\lambda+438}\right) \\ &\quad \left(-\frac{(\lambda-24)(\lambda^2-38\lambda+354)}{(\lambda-18)(\lambda-22)}\right)\left(-\frac{(\lambda-15)(\lambda-21)(\lambda-24)}{\lambda^2-38\lambda+354}\right)\left(-\frac{(\lambda-24)(\lambda^2-34\lambda+276)}{(\lambda-21)(\lambda-15)}\right) \\ &\quad \left(-\frac{(\lambda-20)(\lambda-12)(\lambda-24)}{\lambda^2-34\lambda+276}\right)\left(-\frac{(\lambda-24)(\lambda^2-30\lambda+204)}{(\lambda-20)(\lambda-12)}\right)\left(-\frac{(\lambda-24)(\lambda-19)(\lambda-9)}{\lambda^2-30\lambda+204}\right) \\ &\quad \left(-\frac{(\lambda-24)(\lambda^2-26\lambda+138)}{(\lambda-19)(\lambda-9)}\right)\left(-\frac{(\lambda-6)(\lambda-18)(\lambda-24)}{\lambda^2-26\lambda+138}\right)\left(-\frac{(\lambda-24)(\lambda^2-22\lambda+78)}{(\lambda-6)(\lambda-18)}\right) \\ &\quad \left(-\frac{(\lambda-17)(\lambda-3)(\lambda-24)}{\lambda^2-22\lambda+78}\right)\left(-\frac{(\lambda-24)(\lambda^2-18\lambda+24)}{(\lambda-3)(\lambda-17)}\right)\left(-\frac{(\lambda-16)(\lambda-24)\lambda}{\lambda^2-18\lambda+24}\right) \\ &= (\lambda-24)^{14}(\lambda-16)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 24, \lambda_2 = 16$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 14, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (14\cdot|24|) + (1\cdot|16|) + (1\cdot|0|) \\ &= 352 \end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$

$$
\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)=
\begin{array}{c}
\\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \end{array}\\
\begin{bmatrix}
22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1\\
1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2\\
2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1\\
1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2\\
2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1\\
1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2\\
2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1\\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2\\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1\\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2\\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1\\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2\\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1\\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2\\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1\\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22
\end{bmatrix}
\end{array}
$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right)$ sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (22-\lambda^+)\left(-\frac{(\lambda^+-21)(\lambda^+-23)}{-22+\lambda^+}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-46\lambda^++526\right)}{(\lambda^+-21)(\lambda^+-23)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-26)(\lambda^+-22)}{{\lambda^+}^2-46\lambda^++526}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-50\lambda^++618\right)}{(\lambda^+-26)(\lambda^+-22)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-23)(\lambda^+-29)}{{\lambda^+}^2-50\lambda^++618}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-54\lambda^++716\right)}{(\lambda^+-23)(\lambda^+-29)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-32)(\lambda^+-24)}{{\lambda^+}^2-54\lambda^++716}\right)\left(-\frac{(\lambda^+-20)({\lambda^+}^2-58\lambda^++820)}{(\lambda^+-32)(\lambda^+-24)}\right)\\
&\left(-\frac{(\lambda^+-35)(\lambda^+-25)(\lambda^+-20)}{{\lambda^+}^2-58\lambda^++820}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-66\lambda^++930\right)}{(\lambda^+-35)(\lambda^+-25)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-26)(\lambda^+-38)}{{\lambda^+}^2-66\lambda^++930}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-66\lambda^++1046\right)}{(\lambda^+-26)(\lambda^+-38)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-41)(\lambda^+-27)}{{\lambda^+}^2-66\lambda^++1046}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-70\lambda^++1168\right)}{(\lambda^+-41)(\lambda^+-27)}\right)\\
&\left(-\frac{(\lambda^+-20)(\lambda^+-28)(\lambda^+-44)}{{\lambda^+}^2-70\lambda^++1168}\right)\\
&= (\lambda^+-20)^{14}(\lambda^+-28)(\lambda^+-44)
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 44, \lambda_2^+ = 28$, dan $\lambda_3^+ = 20$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 14$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}$ adalah

$$\begin{aligned} ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{16})}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\ &= (1 \cdot |44|) + (1 \cdot |28|) + (14 \cdot |20|) \\ &= 352 \end{aligned}$$

Dari *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ di Grup Dihedral yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.15 Tabel pola *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$

D_{2n}	n	*DLE*	*DSLE*
D_8	4	$80 = 8(10)$ $= 2 \cdot 4(3 \cdot 4 - 2)$	80
D_{12}	6	$192 = 12 \cdot 16$ $= 2 \cdot 6(3 \cdot 6 - 2)$	192
D_{16}	8	$352 = 16 \cdot 22$ $= 2 \cdot 8(3 \cdot 8 - 2)$	352
$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$2n(3n-2) = 6n^2 - 4n$	$6n^2 - 4n$

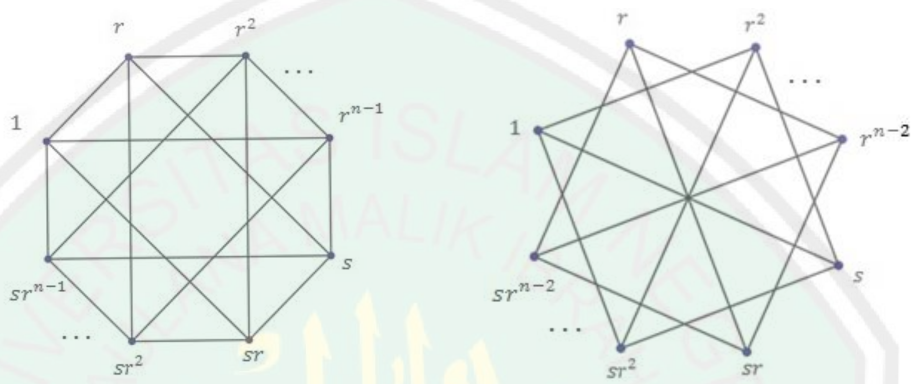
Teorema 3

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r^2, s\rangle = \{1, r^2, \dots, r^{n-2}, s, sr^2, \dots, sr^{n-2}\}$ subgrup normal dari D_{2n} untuk $n \geq 4$ dan n bilangan genap. *DLE* dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

Bukti:

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ untuk $n \geq 3$. Ambil $\langle r^2, s\rangle = \{1, r^2, \dots, r^{n-2}, s, sr^2, \dots, sr^{n-2}\}$ subgrup normal dari grup dihedral D_{2n}. Sesuai definisi graf subgrup dan komplemennya, maka diperoleh graf $\Gamma_{\langle r^2,s\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ yaitu:



Gambar 3.27 Graf $\Gamma_{\langle r^2,s\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$

Sehingga diperoleh matriks *distance* dari $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = \begin{array}{cc} & \begin{array}{ccccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} & \begin{bmatrix} 0 & 1 & 2 & 1 & \dots & 2 & 1 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\ 1 & 0 & 1 & 2 & \dots & 1 & 2 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\ 2 & 1 & 0 & 1 & \dots & 2 & 1 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\ 1 & 2 & 1 & 0 & \dots & 1 & 2 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2 & 1 & 2 & 1 & \dots & 0 & 1 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\ 1 & 2 & 1 & 2 & \dots & 1 & 0 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\ 2 & 1 & 2 & 1 & \dots & 2 & 1 & 0 & 1 & 2 & 1 & \dots & 2 & 1 \\ 1 & 2 & 1 & 2 & \dots & 1 & 2 & 1 & 0 & 1 & 2 & \dots & 1 & 2 \\ 2 & 1 & 2 & 1 & \dots & 2 & 1 & 2 & 1 & 0 & 1 & \dots & 2 & 1 \\ 1 & 2 & 1 & 2 & \dots & 1 & 2 & 1 & 2 & 1 & 0 & \dots & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2 & 1 & 2 & 1 & \dots & 2 & 1 & 2 & 1 & 2 & 1 & \cdots & 0 & 1 \\ 1 & 2 & 1 & 2 & \dots & 1 & 2 & 1 & 2 & 1 & 2 & \dots & 1 & 0 \end{bmatrix} \end{array}$$

dan matriks transmisi $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ adalah:

$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) =$

$$
\begin{array}{c}
\\ 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{bmatrix}
3n-2 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 3n-2 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 3n-2 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 3n-2 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 3n-2 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 3n-2 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 3n-2 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 3n-2 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 3n-2 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 3n-2 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 3n-2 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 3n-2
\end{bmatrix}
\end{array}
$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$ adalah

$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) =$

$$
\begin{array}{c}
\\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{bmatrix}
-1 & 3n-2 & -1 & -2 & \dots & -1 & -2 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
-2 & -1 & 3n-2 & -1 & \dots & -2 & -1 & -2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-1 & -2 & -1 & 3n-2 & \dots & -1 & -2 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2 & -1 & -2 & -1 & \dots & 3n-2 & -1 & -2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & 3n-2 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & 3n-2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & -1 & 3n-2 & -1 & -2 & \dots & -1 & -2 \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & -2 & -1 & 3n-2 & -1 & \dots & -2 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & -1 & -2 & -1 & 3n-2 & \dots & -1 & -2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & -2 & -1 & -2 & -1 & \cdots & 3n-2 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & -1 & -2 & -1 & -2 & \dots & -1 & 3n-2
\end{bmatrix}
\end{array}
$$

Polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$
\begin{array}{c}
\\ 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & r & r^2 & \dots & sr^{n-1} \end{array} \\
\begin{bmatrix}
(3n-2)-\lambda & \dots & \dots & \dots & \dots \\
0 & -\frac{(\lambda-(3n-3))(\lambda-(3n-1))}{-(3n-2)+\lambda} & \dots & \dots & \dots \\
0 & 0 & -\frac{(\lambda-3n)-\left(\lambda^2-(6n-6)\lambda+(9n^2-18n+6)\right)}{(\lambda-(3n-3))(\lambda-(3n-1))} & \dots & \dots \\
\vdots & \vdots & \vdots & & \dots \\
0 & 0 & 0 & \dots & -\frac{(\lambda-2n)(\lambda-3n)\lambda}{\lambda^2-(2n-2)\lambda+3n}
\end{bmatrix}
\end{array}
$$

Maka $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)-\lambda\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda)=\lambda(\lambda-2n)(\lambda-3n)^{2n-2}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh $\lambda_1=0, \lambda_2=2n$, dan $\lambda_3=3n$ dengan mulitiplisitas masing-masing $m(\lambda_1)=1$, $m(\lambda_2)=1$, dan $m(\lambda_3)=2n-2$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) &= m(\lambda_1)|\lambda_1|+m(\lambda_2)|\lambda_2|+m(\lambda_3)|\lambda_3| \\ &= 1\cdot|0|+1\cdot|2n|+2n-2\cdot|3n| \\ &= 2n+6n^2-6n \\ &= 6n^2-4n \end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$ adalah

$\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)=$

$$\begin{array}{cc} & \begin{array}{ccccccccccccc} 1 & r & r^2 & r^3 & \cdots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \cdots & sr^{n-2} & sr^{n-1} \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} & \begin{bmatrix} 3n-2 & 1 & 2 & 1 & \cdots & 2 & 1 & 2 & 1 & 2 & 1 & \cdots & 2 & 1 \\ 1 & 3n-2 & 1 & 2 & \cdots & 1 & 2 & 1 & 2 & 1 & 2 & \cdots & 1 & 2 \\ 2 & 1 & 3n-2 & 1 & \cdots & 2 & 1 & 2 & 1 & 2 & 1 & \cdots & 2 & 1 \\ 1 & 2 & 1 & 3n-2 & \cdots & 1 & 2 & 1 & 2 & 1 & 2 & \cdots & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2 & 1 & 2 & 1 & \cdots & 3n-2 & 1 & 2 & 1 & 2 & 1 & \cdots & 2 & 1 \\ 1 & 2 & 1 & 2 & \cdots & 1 & 3n-2 & 1 & 2 & 1 & 2 & \cdots & 1 & 2 \\ 2 & 1 & 2 & 1 & \cdots & 2 & 1 & 3n-2 & 1 & 2 & 1 & \cdots & 2 & 1 \\ 1 & 2 & 1 & 2 & \cdots & 1 & 2 & 1 & 3n-2 & 1 & 2 & \cdots & 1 & 2 \\ 2 & 1 & 2 & 1 & \cdots & 2 & 1 & 2 & 1 & 3n-2 & 1 & \cdots & 2 & 1 \\ 1 & 2 & 1 & 2 & \cdots & 1 & 2 & 1 & 2 & 1 & 3n-2 & \cdots & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 2 & 1 & 2 & 1 & \cdots & 2 & 1 & 2 & 1 & 2 & 1 & \cdots & 3n-2 & 1 \\ 1 & 2 & 1 & 2 & \cdots & 1 & 2 & 1 & 2 & 1 & 2 & \cdots & 1 & 3n-2 \end{bmatrix} \end{array}$$

Polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)-\lambda^+\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)-\lambda^+\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$
\begin{array}{c|ccccc|}
 & 1 & r & r^2 & \cdots & sr^{n-1} \\
\hline
1 & (3n-2)-\lambda & \cdots & \cdots & \cdots & \cdots \\
r & 0 & -\frac{(\lambda-(3n-3))(\lambda-(3n-1))}{-(3n-2)+\lambda} & \cdots & \cdots & \cdots \\
r^2 & 0 & 0 & -\frac{(\lambda-(3n-4))-(\lambda^2-(6n-2)\lambda+(9n^2-6n-2))}{(\lambda-(3n-3))(\lambda-(3n-1))} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & & \cdots \\
sr^{n-1} & 0 & 0 & 0 & \cdots & -\frac{(\lambda-(6n-4))(\lambda-(4n-4))(\lambda-(3n-4))}{\lambda^2-(10n-10)\lambda+(24n^2-49n+24)}
\end{array}
$$

Maka $\det\left(\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)-\lambda^{+}\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right)$, yaitu:

$$
\begin{aligned}
p(\lambda^{+}) &= (\lambda^{+}-4n+4)(\lambda^{+}-6n+4)(\lambda^{+}-3n+4)^{2n-2} \\
&= (\lambda^{+}-(4n-4))(\lambda^{+}-(6n-4))(\lambda^{+}-(3n-4))^{2n-2}
\end{aligned}
$$

Dengan menetapkan $p(\lambda^{+})=0$, diperoleh $\lambda_1^{+}=4n-4, \lambda_2^{+}=6n-4$, dan $\boldsymbol{\lambda_3^{+}}=3n-4$ dengan multiplisitas masing-masing $m(\lambda_1^{+})=1, m(\lambda_2^{+})=1$, dan $m(\lambda_3^{+})=2n-2$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}$ adalah

$$
\begin{aligned}
ED_L^{+}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) &= m(\lambda_1^{+})|\lambda_1^{+}|+m(\lambda_2^{+})|\lambda_2^{+}|+m(\lambda_3^{+})|\lambda_3^{+}| \\
&= 1\cdot|4n-4|+1\cdot|6n-4|+2n-2\cdot|3n-4| \\
&= 4n-4+6n-4+6n^2-8n-6n+8 \\
&= 6n^2-4n.
\end{aligned}
$$

3.4 *DLE* dan *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}$ dari Grup Dihedral

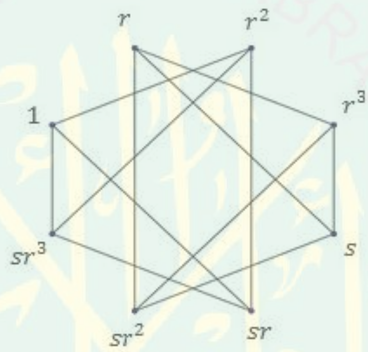
3.4.1 Grup dihedral D_8

Diketahui anggota grup D_8 adalah $\{1,r,r^2,r^3,s,sr,sr^2,sr^3\}$. Subgrup yang dibangun oleh $\langle r^2,sr\rangle$ pada grup dihedral D_8 adalah $\{1,r^2,s,sr^3\}$. Operasi unsur-unsur di D_8 yang hasilnya berada di $\langle r^2,sr\rangle$ seperti pada tabel berikut

Tabel 3.16 Tabel Cayley Grup Dihedral D_8

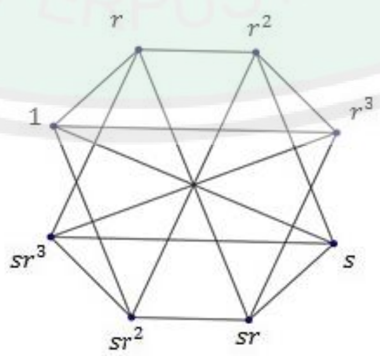
$\circ$	1	r	r^2	r^3	s	sr	sr^2	sr^3
1	1	r	r^2	r^3	s	sr	sr^2	sr^3
r	r	r^2	r^3	1	sr^3	s	sr	sr^2
r^2	r^2	r^3	1	r	sr^2	sr^3	s	sr
r^3	r^3	1	r	r^2	sr	sr^2	sr^3	s
s	s	sr	sr^2	sr^3	1	r	r^2	r^3
sr	sr	sr^2	sr^3	s	r^3	1	r	r^2
sr^2	sr^2	sr^3	s	sr	r^2	r^3	1	r
sr^3	sr^3	s	sr	sr^2	r	r^2	r^3	1

Berdasarkan Tabel 3.16 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2, sr\rangle}(D_8)$ sebagai berikut



Gambar 3.28 Graf $\Gamma_{\langle r^2, sr\rangle}(D_8)$

Dari Gambar 3.28 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^2, sr\rangle}(D_8)$ sebagai berikut



Gambar 3.29 Graf $\overline{\Gamma_{\langle r^2, sr\rangle}(D_8)}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ s\\ sr\\ sr^2\\ sr^3\end{array}\begin{array}{c}\begin{array}{cccccccc}1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3\end{array}\\ \begin{bmatrix}0&1&2&1&1&2&1&2\\1&0&1&2&2&1&2&1\\2&1&0&1&1&2&1&2\\1&2&1&0&2&1&2&1\\1&2&1&2&0&1&2&1\\2&1&2&1&1&0&1&2\\1&2&1&2&2&1&0&1\\2&1&2&1&1&2&1&0\end{bmatrix}\end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ s\\ sr\\ sr^2\\ sr^3\end{array}\begin{array}{c}\begin{array}{cccccccc}1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3\end{array}\\ \begin{bmatrix}10&0&0&0&0&0&0&0\\0&10&0&0&0&0&0&0\\0&0&10&0&0&0&0&0\\0&0&0&10&0&0&0&0\\0&0&0&0&10&0&0&0\\0&0&0&0&0&10&0&0\\0&0&0&0&0&0&10&0\\0&0&0&0&0&0&0&10\end{bmatrix}\end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)-\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ s\\ sr\\ sr^2\\ sr^3\end{array}\begin{array}{c}\begin{array}{cccccccc}1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3\end{array}\\ \begin{bmatrix}10&-1&-2&-1&-1&-2&-1&-2\\-1&10&-1&-2&-2&-1&-2&-1\\-2&-1&10&-1&-1&-2&-1&-2\\-1&-2&-1&10&-2&-1&-2&-1\\-1&-2&-1&-2&10&-1&-2&-1\\-2&-1&-2&-1&-1&10&-1&-2\\-1&-2&-1&-2&-2&-1&10&-1\\-2&-1&-2&-1&-1&-2&-1&10\end{bmatrix}\end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)-\lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)-\lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (10-\lambda)\left(-\frac{(\lambda-9)(\lambda-11)}{-10+\lambda}\right)\left(-\frac{(\lambda-12)(\lambda^2-18\lambda+78)}{(\lambda-9)(\lambda-11)}\right) \\ &\quad \left(-\frac{(\lambda-10)(\lambda-6)(\lambda-12)}{\lambda^2-18\lambda+78}\right)\left(-\frac{(\lambda-12)(\lambda^2-14\lambda+42)}{(\lambda-10)(\lambda-6)}\right)\left(-\frac{(\lambda-12)(\lambda-3)(\lambda-9)}{\lambda^2-14\lambda+42}\right) \\ &\quad \left(-\frac{(\lambda-12)(\lambda^2-10\lambda+12)}{(\lambda-3)(\lambda-9)}\right)\left(-\frac{(\lambda-8)(\lambda-12)\lambda}{\lambda^2-10\lambda+12}\right) \\ &= (\lambda-12)^6(\lambda-8)\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=12, \lambda_2=8$, dan $\lambda_3=0$ dengan multiplisitas masing-masing $m(\lambda_1)=6, m(\lambda_2)=1$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}$ adalah

$$\begin{aligned} ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i| \\ &= (6\cdot|12|)+(1\cdot|8|)+(1\cdot|0|) \\ &= 80 \end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)= \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ s \\ sr \\ sr^2 \\ sr^3 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & r & r^2 & r^3 & s & sr & sr^2 & sr^3 \end{array} \\ \begin{bmatrix} 10 & 1 & 2 & 1 & 1 & 2 & 1 & 2 \\ 1 & 10 & 1 & 2 & 2 & 1 & 2 & 1 \\ 2 & 1 & 10 & 1 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 10 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 10 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 1 & 10 & 1 & 2 \\ 1 & 2 & 1 & 2 & 2 & 1 & 10 & 1 \\ 2 & 1 & 2 & 1 & 1 & 2 & 1 & 10 \end{bmatrix} \end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right)$ sebagai berikut

$$p(\lambda^+) = (10-\lambda^+)\left(-\frac{(\lambda^+-9)(\lambda^+-11)}{-10+\lambda^+}\right)\left(-\frac{(\lambda^+-8)\left({\lambda^+}^2-22\lambda^++118\right)}{(\lambda^+-9)(\lambda^+-11)}\right)$$
$$\left(-\frac{(\lambda^+-10)(\lambda^+-8)(\lambda^+-14)}{{\lambda^+}^2-22\lambda^++118}\right)\left(-\frac{(\lambda^+-8)({\lambda^+}^2-26\lambda^++162)}{(\lambda^+-10)(\lambda^+-14)}\right)$$
$$\left(-\frac{(\lambda^+-11)(\lambda^+-17)(\lambda^+-8)}{{\lambda^+}^2-26\lambda^++162}\right)\left(-\frac{(\lambda^+-8)({\lambda^+}^2-30\lambda^++212)}{(\lambda^+-11)(\lambda^+-17)}\right)$$
$$\left(-\frac{(\lambda^+-20)(\lambda^+-12)(\lambda^+-8)}{{\lambda^+}^2-30\lambda^++212}\right)$$
$$= (\lambda^+-8)^6(\lambda^+-20)(\lambda^+-12)$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 20, \lambda_2^+ = 12$, dan $\lambda_3^+ = 8$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 6$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_8)}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$
$$= (1\cdot|20|) + (1\cdot|12|) + (6\cdot|8|)$$
$$= 80.$$

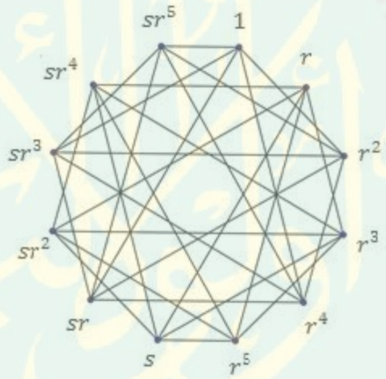
3.4.2 Grup dihedral D_{12}

Diketahui $D_{12} = \{1, r, r^2, \dots, r^5, s, sr, sr^2, \dots, sr^5\}$. Subgrup yang dibangun oleh $\langle r^2, sr\rangle$ pada grup dihedral D_{12} adalah $\{1, r^2, r^4, sr, sr^3, sr^5\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r^2, sr\rangle$ seperti pada tabel berikut

Tabel 3.17 Tabel Cayley Grup Dihedral D_{12}

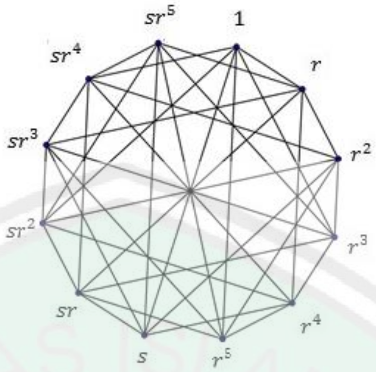
$\circ$	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
1	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
r	r	r^2	r^3	r^4	r^5	1	sr^5	s	sr	sr^2	sr^3	sr^4
r^2	r^2	r^3	r^4	r^5	1	r	sr^4	sr^5	s	sr	sr^2	sr^3
r^3	r^3	r^4	r^5	1	r	r^2	sr^3	sr^4	sr^5	s	sr	sr^2
r^4	r^4	r^5	1	r	r^2	r^3	sr^2	sr^3	sr^4	sr^5	s	sr
r^5	r^5	1	r	r^2	r^3	r^4	sr	sr^2	sr^3	sr^4	sr^5	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	1	r	r^2	r^3	r^4	r^5
sr	sr	sr^2	sr^3	sr^4	sr^5	s	r^5	1	r	r^2	r^3	r^4
sr^2	sr^2	sr^3	sr^4	sr^5	s	sr	r^4	r^5	1	r	r^2	r^3
sr^3	sr^3	sr^4	sr^5	s	sr	sr^2	r^3	r^4	r^5	1	r	r^2
sr^4	sr^4	sr^5	s	sr	sr^2	sr^3	r^2	r^3	r^4	r^5	1	r
sr^5	sr^5	s	sr	sr^2	sr^3	sr^4	r	r^2	r^3	r^4	r^5	1

Berdasarkan Tabel 3.17 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2, sr\rangle}(D_{12})$ sebagai berikut



Gambar 3.30 Graf $\Gamma_{\langle r^2, sr\rangle}(D_{12})$

Dari Gambar 3.30 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^2,sr\rangle}(D_{12})$ sebagai berikut



Gambar 3.31 Graf $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{12})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 0 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 0 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \end{bmatrix} \end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)} - \boldsymbol{D}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)}$ adalah

$$\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)} = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 16 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 \\ -1 & 16 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 \\ -2 & -1 & 16 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 \\ -1 & -2 & -1 & 16 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 \\ -2 & -1 & -2 & -1 & 16 & -1 & -1 & -2 & -1 & -2 & -1 & -2 \\ -1 & -2 & -1 & -2 & -1 & 16 & -2 & -1 & -2 & -1 & -2 & -1 \\ -1 & -2 & -1 & -2 & -1 & -2 & 16 & -1 & -2 & -1 & -2 & -1 \\ -2 & -1 & -2 & -1 & -2 & -1 & -1 & 16 & -1 & -2 & -1 & -2 \\ -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & 16 & -1 & -2 & -1 \\ -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & 16 & -1 & -2 \\ -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & 16 & -1 \\ -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & 16 \end{bmatrix} \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)} - \lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)} - \lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\overline{\left(\Gamma_{\langle r^2,sr\rangle}(D_{12})\right)}$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (16-\lambda)\left(-\frac{(\lambda-15)(\lambda-17)}{-16+\lambda}\right)\left(-\frac{(\lambda-18)(\lambda^2-30\lambda+222)}{(\lambda-15)(\lambda-17)}\right)\left(-\frac{(\lambda-16)(\lambda-12)(\lambda-18)}{\lambda^2-30\lambda+222}\right) \\ &\quad \left(-\frac{(\lambda-18)(\lambda^2-26\lambda+162)}{(\lambda-12)(\lambda-16)}\right)\left(-\frac{(\lambda-15)(\lambda-18)(\lambda-9)}{\lambda^2-26\lambda+162}\right)\left(-\frac{(\lambda-18)(\lambda^2-22\lambda+108)}{(\lambda-15)(\lambda-9)}\right) \\ &\quad \left(-\frac{(\lambda-6)(\lambda-18)(\lambda-14)}{\lambda^2-22\lambda+108}\right)\left(-\frac{(\lambda-18)(\lambda^2-18\lambda+60)}{(\lambda-6)(\lambda-14)}\right)\left(-\frac{(\lambda-13)(\lambda-3)(\lambda-18)}{\lambda^2-18\lambda+60}\right) \\ &\quad \left(-\frac{(\lambda-18)(\lambda^2-14\lambda+18)}{(\lambda-13)(\lambda-3)}\right)\left(-\frac{(\lambda-12)(\lambda-18)\lambda}{\lambda^2-14\lambda+18}\right) \\ &= (\lambda-12)(\lambda-18)^{10}\lambda \end{aligned}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 18, \lambda_2 = 12$, dan $\lambda_3 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 10, m(\lambda_2) = 1$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$

$$= (10\cdot|18|) + (1\cdot|12|) + (1\cdot|0|)$$

$$= 192$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma(D_{12})}\right) + \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right)$ adalah

$$\boldsymbol{D_L}^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \end{array} \begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array} \\ \begin{bmatrix} 16 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 16 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 16 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 16 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 16 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 16 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 & 1 & 2 & 16 & 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 16 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 16 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 16 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 16 & 1 \\ 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 16 \end{bmatrix} \end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{12})}\right)$ sebagai berikut

$$p(\lambda^+) = (16-\lambda^+)\left(-\frac{(\lambda^+-15)(\lambda^+-17)}{-16+\lambda^+}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-34\lambda^++286\right)}{(\lambda^+-15)(\lambda^+-17)}\right)$$
$$\left(-\frac{(\lambda^+-20)(\lambda^+-16)(\lambda^+-14)}{{\lambda^+}^2-34\lambda^++286}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-38\lambda^++354\right)}{(\lambda^+-20)(\lambda^+-16)}\right)$$
$$\left(-\frac{(\lambda^+-17)(\lambda^+-23)(\lambda^+-14)}{{\lambda^+}^2-38\lambda^++354}\right)\left(-\frac{(\lambda^+-14)({\lambda^+}^2-42\lambda^++428)}{(\lambda^+-17)(\lambda^+-23)}\right)$$
$$\left(-\frac{(\lambda^+-26)(\lambda^+-18)(\lambda^+-14)}{{\lambda^+}^2-42\lambda^++428}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-46\lambda^++508\right)}{(\lambda^+-26)(\lambda^+-18)}\right)$$
$$\left(-\frac{(\lambda^+-14)(\lambda^+-29)(\lambda^+-19)}{{\lambda^+}^2-46\lambda^++508}\right)\left(-\frac{(\lambda^+-14)\left({\lambda^+}^2-50\lambda^++594\right)}{(\lambda^+-29)(\lambda^+-19)}\right)$$
$$\left(-\frac{(\lambda^+-20)(\lambda^+-32)(\lambda^+-14)}{{\lambda^+}^2-50\lambda^++594}\right)$$
$$= (\lambda^+-32)(\lambda^+-20)(\lambda^+-14)^{10}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 32, \lambda_2^+ = 20$, dan $\lambda_3^+ = 14$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1$, dan $m(\lambda_3^+) = 10$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2, sr\rangle}(D_{12})}$ adalah

$$
\begin{aligned}
ED_L^+\left(\overline{\Gamma_{\langle r^2, sr\rangle}(D_{12})}\right) &= \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+| \\
&= (1 \cdot |32|) + (1 \cdot |20|) + (10 \cdot |14|) \\
&= 192.
\end{aligned}
$$

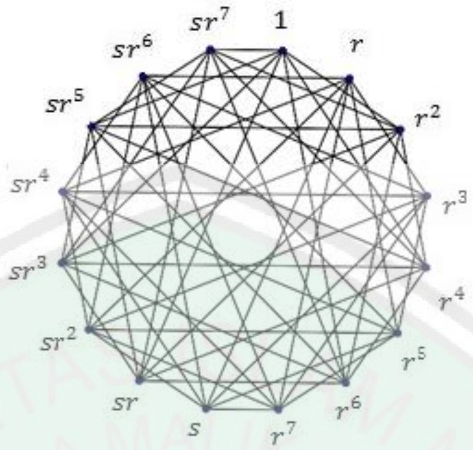
3.4.3 Grup dihedral D_{16}

Diketahui $D_{16} = \{1, r, r^2, \ldots, r^7, s, sr, sr^2, sr^3, \ldots, sr^7\}$. Subgrup yang dibangun oleh $\langle r^2, sr\rangle$ pada grup dihedral D_{16} adalah $\{1, r^2, r^4, r^6, sr, sr^3, sr^5, sr^7\}$. Operasi unsur-unsur di D_{16} yang hasilnya berada di $\langle r^2, sr\rangle$ seperti pada tabel berikut

Tabel 3.18 Tabel Cayley Grup Dihedral D_{16}

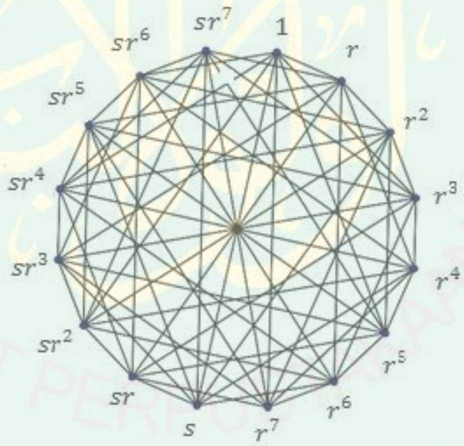
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r	r	r^2	r^3	r^4	r^5	r^6	r^7	1	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^2	r^2	r^3	r^4	r^5	r^6	r^7	1	r	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5
r^3	r^3	r^4	r^5	r^6	r^7	1	r	r^2	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4
r^4	r^4	r^5	r^6	r^7	1	r	r^2	r^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3
r^5	r^5	r^6	r^7	1	r	r^2	r^3	r^4	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2
r^6	r^6	r^7	1	r	r^2	r^3	r^4	r^5	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr
r^7	r^7	1	r	r^2	r^3	r^4	r^5	r^6	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	r^7	1	r	r^2	r^3	r^4	r^5	r^6
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	r^6	r^7	1	r	r^2	r^3	r^4	r^5
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	r^5	r^6	r^7	1	r	r^2	r^3	r^4
sr^4	sr^4	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	r^4	r^5	r^6	r^7	1	r	r^2	r^3
sr^5	sr^5	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	r^3	r^4	r^5	r^6	r^7	1	r	r^2
sr^6	sr^6	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	r^2	r^3	r^4	r^5	r^6	r^7	1	r
sr^7	sr^7	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r	r^2	r^3	r^4	r^5	r^6	r^7	1

Berdasarkan Tabel 3.18 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^2,sr\rangle}\,(D_{16})$ sebagai berikut



Gambar 3.32 Graf $\Gamma_{\langle r^2,sr\rangle}(D_{16})$

Dari Gambar 3.32 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^2,sr\rangle}\,(D_{16})$ sebagai berikut



Gambar 3.33 Graf $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$ sebagai berikut

$$
\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
\hline
1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^2 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^3 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^4 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^5 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
r^6 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
r^7 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
s & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
sr & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 2 \\
sr^2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 \\
sr^3 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 \\
sr^4 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 \\
sr^5 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 \\
sr^6 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\
sr^7 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0
\end{array}
$$

$$
\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
\hline
1 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^2 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^3 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^4 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^5 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^6 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
sr & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\
sr^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 & 0 \\
sr^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 & 0 \\
sr^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 & 0 \\
sr^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 & 0 \\
sr^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22 & 0 \\
sr^7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22
\end{array}
$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$ adalah

$$
\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) = \begin{array}{c|cccccccccccccccc}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 \\
\hline
1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^2 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^3 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^4 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^5 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
r^6 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 \\
r^7 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
s & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & 22 & -1 & -2 & -1 & -2 & -1 & -2 & -1 \\
sr & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & 22 & -1 & -2 & -1 & -2 & -1 & -2 \\
sr^2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & 22 & -1 & -2 & -1 & -2 & -1 \\
sr^3 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & 22 & -1 & -2 & -1 & -2 \\
sr^4 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & 22 & -1 & -2 & -1 \\
sr^5 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & 22 & -1 & -2 \\
sr^6 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -2 & -1 & -2 & -1 & -2 & -1 & 22 & -1 \\
sr^7 & -2 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & -1 & -2 & -1 & -2 & -1 & -2 & -1 & 22
\end{array}
$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)-\lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)-\lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$ sebagai berikut

$$\begin{aligned}p(\lambda) &= (22-\lambda)\left(-\frac{(\lambda-21)(\lambda-23)}{-22+\lambda}\right)\left(-\frac{(\lambda-24)(\lambda^2-42\lambda+438)}{(\lambda-21)(\lambda-23)}\right)\left(-\frac{(\lambda-22)(\lambda-18)(\lambda-24)}{\lambda^2-42\lambda+438}\right)\\
&\quad\left(-\frac{(\lambda-24)(\lambda^2-38\lambda+354)}{(\lambda-18)(\lambda-22)}\right)\left(-\frac{(\lambda-15)(\lambda-21)(\lambda-24)}{\lambda^2-38\lambda+354}\right)\left(-\frac{(\lambda-24)(\lambda^2-34\lambda+276)}{(\lambda-21)(\lambda-15)}\right)\\
&\quad\left(-\frac{(\lambda-20)(\lambda-12)(\lambda-24)}{\lambda^2-34\lambda+276}\right)\left(-\frac{(\lambda-24)(\lambda^2-30\lambda+204)}{(\lambda-20)(\lambda-12)}\right)\left(-\frac{(\lambda-24)(\lambda-19)(\lambda-9)}{\lambda^2-30\lambda+204}\right)\\
&\quad\left(-\frac{(\lambda-24)(\lambda^2-26\lambda+138)}{(\lambda-19)(\lambda-9)}\right)\left(-\frac{(\lambda-6)(\lambda-18)(\lambda-24)}{\lambda^2-26\lambda+138}\right)\left(-\frac{(\lambda-24)(\lambda^2-22\lambda+78)}{(\lambda-6)(\lambda-18)}\right)\\
&\quad\left(-\frac{(\lambda-17)(\lambda-3)(\lambda-18)}{\lambda^2-22\lambda+78}\right)\left(-\frac{(\lambda-24)(\lambda^2-18\lambda+24)}{(\lambda-3)(\lambda-17)}\right)\left(-\frac{(\lambda-16)(\lambda-24)\lambda}{\lambda^2-18\lambda+24}\right)\\
&= (\lambda-24)^{14}(\lambda-16)\lambda\end{aligned}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=24$, $\lambda_2=16$, dan $\lambda_3=0$ dengan multiplisitas masing-masing $m(\lambda_1)=14, m(\lambda_2)=1$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}$ adalah

$$\begin{aligned}ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) &= \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|\\
&= (14\cdot|24|)+(1\cdot|16|)+(1\cdot|0|)\\
&= 352\end{aligned}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$ adalah

$$
\boldsymbol{D_L}^{+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)=
\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccc}
1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7
\end{array}\\
\begin{bmatrix}
22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 22 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 22 & 1 & 2 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 22 & 1 & 2 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 22 & 1 & 2 & 1 & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 22 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 22 & 1 & 2 \\
1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 22 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 22
\end{bmatrix}
\end{array}
$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^{+}}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right)$ sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (22-\lambda^+)\left(-\frac{(\lambda^+-21)(\lambda^+-23)}{-22+\lambda^+}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-46\lambda^++526\right)}{(\lambda^+-21)(\lambda^+-23)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-26)(\lambda^+-22)}{{\lambda^+}^2-46\lambda^++526}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-50\lambda^++618\right)}{(\lambda^+-26)(\lambda^+-22)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-23)(\lambda^+-29)}{{\lambda^+}^2-50\lambda^++618}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-54\lambda^++716\right)}{(\lambda^+-23)(\lambda^+-29)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-32)(\lambda^+-24)}{{\lambda^+}^2-54\lambda^++716}\right)\left(-\frac{(\lambda^+-20)({\lambda^+}^2-58\lambda^++820)}{(\lambda^+-32)(\lambda^+-24)}\right)\\
&\quad\left(-\frac{(\lambda^+-35)(\lambda^+-25)(\lambda^+-20)}{{\lambda^+}^2-58\lambda^++820}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-62\lambda^++930\right)}{(\lambda^+-35)(\lambda^+-25)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-26)(\lambda^+-38)}{{\lambda^+}^2-62\lambda^++930}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-66\lambda^++1046\right)}{(\lambda^+-26)(\lambda^+-38)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-41)(\lambda^+-27)}{{\lambda^+}^2-66\lambda^++1046}\right)\left(-\frac{(\lambda^+-20)\left({\lambda^+}^2-70\lambda^++1168\right)}{(\lambda^+-20)(\lambda^+-41)}\right)\\
&\quad\left(-\frac{(\lambda^+-20)(\lambda^+-28)(\lambda^+-44)}{{\lambda^+}^2-70\lambda^++1168}\right)\\
&= (\lambda^+-20)^{14}(\lambda^+-28)(\lambda^+-44)
\end{aligned}
$$

Dengan menetapkan $p(\lambda^+)=0$, diperoleh nilai eigen $\lambda_1^+=44, \lambda_2^+=28$, dan $\lambda_3^+=20$ dengan multiplisitas masing-masing $m(\lambda_1^+)=1, m(\lambda_2^+)=1$, dan $m(\lambda_3^+)=14$.

Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{16})}\right) = \sum_{i=1}^{3} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1 \cdot |44|) + (1 \cdot |28|) + (14 \cdot |20|)$$

$$= 352.$$

Dari *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}\,(D_{2n})}$ di Grup Dihedral yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.19 Tabel pola *DLE* dan *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}\,(D_{2n})}$

D_{2n}	n	*DLE*	*DSLE*
D_8	4	$80 = 8(10)$ $= 2 \cdot 4(3 \cdot 4 - 2)$	80
D_{12}	6	$192 = 12 \cdot 16$ $= 2 \cdot 6(3 \cdot 6 - 2)$	192
D_{16}	8	$352 = 16 \cdot 22$ $= 2 \cdot 8(3 \cdot 8 - 2)$	352
$\vdots$	$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$2n(3n-2) = 6n^2 - 4n$	$6n^2 - 4n$

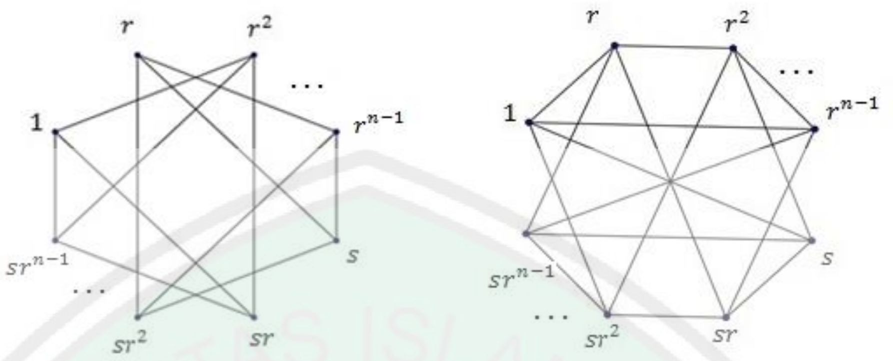
Teorema 4

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r^2, sr\rangle = \{1, r^2, \dots, r^{n-2}, sr, sr^3, \dots, sr^{n-1}\}$ subgrup normal dari D_{2n} untuk $n \geq 4$ dan n bilangan genap. *DLE* dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

Bukti:

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ untuk $n \geq 3$. Ambil $\langle r^2, sr\rangle = \{1, r^2, \dots, r^{n-2}, sr, sr^3, \dots, sr^{n-1}\}$ subgrup normal dari grup dihedral D_{2n}. Sesuai definisi graf subgrup dan komplemennya, maka diperoleh graf $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}$ yaitu:



Gambar 3.34 Graf $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}$

Sesuai langkah memperoleh matriks distance dan transmisi pada Teorema 3, maka matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) =$$

$$\begin{array}{c c}
 & \begin{array}{ccccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
3n-2 & -1 & -2 & -1 & \dots & -2 & -1 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
-1 & 3n-2 & -1 & -2 & \dots & -1 & -2 & -2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-2 & -1 & 3n-2 & -1 & \dots & -2 & -1 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
-1 & -2 & -1 & 3n-2 & \dots & -1 & -2 & -2 & -1 & -2 & -1 & \dots & -2 & -1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-2 & -1 & -2 & -1 & \dots & 3n-2 & -1 & -1 & -2 & -1 & -2 & \dots & -1 & -2 \\
-1 & -2 & -1 & -2 & \dots & -1 & 3n-2 & -2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & 3n-2 & -1 & -2 & -1 & \dots & -2 & -1 \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & -1 & 3n-2 & -1 & -2 & \dots & -1 & -2 \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & -2 & -1 & 3n-2 & -1 & \dots & -2 & -1 \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & -1 & -2 & -1 & 3n-2 & \dots & -1 & -2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-1 & -2 & -1 & -2 & \dots & -1 & -2 & -2 & -1 & -2 & -1 & \cdots & 3n-2 & -1 \\
-2 & -1 & -2 & -1 & \dots & -2 & -1 & -1 & -2 & -1 & -2 & \dots & -1 & 3n-2
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \lambda \boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$
\begin{array}{c|ccccc}
 & 1 & r & r^2 & \cdots & sr^{n-1} \\
\hline
1 & (3n-2)-\lambda & \cdots & \cdots & \cdots & \cdots \\
r & 0 & -\frac{(\lambda-(3n-3))(\lambda-(3n-1))}{-(3n-2)+\lambda} & \cdots & \cdots & \cdots \\
r^2 & 0 & 0 & -\frac{(\lambda-3n)-\left(\lambda^2-(6n-6)\lambda+(9n^2-18n+6)\right)}{(\lambda-(3n-3))(\lambda-(3n-1))} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & & \cdots \\
sr^{n-1} & 0 & 0 & 0 & \cdots & -\frac{(\lambda-2n)(\lambda-3n)\lambda}{\lambda^2-(2n-2)\lambda+3n}
\end{array}
$$

Maka $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)-\lambda\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda)=\lambda(\lambda-2n)(\lambda-3n)^{2n-2}$$

Dengan menetapkan $p(\lambda)=0$ maka diperoleh $\lambda_1=0, \lambda_2=2n,$ dan $\lambda_3=3n$ dengan multiplisitas masing-maisng $m(\lambda_1)=1,$ $m(\lambda_2)=1,$ dan $m(\lambda_3)=2n-2$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}$ adalah

$$
\begin{aligned}
ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) &= m(\lambda_1)|\lambda_1|+m(\lambda_2)|\lambda_2|+m(\lambda_3)|\lambda_3| \\
&= 1\cdot|0|+1\cdot|2n|+2n-2\cdot|3n| \\
&= 2n+6n^2-6n \\
&= 6n^2-4n
\end{aligned}
$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)+\boldsymbol{D}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$ adalah

$\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) =$

$$\begin{array}{c|cccccccccccccc}
 & 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \\
\hline
1 & 3n-2 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\
r & 1 & 3n-2 & 1 & 2 & \dots & 1 & 2 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\
r^2 & 2 & 1 & 3n-2 & 1 & \dots & 2 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\
r^3 & 1 & 2 & 1 & 3n-2 & \dots & 1 & 2 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
r^{n-2} & 2 & 1 & 2 & 1 & \dots & 3n-2 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 2 \\
r^{n-1} & 1 & 2 & 1 & 2 & \dots & 1 & 3n-2 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \\
s & 1 & 2 & 1 & 2 & \dots & 1 & 2 & 3n-2 & 1 & 2 & 1 & \dots & 2 & 1 \\
sr & 2 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 3n-2 & 1 & 2 & \dots & 1 & 2 \\
sr^2 & 1 & 2 & 1 & 2 & \dots & 1 & 2 & 2 & 1 & 3n-2 & 1 & \dots & 2 & 1 \\
sr^3 & 2 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 2 & 1 & 3n-2 & \dots & 1 & 2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
sr^{n-2} & 1 & 2 & 1 & 2 & \dots & 1 & 2 & 2 & 1 & 2 & 1 & \cdots & 3n-2 & 1 \\
sr^{n-1} & 2 & 1 & 2 & 1 & \dots & 2 & 1 & 1 & 2 & 1 & 2 & \dots & 1 & 3n-2
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \lambda^+\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \lambda^+\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$\begin{array}{c|ccccc}
 & 1 & r & r^2 & \dots & sr^{n-1} \\
\hline
1 & (3n-2)-\lambda & \dots & \dots & \dots & \dots \\
r & 0 & -\frac{(\lambda-(3n-3))(\lambda-(3n-1))}{-(3n-2)+\lambda} & \dots & \dots & \dots \\
r^2 & 0 & 0 & -\frac{(\lambda-(3n-4))-\left(\lambda^2-(6n-2)\lambda+(9n^2-6n-2)\right)}{(\lambda-(3n-3))(\lambda-(3n-1))} & \dots & \dots \\
\vdots & \vdots & \vdots & \vdots & & \dots \\
sr^{n-1} & 0 & 0 & 0 & \dots & -\frac{(\lambda-(6n-4))(\lambda-(4n-4))(\lambda-(3n-4))}{\lambda^2-(10n-10)\lambda+(24n^2-49n+24)}
\end{array}$$

Maka $\det\left(\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) - \lambda^+\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L^+}\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right)$, yaitu:

$$\begin{aligned}
p(\lambda^+) &= (\lambda^+ - 4n + 4)(\lambda^+ - 6n + 4)(\lambda^+ - 3n + 4)^{2n-2} \\
&= (\lambda^+ - (4n-4))(\lambda^+ - (6n-4))(\lambda^+ - (3n-4))^{2n-2}
\end{aligned}$$

Dengan menetapkan $p(\lambda^+) = 0$ maka diperoleh $\lambda_1^+ = 4n-4, \lambda_2^+ = 6n-4,$ dan $\lambda_3^+ = 3n-4$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1,$ dan $m(\lambda_3^+) = 2n-2$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}$ adalah

$$
\begin{aligned}
ED_L^{+}\left(\overline{\Gamma_{\langle r^2, sr\rangle}(D_{2n})}\right) &= m(\lambda_1^{+})|\lambda_1^{+}| + m(\lambda_2^{+})|\lambda_2^{+}| + m(\lambda_3^{+})|\lambda_3^{+}| \\
&= (1 \cdot |4n-4|) + (1 \cdot |6n-4|) + (2n-2 \cdot |3n-4|\ ) \\
&= 4n - 4 + 6n - 4 + 6n^2 - 8n - 6n + 8 \\
&= 6n^2 - 4n.
\end{aligned}
$$

3.5 *DLE* dan *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ dari Grup Dihedral

3.5.1 *DLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ dari Grup Dihedral

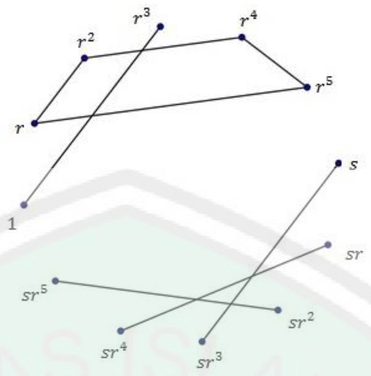
3.5.1.1 Grup Dihedral D_{12}

Diketahui $D_{12} = \{1, r, r^2, \dots, r^5, s, sr, sr^2, \dots, sr^5\}$. Subgrup yang dibangun oleh $\langle r^3\rangle$ pada grup dihedral D_{12} adalah $\{1, r^3\}$. Operasi unsur-unsur di D_{12} yang hasilnya berada di $\langle r^3\rangle$ seperti pada tabel berikut

Tabel 3.20 Tabel Cayley Grup Dihedral D_{12}

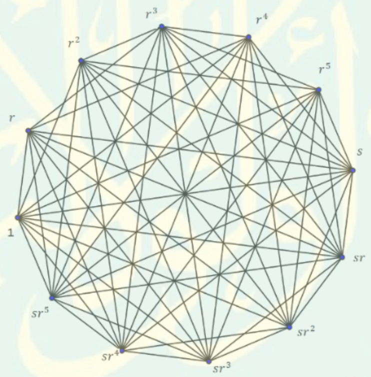
$\circ$	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
1	1	r	r^2	r^3	r^4	r^5	s	sr	sr^2	sr^3	sr^4	sr^5
r	r	r^2	r^3	r^4	r^5	1	sr^5	s	sr	sr^2	sr^3	sr^4
r^2	r^2	r^3	r^4	r^5	1	r	sr^4	sr^5	s	sr	sr^2	sr^3
r^3	r^3	r^4	r^5	1	r	r^2	sr^3	sr^4	sr^5	s	sr	sr^2
r^4	r^4	r^5	1	r	r^2	r^3	sr^2	sr^3	sr^4	sr^5	s	sr
r^5	r^5	1	r	r^2	r^3	r^4	sr	sr^2	sr^3	sr^4	sr^5	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	1	r	r^2	r^3	r^4	r^5
sr	sr	sr^2	sr^3	sr^4	sr^5	s	r^5	1	r	r^2	r^3	r^4
sr^2	sr^2	sr^3	sr^4	sr^5	s	sr	r^4	r^5	1	r	r^2	r^3
sr^3	sr^3	sr^4	sr^5	s	sr	sr^2	r^3	r^4	r^5	1	r	r^2
sr^4	sr^4	sr^5	s	sr	sr^2	sr^3	r^2	r^3	r^4	r^5	1	r
sr^5	sr^5	s	sr	sr^2	sr^3	sr^4	r	r^2	r^3	r^4	r^5	1

Berdasarkan Tabel 3.20 dan definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{12})$ sebagai berikut



Gambar 3.35 Graf $\Gamma_{\langle r^3 \rangle}(D_{12})$

Dari Gambar 3.35 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{12})$ sebagai berikut



Gambar 3.36 Graf $\overline{\Gamma_{\langle r^3 \rangle}(D_{12})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{12})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{12})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5 \end{array}\begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array}\\ \begin{bmatrix} 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 2 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2\\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 \end{bmatrix}\end{array}$$

$$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5 \end{array}\begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array}\\ \begin{bmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 \end{bmatrix}\end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)-\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)$ adalah

$$\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)=\begin{array}{c} \\ 1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5 \end{array}\begin{array}{c} \begin{array}{cccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 \end{array}\\ \begin{bmatrix} 12 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & 13 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & -2 & 13 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1\\ -2 & -1 & -1 & 12 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & -1 & -2 & -1 & 13 & -2 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & -2 & -1 & -1 & -2 & 13 & -1 & -1 & -1 & -1 & -1 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & 12 & -1 & -1 & -2 & -1 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 12 & -1 & -1 & -2 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 12 & -1 & -1 & 2\\ -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 12 & 1 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 12 & -1\\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 12 \end{bmatrix}\end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)-\lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)-\lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right)$ sebagai berikut

$$
\begin{aligned}
p(\lambda) &= (12-\lambda)\left(-\frac{\lambda^2-25\lambda+155}{\lambda-12}\right)\left(-\frac{(\lambda-13)(\lambda-15)(\lambda-10)}{\lambda^2-25\lambda+155}\right)\left(-\frac{(\lambda-14)(\lambda^2-21\lambda+106)}{(\lambda-13)(\lambda-10)}\right) \\
&\quad \left(-\frac{(\lambda-14)(\lambda^3-35\lambda^2+392\lambda-1388)}{(\lambda^2-21\lambda+106)(\lambda-15)}\right)\left(-\frac{(\lambda-12)(\lambda-14)(\lambda-16)(\lambda-6)}{\lambda^3-35\lambda^2+392\lambda-1388}\right) \\
&\quad \left(-\frac{\lambda^2-18\lambda+66}{(\lambda-6)}\right)\left(-\frac{(\lambda-13)(\lambda^2-17\lambda+54)}{\lambda^2-18\lambda+66}\right)\left(-\frac{(\lambda-13)(\lambda^2-16\lambda+42)}{\lambda^2-17\lambda+54}\right) \\
&\quad \left(-\frac{(\lambda-14)(\lambda-12)(\lambda^2-15\lambda+28)}{(\lambda-13)(\lambda^2-16\lambda+42)}\right)\left(-\frac{(\lambda-14)(\lambda-12)(\lambda^2-14\lambda+14)}{(\lambda-13)(\lambda^2-15\lambda+28)}\right)\left(-\frac{\lambda(\lambda-14)(\lambda-12)}{\lambda^2-14\lambda+14}\right) \\
&= \lambda\,(\lambda-16)(\lambda-12)^4(\lambda-14)^6
\end{aligned}
$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1=16, \lambda_2=12, \lambda_3=14$, dan $\lambda_4=0$ dengan multiplisitas masing-masing $m(\lambda_1)=1, m(\lambda_2)=4, m(\lambda_3)=6$, dan $m(\lambda_3)=1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{12})}$ adalah

$$
\begin{aligned}
ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{12})}\right) &= \sum_{i=1}^{4} m(\lambda_i)|\lambda_i| \\
&= (1\cdot|16|)+(4\cdot|12|)+(6\cdot|14|)+(1\cdot|0|) \\
&= 148.
\end{aligned}
$$

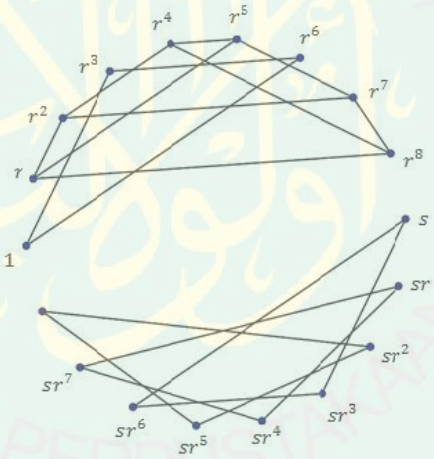
3.5.1.2 Grup Dihedral D_{18}

Diketahui $D_{18}=\{1,r,r^2,\dots,r^8,s,sr,sr^2,\dots,sr^8\}$. Subgrup yang dibangun oleh $\langle r^3\rangle$ pada grup dihedral D_{18} adalah $\{1,r^3,r^6\}$. Operasi unsur-unsur di D_{18} yang hasilnya berada di $\langle r^3\rangle$ seperti tabel berikut

Tabel 3.21 Tabel Cayley Grup Dihedral D_{18}

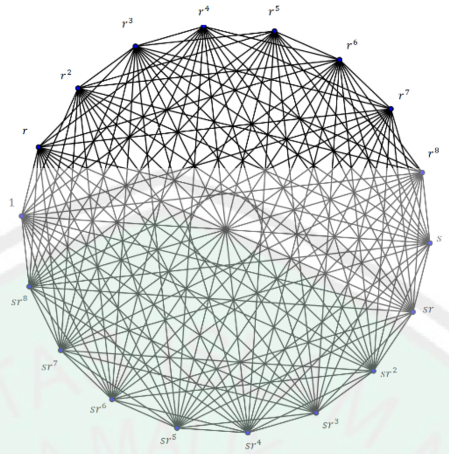
$\circ$	$\mathbf{1}$	$\boldsymbol{r}$	$\boldsymbol{r^2}$	$\boldsymbol{r^3}$	$\boldsymbol{r^4}$	$\boldsymbol{r^5}$	$\boldsymbol{r^6}$	$\boldsymbol{r^7}$	$\boldsymbol{r^8}$	$\boldsymbol{s}$	$\boldsymbol{sr}$	$\boldsymbol{sr^2}$	$\boldsymbol{sr^3}$	$\boldsymbol{sr^4}$	$\boldsymbol{sr^5}$	$\boldsymbol{sr^6}$	$\boldsymbol{sr^7}$	$\boldsymbol{sr^8}$
$\mathbf{1}$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8
$\boldsymbol{r}$	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	1	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
$\boldsymbol{r^2}$	r^2	r^3	r^4	r^5	r^6	r^7	r^8	1	r	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
$\boldsymbol{r^3}$	r^3	r^4	r^5	r^6	r^7	r^8	1	r	r^2	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5
$\boldsymbol{r^4}$	r^4	r^5	r^6	r^7	r^8	1	r	r^2	r^3	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4
$\boldsymbol{r^5}$	r^5	r^6	r^7	r^8	1	r	r^2	r^3	r^4	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3
$\boldsymbol{r^6}$	r^6	r^7	r^8	1	r	r^2	r^3	r^4	r^5	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2
$\boldsymbol{r^7}$	r^7	r^8	1	r	r^2	r^3	r^4	r^5	r^6	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr
$\boldsymbol{r^8}$	r^8	1	r	r^2	r^3	r^4	r^5	r^6	r^7	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s
$\boldsymbol{s}$	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8
$\boldsymbol{sr}$	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s	r^8	1	r	r^2	r^3	r^4	r^5	r^6	r^7
$\boldsymbol{sr^2}$	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr	r^7	r^8	1	r	r^2	r^3	r^4	r^5	r^6
$\boldsymbol{sr^3}$	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2	r^6	r^7	r^8	1	r	r^2	r^3	r^4	r^5
$\boldsymbol{sr^4}$	sr^4	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3	r^5	r^6	r^7	r^8	1	r	r^2	r^3	r^4
$\boldsymbol{sr^5}$	sr^5	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4	r^4	r^5	r^6	r^7	r^8	1	r	r^2	r^3
$\boldsymbol{sr^6}$	sr^6	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5	r^3	r^4	r^5	r^6	r^7	r^8	1	r	r^2
$\boldsymbol{sr^7}$	sr^7	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r^2	r^3	r^4	r^5	r^6	r^7	r^8	1	r
$\boldsymbol{sr^8}$	sr^8	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	1

Dari definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{18})$ sebagai berikut



Gambar 3.37 Graf $\Gamma_{\langle r^3 \rangle}(D_{18})$

Dari Gambar 3.37 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{18})$ sebagai berikut



Gambar 3.38 Graf $\overline{\Gamma_{\langle r^3\rangle}(D_{18})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)$dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)$ sebagai berikut

$$\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right) = \begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$
\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)=
\begin{array}{c}
\begin{matrix} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 \end{matrix} \\
\begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \end{matrix}
\begin{bmatrix}
19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19
\end{bmatrix}
\end{array}
$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)=\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)-\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)$ adalah

$$
\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)=
\begin{array}{c}
\begin{matrix} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 \end{matrix} \\
\begin{matrix} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \end{matrix}
\begin{bmatrix}
19 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 20 & -2 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -2 & 20 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -1 & -1 & 19 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -2 & -1 & 20 & -2 & -1 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -2 & -1 & -1 & -2 & 20 & -1 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2 & -1 & -1 & -2 & -1 & -1 & 19 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -2 & -1 & -1 & -2 & -1 & 20 & -2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -2 & -1 & -1 & -2 & -1 & -1 & -2 & 20 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 19 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 19 & -1 & -1 & -2 & -1 & -1 & -2 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 19 & -1 & -1 & -2 & -1 & -1 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 19 & -1 & -1 & -2 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 19 & -1 & -1 & -2 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & 19 & -1 & -1 & -2 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & 19 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & 19 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -2 & -1 & -1 & -2 & -1 & -1 & 19
\end{bmatrix}
\end{array}
$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)-\lambda\boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)-\lambda\boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right)$ sebagai berikut

$$p(\lambda) = (19-\lambda)\left(-\frac{\lambda^2-39\lambda+379}{\lambda-19}\right)\left(-\frac{(\lambda-22)(\lambda-17)(\lambda-20)}{\lambda^2-39\lambda+379}\right)$$
$$\left(-\frac{(\lambda-21)(\lambda^2-35\lambda+302)}{(\lambda-17)(\lambda-20)}\right)\left(-\frac{(\lambda-21)(\lambda^3-56\lambda^2+1029\lambda-6190)}{(\lambda^2-35\lambda+302)(\lambda-22)}\right)$$
$$\left(-\frac{(\lambda-23)(\lambda-13)(\lambda-19)(\lambda-21)}{\lambda^3-56\lambda^2+1029\lambda-6190}\right)\left(-\frac{(\lambda-21)(\lambda^2-30\lambda+213)}{(\lambda-13)(\lambda-19)}\right)$$
$$\left(-\frac{(\lambda-21)(\lambda^3-52\lambda^2+858\lambda-4419)}{(\lambda-21)(\lambda^2-30\lambda+213)}\right)\left(-\frac{(\lambda-24)(\lambda-18)(\lambda-9)(\lambda-21)}{\lambda^3-52\lambda^2+858\lambda-4419}\right)$$
$$\left(-\frac{\lambda^2-28\lambda+162}{\lambda-9}\right)\left(-\frac{(\lambda-20)(\lambda^2-27\lambda+144)}{\lambda^2-28\lambda+162}\right)$$
$$\left(-\frac{(\lambda-20)(\lambda^2-26\lambda+126)}{\lambda^2-27\lambda+144}\right)\left(-\frac{(\lambda-21)(\lambda^3-44\lambda^2+581\lambda-2016)}{(\lambda-20)(\lambda^2-26\lambda+126)}\right)$$
$$\left(-\frac{(\lambda-21)(\lambda-19)(\lambda^3-43\lambda^2+542\lambda-1638)}{(\lambda-20)(\lambda^3-44\lambda^2+581\lambda-2016)}\right)$$
$$\left(-\frac{(\lambda-21)(\lambda-19)(\lambda^2-22\lambda+63)}{\lambda^3-43\lambda^2+542\lambda-1638}\right)\left(-\frac{(\lambda-18)(\lambda-21)(\lambda^2-21\lambda+42)}{(\lambda-19)(\lambda^2-22\lambda+63)}\right)$$
$$\left(-\frac{(\lambda-18)(\lambda-21)(\lambda^2-20\lambda-21)}{(\lambda-19)(\lambda^2-21\lambda+42)}\right)\left(-\frac{\lambda(\lambda-18)(\lambda-21)}{\lambda^2-20\lambda+21}\right)$$
$$= \lambda(\lambda-24)(\lambda-18)^4(\lambda-21)^{12}$$

Dengan menetapkan $p(\lambda)=0$, diperoleh nilai eigen $\lambda_1 = 24$, $\lambda_2 = 21, \lambda_3 = 18$, dan $\lambda_4 = 0$ dengan multiplisitas masing-masing $m(\lambda_1) = 1, m(\lambda_2) = 12, m(\lambda_3) = 4$, dan $m(\lambda_3) = 1$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{18})}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{18})}\right) = \sum_{i=1}^{4} m(\lambda_i)|\lambda_i|$$
$$= (1\cdot|24|) + (12\cdot|21|) + (4\cdot|18|) + (1\cdot|0|)$$
$$= 348.$$

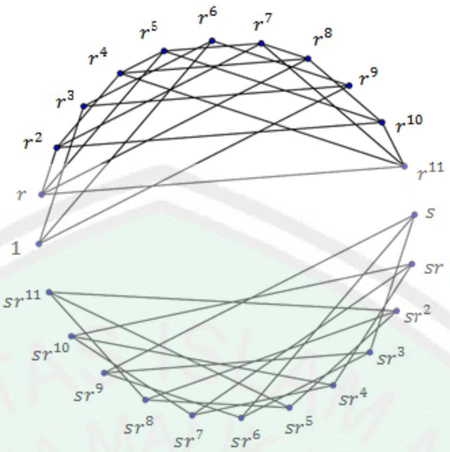
3.5.1.3 Grup Dihedral D_{24}

Diketahui anggota grup $D_{24} = \{1, r, r^2, \dots, r^{11}, s, sr, sr^2, \dots, sr^{11}\}$. Subgrup yang dibangun oleh $\langle r^3 \rangle$ pada grup dihedral D_{24} adalah $\{1, r^3, r^6, r^9\}$. Operasi unsur-unsur di D_{24} yang hasilnya berada di $\langle r^3 \rangle$ seperti pada tabel berikut

Tabel 3.22 Tabel Cayley Grup Dihedral D_{24}

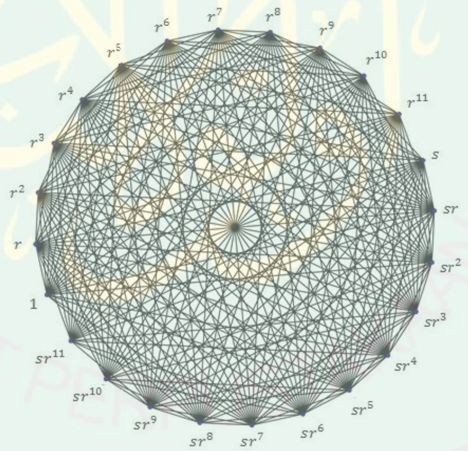
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}
r	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}
r^2	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9
r^3	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8
r^4	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r^5	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^6	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5
r^7	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4
r^8	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3
r^9	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2
r^{10}	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr
r^{11}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8
sr^4	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr^5	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5	r^6
sr^6	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4	r^5
sr^7	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3	r^4
sr^8	sr^8	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2	r^3
sr^9	sr^9	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r	r^2
sr^{10}	sr^{10}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1	r
sr^{11}	sr^{11}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	1

Dari definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{24})$ sebagai berikut



Gambar 3.39 Graf $\Gamma_{\langle r^3 \rangle}(D_{24})$

Dari Gambar 3.39 dapat diperoleh komplemen graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{24})$ sebagai berikut



Gambar 3.40 Graf $\overline{\Gamma_{\langle r^3 \rangle}(D_{24})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{24})}\right)$ dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{24})}\right)$ sebagai berikut

$$D\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right)=\begin{array}{c}\\ \begin{matrix}1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ r^6\\ r^7\\ r^8\\ r^9\\ r^{10}\\ r^{11}\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5\\ sr^6\\ sr^7\\ sr^8\\ sr^9\\ sr^{10}\\ sr^{11}\end{matrix}\end{array}\begin{array}{c}\begin{matrix}1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11}\end{matrix}\\ \begin{bmatrix}
0&1&1&2&1&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&0&2&1&1&2&1&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1\\
1&2&0&1&2&1&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1\\
2&1&1&0&1&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&2&1&0&2&1&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1\\
1&2&1&1&2&0&1&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1\\
2&1&1&2&1&1&0&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&2&1&1&2&1&0&2&1&1&2&1&1&1&1&1&1&1&1&1&1&1&1\\
1&2&1&1&2&1&1&2&0&1&2&1&1&1&1&1&1&1&1&1&1&1&1&1\\
2&1&1&2&1&1&2&1&1&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&2&1&1&2&1&1&2&1&0&2&1&1&1&1&1&1&1&1&1&1&1&1\\
1&2&1&1&2&1&1&2&1&1&2&0&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&2&1&1&2&1&1&2&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&2&1&1&2&1&1&2&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&2&1&1&2&1&1&2\\
1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&0&1&1&2&1&1&2&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&0&1&1&2&1&1&2&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&0&1&1&2&1&1&2\\
1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&0&1&1&2&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&0&1&1&2&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&0&1&1&2\\
1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&2&1&1&0&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&2&1&1&0&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&2&1&1&2&1&1&2&1&1&0
\end{bmatrix}\end{array}$$

$$T_r\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right)=\begin{array}{c}\\ \begin{matrix}1\\ r\\ r^2\\ r^3\\ r^4\\ r^5\\ r^6\\ r^7\\ r^8\\ r^9\\ r^{10}\\ r^{11}\\ s\\ sr\\ sr^2\\ sr^3\\ sr^4\\ sr^5\\ sr^6\\ sr^7\\ sr^8\\ sr^9\\ sr^{10}\\ sr^{11}\end{matrix}\end{array}\begin{array}{c}\begin{matrix}1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11}\end{matrix}\\ \begin{bmatrix}
26&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&26&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&26&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&27&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&26
\end{bmatrix}\end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) = \boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right)$ adalah

$$D_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) = \begin{array}{cc} & \begin{array}{cccccccccccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11} \end{array} \\ \begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ r^9 \\ r^{10} \\ r^{11} \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \\ sr^9 \\ sr^{10} \\ sr^{11} \end{array} & \left[\begin{array}{cccccccccccccccccccccccc}
-26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -27 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & -27 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & -27 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & -27 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & -27 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & -27 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & -27 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & -27 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & -26
\end{array}\right] \end{array}$$

Setelah mendapatkan matriks *distance Laplacian* maka dicari polinomial karakteristik dari matriks tersebut dengan rumus $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) - \lambda \boldsymbol{I}\right)$. Matriks $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) - \lambda \boldsymbol{I}$ akan direduksi untuk memperoleh polinomial karakteristik. Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right)$ sebagai berikut

$$\begin{aligned} p(\lambda) &= (26-\lambda)\left(-\frac{\lambda^2-53\lambda+701}{\lambda-26}\right)\left(-\frac{(\lambda-24)(\lambda-17)(\lambda-29)}{\lambda^2-53\lambda+701}\right) \\ &\quad \left(-\frac{(\lambda-28)(\lambda^2-49\lambda+596)}{(\lambda-27)(\lambda-24)}\right)\left(-\frac{(\lambda-28)(\lambda^3-77\lambda^2+1960\lambda-16480)}{(\lambda^2-49\lambda+596)(\lambda-29)}\right) \\ &\quad \left(-\frac{(\lambda-26)(\lambda-28)(\lambda-30)(\lambda-20)}{\lambda^3-77\lambda^2+1960\lambda-16480}\right)\left(-\frac{(\lambda-28)(\lambda^2-44\lambda+472)}{(\lambda-20)(\lambda-26)}\right) \\ &\quad \left(-\frac{(\lambda-28)(\lambda^3-73\lambda^2+1733\lambda-13316)}{(\lambda-30)(\lambda^2-44\lambda+472)}\right)\left(-\frac{(\lambda-25)(\lambda-16)(\lambda-28)(\lambda-31)}{\lambda^3-73\lambda^2+1733\lambda-13316}\right) \\ &\quad \left(-\frac{(\lambda-28)(\lambda^2-39\lambda+356)}{(\lambda-16)(\lambda-25)}\right)\left(-\frac{(\lambda-28)(\lambda^3-69\lambda^2+1504\lambda-10160)}{(\lambda-31)(\lambda^2-39\lambda+356)}\right) \\ &\quad \left(-\frac{(\lambda-12)(\lambda-24)(\lambda-28)(\lambda-32)}{\lambda^3-69\lambda^2+1504\lambda-10160}\right)\left(-\frac{\lambda^2-38\lambda+300}{\lambda-12}\right) \\ &\quad \left(-\frac{(\lambda-27)(\lambda^2-37\lambda+276)}{\lambda^2-38\lambda+300}\right)\left(-\frac{(\lambda-27)(\lambda^2-36\lambda+252)}{\lambda^2-37\lambda+276}\right) \end{aligned}$$

$$\left(-\frac{(\lambda-28)(\lambda^3-61\lambda^2+1136\lambda-5880)}{(\lambda-27)(\lambda^2-36\lambda+252)}\right)$$
$$\left(-\frac{(\lambda-28)(\lambda-26)(\lambda^3-60\lambda^2+1084\lambda-5208)}{(\lambda-28)(\lambda^3-61\lambda^2+1136\lambda-5880)}\right)$$
$$\left(-\frac{(\lambda-28)(\lambda-26)(\lambda^2-32\lambda+168)}{\lambda^3-60\lambda^2+1084\lambda-5208}\right)\left(-\frac{(\lambda-28)(\lambda^3-56\lambda^2+916\lambda+3528)}{(\lambda-26)(\lambda^2-32\lambda+168)}\right)$$
$$\left(-\frac{(\lambda-25)(\lambda-28)(\lambda^3-55\lambda^2+864\lambda+2856)}{(\lambda-26)(\lambda^3-56\lambda^2+916\lambda+3528)}\right)$$
$$\left(-\frac{(\lambda-25)(\lambda-28)(\lambda^2-28\lambda+84)}{\lambda^3-55\lambda^2+864\lambda-2856}\right)$$
$$= \lambda(\lambda-32)(\lambda-24)^4(\lambda-28)^{18}$$

Dengan menetapkan $p(\lambda) = 0$, diperoleh nilai eigen $\lambda_1 = 32, \lambda_2 = 24$, dan $\lambda_3 = 28$ dengan multiplisitas masing-masing $m(\lambda_1) = 1, m(\lambda_2) = 4$, dan $m(\lambda_3) = 18$.

Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{24})}$ adalah

$$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) = \sum_{i=1}^{3} m(\lambda_i)|\lambda_i|$$
$$= (1\cdot|32|) + (4\cdot|24|) + (18\cdot|28|)$$
$$= 632.$$

Dari *DLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ di Grup Dihedral yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.23 Tabel pola *DLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$

D_{2n}	n	DLE
D_{12}	6	$148 = 6\left(\frac{14\cdot 6-10}{3}\right) = \frac{14(6)^2-10(6)}{3}$
D_{18}	9	$348 = 9\left(\frac{14\cdot 9-10}{3}\right) = \frac{14(9)^2-10(9)}{3}$
D_{24}	12	$632 = 12\left(\frac{14\cdot 12-10}{3}\right) = \frac{14(12)^2-10(12)}{3}$
$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$n\left(\frac{14n-10}{3}\right) = \frac{14n^2-10n}{3}$

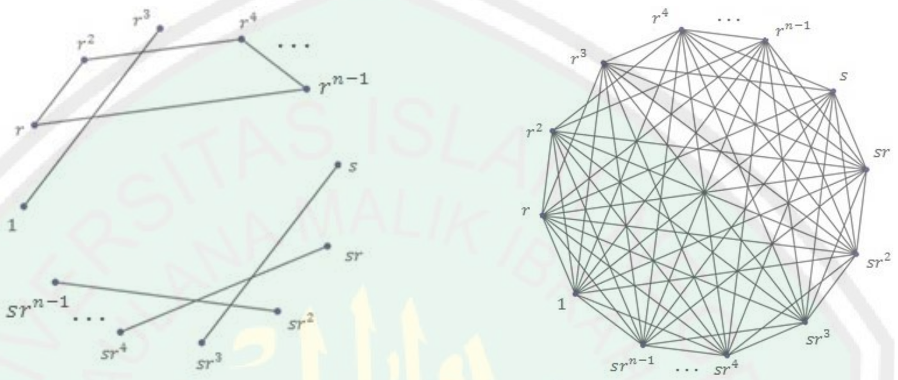
Teorema 5

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r^3\rangle = \{1, r^3, \dots, r^{n-3}\}$ subgrup normal dari dari D_{2n} untuk $n \geq 6$ dan n bilangan kelipatan 3. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) = \frac{14n^2-10n}{3}$$

Bukti:

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ untuk n kelipatan 3 dengan $n \geq 6$. Ambil $\langle r^3 \rangle = \{1, r^3, \dots, r^{n-3}\}$ subgrup normal dari D_{2n}. Sesuai definisi graf subgrup dan komplemennya, maka diperoleh graf $\Gamma_{\langle r^3 \rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}$ yaitu:



Gambar 3.41 Graf $\Gamma_{\langle r^3 \rangle}(D_{2n})$ dan $\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}$

Sehingga diperoleh matriks *distance* dari $\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}$ sebagai berikut

$$
\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}\right) =
\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccc}
1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1}
\end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 2 & \dots & 1 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 0 & 2 & 1 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 0 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
2 & 1 & 1 & 0 & \dots & 1 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
1 & 1 & 2 & 1 & \dots & 0 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 2 & 1 & 1 & \dots & 2 & 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 0 & 1 & 1 & 2 & \dots & 1 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 0 & 1 & 1 & \dots & 2 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 & 0 & 1 & \dots & 1 & 2 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 1 & 0 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 1 & \cdots & 0 & 1 \\
1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 & 2 & 1 & \dots & 1 & 0
\end{bmatrix}
\end{array}
$$

dan matriks transmisi $\overline{\Gamma_{\langle r^3 \rangle}(D_{2n})}$ adalah:

$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) =$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
7n-2 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 7n-1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 7n-1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 7n-2 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 7n-1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 7n-1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 7n-2 & 0 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 7n-2 & 0 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 7n-2 & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 7n-2 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 7n-2 & 0 \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 7n-2
\end{bmatrix}
\end{array}$$

Matriks *distance Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) =$ $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)$ adalah

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ \vdots \\ r^{n-2} \\ r^{n-1} \\ s \\ sr \\ sr^2 \\ sr^3 \\ \vdots \\ sr^{n-2} \\ sr^{n-1} \end{array} &
\begin{bmatrix}
7n-2 & -1 & -1 & -2 & \dots & -1 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & 7n-1 & -2 & -1 & \dots & -1 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -2 & 7n-1 & -1 & \dots & -2 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-2 & -1 & -1 & 7n-2 & \dots & -1 & -1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-1 & -1 & -2 & -1 & \dots & 7n-1 & -2 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -2 & -1 & -1 & \dots & -2 & 7n-1 & -1 & -1 & -1 & -1 & \dots & -1 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & 7n-2 & -1 & -1 & -2 & \dots & -1 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & 7n-2 & -1 & -1 & \dots & -2 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & -1 & 7n-2 & -1 & \dots & -1 & -2 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -2 & -1 & -1 & 7n-2 & \dots & -1 & -1 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & -2 & -1 & -1 & \cdots & 7n-2 & -1 \\
-1 & -1 & -1 & -1 & \dots & -1 & -1 & -1 & -1 & -2 & -1 & \dots & -1 & 7n-2
\end{bmatrix}
\end{array}$$

Polinomial karakteristik $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \lambda\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \lambda\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 1 & r & r^2 & \cdots & sr^{n-1} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1} \end{array} &
\begin{bmatrix}
0 & -\frac{\lambda^2-\left(\frac{14}{3}n-3\right)\lambda+\frac{98}{18}n^2-7n+1}{\lambda-(\frac{7}{3}n-2)} & \cdots & \cdots & \cdots \\
0 & 0 & -\frac{\left(\lambda-\left(\frac{7}{3}n-1\right)\right)\left(\lambda-\left(\frac{7}{3}n+1\right)\right)\left(\lambda-\left(\frac{7}{3}n-4\right)\right)}{\lambda^2-\left(\frac{14}{3}n-3\right)\lambda+\frac{98}{18}n^2-7n+1} & \cdots & \cdots \\
\vdots & \vdots & \vdots & & \cdots \\
0 & 0 & 0 & \cdots & -\frac{\lambda\left(\lambda-\left(\frac{7}{3}n\right)\right)(\lambda-(2n))}{\lambda^2-(2n+2)\lambda+\frac{7}{3}n}
\end{bmatrix}
\end{array}
$$

Maka $\det\left(\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)-\lambda\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D_L}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda) = \lambda\left(\lambda-\frac{8}{3}n\right)(\lambda-2n)^4\left(\lambda-\frac{7}{3}n\right)^{2n-6}$$

Dengan menetapkan $p(\lambda) = 0$ maka diperoleh $\lambda_1 = 0,\ \lambda_2 = \frac{8}{3}n,\ \lambda_3 = 2n$, dan $\lambda_4 = \frac{7}{3}n$ dengan multiplisitas masing-maisng $m(\lambda_1) = 1, m(\lambda_2) = 1,\ m(\lambda_3) = 4$, dan $m(\lambda_4) = 2n-6$. Maka *DLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}$ adalah

$$
\begin{aligned}
ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) &= m(\lambda_1)|\lambda_1| + m(\lambda_2)|\lambda_2| + m(\lambda_3)|\lambda_3| + m(\lambda_3)|\lambda_3| \\
&= (1\cdot|0|) + \left(1\cdot\left|\frac{8}{3}n\right|\right) + (4\cdot|2n|) + \left((2n-6)\cdot\left|\frac{7}{3}n\right|\right) \\
&= \frac{8}{3}n + 8n + \frac{14}{3}n^2 - 14n \\
&= \frac{14n^2-10n}{3}
\end{aligned}
$$

3.5.2 *DSLE* dari Komplemen Graf Subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ dari Grup Dihedral

3.5.2.1 Grup Dihedral D_{18}

Diketahui $D_{18} = \{1, r, r^2, \dots, r^8, s, sr, sr^2, \dots, sr^8\}$. Subgrup yang dibangun oleh $\langle r^3\rangle$ pada grup dihedral D_{18} adalah $\{1, r^3, r^6\}$. Operasi unsur-unsur di D_{18} yang hasilnya berada di $\langle r^3\rangle$ seperti pada tabel sebelumnya. Dengan cara yang sama pada sebelumnya diperoleh matriks *distance signless Laplacian* $\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)} + \boldsymbol{D}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)}$ yaitu

$$\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)} = \begin{array}{c|cccccccccccccccccc|}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 \\
\hline
1 & 19 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r & 1 & 20 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^2 & 1 & 2 & 20 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^3 & 2 & 1 & 1 & 19 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^4 & 1 & 1 & 2 & 1 & 20 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^5 & 1 & 2 & 1 & 1 & 2 & 20 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^6 & 2 & 1 & 1 & 2 & 1 & 1 & 19 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^7 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 20 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^8 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 20 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
s & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 19 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
sr & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 19 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
sr^2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 19 & 1 & 1 & 2 & 1 & 1 & 2 \\
sr^3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 19 & 1 & 1 & 2 & 1 & 1 \\
sr^4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 19 & 1 & 1 & 2 & 1 \\
sr^5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 19 & 1 & 1 & 2 \\
sr^6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 19 & 1 & 1 \\
sr^7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 19 & 1 \\
sr^8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 19
\end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^{+}}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)}$ sebagai berikut

$$p(\lambda^+) = (19-\lambda^+)\left(-\frac{\lambda^{+2}-39\lambda^{+}+379}{\lambda^{+}-19}\right)\left(-\frac{(\lambda^{+}-18)\left(\lambda^{+2}-41\lambda^{+}+416\right)}{\lambda^{+2}-39\lambda^{+}+379}\right)$$

$$\left(-\frac{(\lambda^{+}-17)\left(\lambda^{+2}-43\lambda^{+}+458\right)}{\lambda^{+2}-41\lambda^{+}+416}\right)\left(-\frac{(\lambda^{+}-19)\left(\lambda^{+3}-62\lambda^{+2}+1267\lambda^{+}-8546\right)}{(\lambda^{+}-18)\left(\lambda^{+2}-43\lambda^{+}+458\right)}\right)$$

$$\left(-\frac{(\lambda^{+}-17)(\lambda^{+}-19)\left(\lambda^{+2}-46\lambda^{+}+517\right)}{\lambda^{+3}-62\lambda^{+2}+1267\lambda^{+}-8546}\right)\left(-\frac{(\lambda^{+}-17)\left(\lambda^{+2}-48\lambda^{+}+563\right)}{\lambda^{+2}-46\lambda^{+}+517}\right)$$

$$\left(-\frac{(\lambda^+-19)\left({\lambda^+}^3-66{\lambda^+}^2+1412\lambda^+-9825\right)}{(\lambda^+-17)\left({\lambda^+}^2-48\lambda^++563\right)}\right)$$

$$\left(-\frac{(\lambda^+-16)(\lambda^+-19)\left({\lambda^+}^2-51\lambda^++626\right)}{{\lambda^+}^3-66{\lambda^+}^2+1412\lambda^+-9825}\right)\left(-\frac{{\lambda^+}^3-70{\lambda^+}^2+1586\lambda^+-11708}{{\lambda^+}^2-51\lambda^++626}\right)$$

$$\left(-\frac{(\lambda^+-18)\left({\lambda^+}^3-71{\lambda^+}^2+1628\lambda^+-12148\right)}{{\lambda^+}^3-70{\lambda^+}^2+1586\lambda^+-11708}\right)$$

$$\left(-\frac{(\lambda^+-18)\left({\lambda^+}^3-72{\lambda^+}^2+1670\lambda^+-12588\right)}{{\lambda^+}^3-71{\lambda^+}^2+1628\lambda^+-12148}\right)$$

$$\left(-\frac{(\lambda^+-17)(\lambda^+-19)\left({\lambda^+}^4-93{\lambda^+}^3+3097{\lambda^+}^2-45472\lambda^++246652\right)}{(\lambda^+-18)\left({\lambda^+}^3-72{\lambda^+}^2+1670\lambda^+-12588\right)}\right)$$

$$\left(-\frac{(\lambda^+-21)(\lambda^+-19)\left({\lambda^+}^4-93{\lambda^+}^3+3156{\lambda^+}^2-46626\lambda^++254132\right)}{(\lambda^+-18)\left({\lambda^+}^4-93{\lambda^+}^3+3097{\lambda^+}^2-45472\lambda^++246652\right)}\right)$$

$$\left(-\frac{(\lambda^+-17)(\lambda^+-19)\left({\lambda^+}^3-76{\lambda^+}^2+1847\lambda^+-14534\right)}{{\lambda^+}^4-93{\lambda^+}^3+3156{\lambda^+}^2-46626\lambda^++254132}\right)$$

$$\left(-\frac{(\lambda^+-20)(\lambda^+-17)\left({\lambda^+}^3-77{\lambda^+}^2+1886\lambda^+-14908\right)}{(\lambda^+-19)\left({\lambda^+}^3-76{\lambda^+}^2+1847\lambda^+-14534\right)}\right)$$

$$\left(-\frac{(\lambda^+-20)(\lambda^+-17)\left({\lambda^+}^3-78{\lambda^+}^2+1925\lambda^+-15282\right)}{(\lambda^+-19)\left({\lambda^+}^3-77{\lambda^+}^2+1886\lambda^+-14908\right)}\right)$$

$$\left(-\frac{(\lambda^+-20)(\lambda^+-17)\left({\lambda^+}^2-60\lambda^+-824\right)}{{\lambda^+}^3-78{\lambda^+}^2+1925\lambda^+-15282}\right)$$

$$= (\lambda^+-16)(\lambda^+-38{,}7178)(\lambda^+-21{,}2822)(\lambda^+-20)^3(\lambda^+-19)^4$$
$$(\lambda^+-17)^8$$

Dengan menetapkan $p(\lambda^+)=0$, diperoleh nilai eigen $\lambda_1^+=38{,}7178, \lambda_2^+=21{,}2822, \lambda_3^+=19, \lambda_4^+=17, \lambda_5^+=20,$ dan $\lambda_6^+=16$ dengan multiplisitas masing-masing $m(\lambda_1^+)=1, m(\lambda_2^+)=1, m(\lambda_3^+)=4, m(\lambda_4^+)=8, m(\lambda_5^+)=3,$ dan $m(\lambda_6^+)=1$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{18})}$ adalah

$$ED_L^{+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{18})\right)} = \sum_{i=1}^{6} m(\lambda_i^{+})|\lambda_i^{+}|$$

$$= (1\cdot|38{,}7178|) + (1\cdot|21{,}2822|) + (4\cdot|19|) + (8\cdot|17|) + (3\cdot|20|) + (1\cdot|16|)$$

$$= 348.$$

3.5.2.2 Grup Dihedral D_{24}

Diketahui anggota grup $D_{24} = \{1, r, r^2, \ldots, r^{11}, s, sr, sr^2, \ldots, sr^{11}\}$. Subgrup yang dibangun oleh $\langle r^3\rangle$ pada grup dihedral D_{24} adalah $\{1, r^3, r^6, r^9\}$. Operasi unsur-unsur di D_{24} yang hasilnya berada di $\langle r^3\rangle$ seperti pada tabel sebelumnya. Dengan cara yang sama pada sebelumnya diperoleh matriks *distance signless Laplacian* $\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{24})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{24})\right)} + \boldsymbol{D}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{24})\right)}$ yaitu

$$\boldsymbol{D_L}^{+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{24})\right)} = \begin{array}{c|cccccccccccccccccccccccc|}
 & 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11} \\
1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r & 1 & 27 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^2 & 1 & 2 & 27 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^3 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^4 & 1 & 1 & 2 & 1 & 27 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^5 & 1 & 2 & 1 & 1 & 2 & 27 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^6 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^7 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 27 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^8 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 27 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^9 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^{10} & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 27 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
r^{11} & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 27 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
s & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
sr & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
sr^2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
sr^3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
sr^4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
sr^5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 & 2 \\
sr^6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 & 1 \\
sr^7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 & 1 \\
sr^8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 & 2 \\
sr^9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 & 1 \\
sr^{10} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26 & 1 \\
sr^{11} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 26
\end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right)$ sebagai berikut

$$
\begin{aligned}
p(\lambda^+) &= (26-\lambda^+)\left(-\frac{{\lambda^+}^2-53\lambda^++701}{\lambda^+-26}\right)\left(-\frac{(\lambda^+-25)\left({\lambda^+}^2-55\lambda^++752\right)}{{\lambda^+}^2-53\lambda^++701}\right)\\
&\left(-\frac{(\lambda^+-24)\left({\lambda^+}^2-57\lambda^++808\right)}{{\lambda^+}^2-55\lambda^++752}\right)\left(-\frac{(\lambda^+-26)\left({\lambda^+}^3-83{\lambda^+}^2+2282\lambda^+-20796\right)}{(\lambda^+-25)\left({\lambda^+}^2-57\lambda^++808\right)}\right)\\
&\left(-\frac{(\lambda^+-26)(\lambda^+-24)\left({\lambda^+}^2-60\lambda^++888\right)}{{\lambda^+}^3-83{\lambda^+}^2+2282\lambda^+-20796}\right)\left(-\frac{(\lambda^+-24)\left({\lambda^+}^2-62\lambda^++948\right)}{{\lambda^+}^2-60\lambda^++888}\right)\\
&\left(-\frac{(\lambda^+-26)\left({\lambda^+}^3-87{\lambda^+}^2+2483\lambda^+-23286\right)}{(\lambda^+-24)\left({\lambda^+}^2-62\lambda^++948\right)}\right)\\
&\left(-\frac{(\lambda^+-24)(\lambda^+-23)\left({\lambda^+}^2-65\lambda^++1032\right)}{{\lambda^+}^3-87{\lambda^+}^2+2483\lambda^+-23286}\right)\left(-\frac{(\lambda^+-24)\left({\lambda^+}^2-67\lambda^++1096\right)}{{\lambda^+}^2-65\lambda^++1032}\right)\\
&\left(-\frac{(\lambda^+-26)\left({\lambda^+}^3-91{\lambda^+}^2+2682\lambda^+-25672\right)}{(\lambda^+-24)\left({\lambda^+}^2-67\lambda^++1096\right)}\right)\\
&\left(-\frac{(\lambda^+-22)(\lambda^+-26)\left({\lambda^+}^2-70\lambda^++1184\right)}{{\lambda^+}^3-91{\lambda^+}^2+2682\lambda^+-25672}\right)\left(-\frac{{\lambda^+}^3-96{\lambda^+}^2+2992\lambda^+-30440}{{\lambda^+}^2-70\lambda^++1184}\right)\\
&\left(-\frac{(\lambda^+-25)\left({\lambda^+}^3-97{\lambda^+}^2+3050\lambda^+-31280\right)}{{\lambda^+}^3-96{\lambda^+}^2+2992\lambda^+-30440}\right)\\
&\left(-\frac{(\lambda^+-25)\left({\lambda^+}^3-98{\lambda^+}^2+3108\lambda^+-32120\right)}{{\lambda^+}^3-97{\lambda^+}^2+3050\lambda^+-31280}\right)\\
&\left(-\frac{(\lambda^+-24)\left({\lambda^+}^4-125{\lambda^+}^3+5738{\lambda^+}^2-115160\lambda^++855280\right)}{(\lambda^+-25)\left({\lambda^+}^3-98{\lambda^+}^2+3108\lambda^+-32120\right)}\right)\\
&\left(-\frac{(\lambda^+-26)(\lambda^+-24)\left({\lambda^+}^4-126{\lambda^+}^3+5820{\lambda^+}^2-117392\lambda^++875440\right)}{(\lambda^+-25)\left({\lambda^+}^4-125{\lambda^+}^3+5738{\lambda^+}^2-115160\lambda^++855280\right)}\right)\\
&\left(-\frac{(\lambda^+-26)(\lambda^+-24)\left({\lambda^+}^3-102{\lambda^+}^2+3352\lambda^+-35824\right)}{{\lambda^+}^4-126{\lambda^+}^3+5820{\lambda^+}^2-117392\lambda^++875440}\right)\\
&\left(-\frac{(\lambda^+-24)\left({\lambda^+}^4-130{\lambda^+}^3+6188{\lambda^+}^2-128560\lambda^++987408\right)}{(\lambda^+-26)\left({\lambda^+}^3-102{\lambda^+}^2+3352\lambda^+-35824\right)}\right)
\end{aligned}
$$

$$\left(-\frac{(\lambda^{+}-27)(\lambda^{+}-24)\left({\lambda^{+}}^{4}-131{\lambda^{+}}^{3}+6270{\lambda^{+}}^{2}-130792\lambda^{+}+1007568\right)}{(\lambda^{+}-26)\left({\lambda^{+}}^{4}-130{\lambda^{+}}^{3}+6188{\lambda^{+}}^{2}-128560\lambda^{+}+987408\right)}\right)$$

$$\left(-\frac{(\lambda^{+}-27)(\lambda^{+}-24)\left({\lambda^{+}}^{3}-106{\lambda^{+}}^{2}-3596\lambda^{+}-39528\right)}{{\lambda^{+}}^{4}-131{\lambda^{+}}^{3}+6270{\lambda^{+}}^{2}-130792\lambda^{+}+1007568}\right)$$

$$\left(-\frac{(\lambda^{+}-28)(\lambda^{+}-24)\left({\lambda^{+}}^{3}-107{\lambda^{+}}^{2}-3650\lambda^{+}+40248\right)}{(\lambda^{+}-27)\left({\lambda^{+}}^{3}-106{\lambda^{+}}^{2}-3596\lambda^{+}-39528\right)}\right)$$

$$\left(-\frac{(\lambda^{+}-24)(\lambda^{+}-28)\left({\lambda^{+}}^{3}-108{\lambda^{+}}^{2}+3704\lambda^{+}-40968\right)}{(\lambda^{+}-27)\left({\lambda^{+}}^{3}-107{\lambda^{+}}^{2}-3650\lambda^{+}+40248\right)}\right)$$

$$\left(-\frac{(\lambda^{+}-24)(\lambda^{+}-28)\left({\lambda^{+}}^{2}-82\lambda^{+}+1544\right)}{{\lambda^{+}}^{3}-108{\lambda^{+}}^{2}+3704\lambda^{+}-40968}\right)$$

$$= (\lambda^{+}-22)(\lambda^{+}-52{,}7047)(\lambda^{+}-29{,}2953)(\lambda^{+}-28)^{3}(\lambda^{+}-26)^{6}(\lambda^{+}-24)^{12}$$

Dengan menetapkan $p(\lambda^{+}) = 0$, diperoleh nilai eigen $\lambda_1^{+} = 52{,}7047, \lambda_2^{+} = 29{,}2953, \lambda_3^{+} = 28, \lambda_4^{+} = 24, \lambda_5^{+} = 22$, dan $\lambda_6^{+} = 26$ dengan multiplisitas masing-masing $m(\lambda_1^{+}) = 1, m(\lambda_2^{+}) = 1, m(\lambda_3^{+}) = 3, m(\lambda_4^{+}) = 12, m(\lambda_5^{+}) = 1$, dan $m(\lambda_6^{+}) = 6$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{24})}$ adalah

$$\begin{aligned} ED_L^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{24})}\right) &= \sum_{i=1}^{6} m(\lambda_i^{+})|\lambda_i^{+}| \\ &= (1\cdot|52{,}7047|) + (1\cdot|29{,}2953|) + (3\cdot|28|) + (12\cdot|24|) + (1\cdot|22|) + (6\cdot|26|) \\ &= 632. \end{aligned}$$

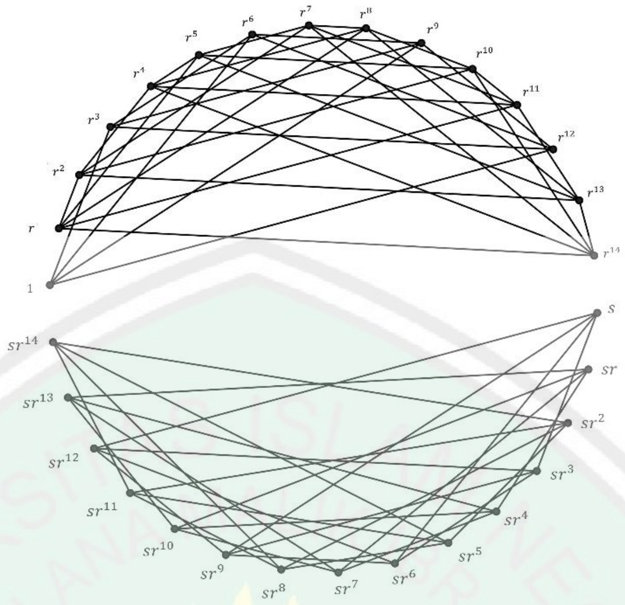
3.5.2.3 Grup Dihedral D_{30}

Diketahui $D_{30} = \{1, r, r^2, \ldots, r^{14}, s, sr, sr^2, \ldots, sr^{14}\}$. Subgrup yang dibangun oleh $\langle r^3 \rangle$ pada grup dihedral D_{30} adalah $\{1, r^3, r^6, r^9, r^{12}\}$. Operasi unsur-unsur di D_{30} yang hasilnya berada di $\langle r^3 \rangle$ seperti tabel berikut

Tabel 3.24 Tabel Cayley Grup Dihedral D_{30}

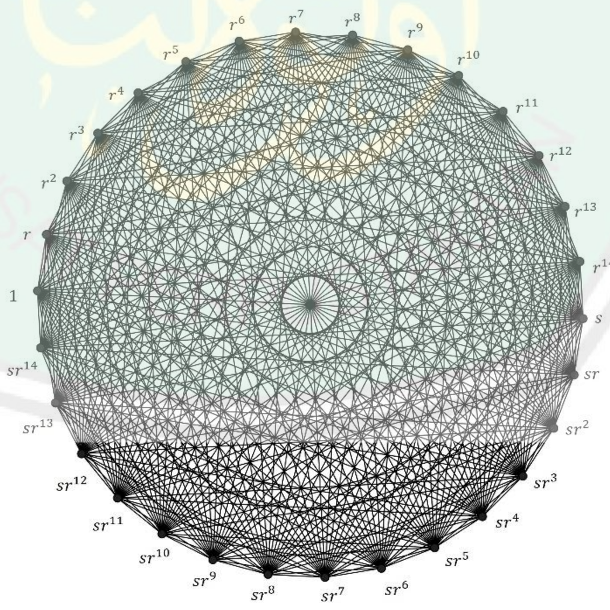
$\circ$	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}
1	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}
r	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}
r^2	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}
r^3	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}
r^4	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}
r^5	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9
r^6	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8
r^7	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7
r^8	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6
r^9	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5
r^{10}	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4
r^{11}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3
r^{12}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2
r^{13}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr
r^{14}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s
s	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}
sr	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}
sr^2	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}
sr^3	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}
sr^4	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}
sr^5	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9
sr^6	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8
sr^7	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6	r^7
sr^8	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5	r^6
sr^9	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4	r^5
sr^{10}	sr^{10}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3	r^4
sr^{11}	sr^{11}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2	r^3
sr^{12}	sr^{12}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r	r^2
sr^{13}	sr^{13}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1	r
sr^{14}	sr^{14}	s	sr	sr^2	sr^3	sr^4	sr^5	sr^6	sr^7	sr^8	sr^9	sr^{10}	sr^{11}	sr^{12}	sr^{13}	r	r^2	r^3	r^4	r^5	r^6	r^7	r^8	r^9	r^{10}	r^{11}	r^{12}	r^{13}	r^{14}	1

Dari definisi graf subgrup, maka diperoleh graf subgrup $\Gamma_{\langle r^3 \rangle}(D_{30})$ sebagai berikut



Gambar 3 .42 Graf $\Gamma_{\langle r^3 \rangle}(D_{30})$

Dari Gambar 3.42 dapat diperoleh komplemen graf subgrup $\overline{\Gamma_{\langle r^3 \rangle}(D_{30})}$ sebagai berikut



Gambar 3.43 Graf $\overline{\Gamma_{\langle r^3 \rangle}(D_{30})}$

Matriks *distance* $\boldsymbol{D}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{30})}\right)$dan matriks transmisi $\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{30})}\right)$ sebagai berikut

$\left(\overline{\Gamma_{\langle r^3\rangle}(D_{30})}\right) =$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccccccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & r^{12} & r^{13} & r^{14} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11} & sr^{12} & sr^{13} & sr^{14} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ r^9 \\ r^{10} \\ r^{11} \\ r^{12} \\ r^{13} \\ r^{14} \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \\ sr^9 \\ sr^{10} \\ sr^{11} \\ sr^{12} \\ sr^{13} \\ sr^{14} \end{array} &
\begin{bmatrix}
0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0
\end{bmatrix}
\end{array}$$

$\boldsymbol{T_r}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{30})}\right) =$

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccccccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & r^{12} & r^{13} & r^{14} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11} & sr^{12} & sr^{13} & sr^{14} \end{array} \\
\begin{array}{c} 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ r^9 \\ r^{10} \\ r^{11} \\ r^{12} \\ r^{13} \\ r^{14} \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \\ sr^9 \\ sr^{10} \\ sr^{11} \\ sr^{12} \\ sr^{13} \\ sr^{14} \end{array} &
\begin{bmatrix}
0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 34 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 33
\end{bmatrix}
\end{array}$$

Matriks *distance signless Laplacian* yang diperoleh dari perhitungan $\boldsymbol{D_L}^+\overline{\left(\Gamma_{\langle r^3\rangle}(D_{30})\right)} = \boldsymbol{T_r}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{30})\right)} + \boldsymbol{D}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{30})\right)}$ adalah

$$\boldsymbol{D_L}^+\overline{\left(\Gamma_{\langle r^3\rangle}(D_{30})\right)} =$$

$$\begin{array}{c} \\ 1 \\ r \\ r^2 \\ r^3 \\ r^4 \\ r^5 \\ r^6 \\ r^7 \\ r^8 \\ r^9 \\ r^{10} \\ r^{11} \\ r^{12} \\ r^{13} \\ r^{14} \\ s \\ sr \\ sr^2 \\ sr^3 \\ sr^4 \\ sr^5 \\ sr^6 \\ sr^7 \\ sr^8 \\ sr^9 \\ sr^{10} \\ sr^{11} \\ sr^{12} \\ sr^{13} \\ sr^{14} \end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccccccccccccccccc} 1 & r & r^2 & r^3 & r^4 & r^5 & r^6 & r^7 & r^8 & r^9 & r^{10} & r^{11} & r^{12} & r^{13} & r^{14} & s & sr & sr^2 & sr^3 & sr^4 & sr^5 & sr^6 & sr^7 & sr^8 & sr^9 & sr^{10} & sr^{11} & sr^{12} & sr^{13} & sr^{14} \end{array} \\
\begin{bmatrix}
33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 34 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 34 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 34 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 34 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 34 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 34 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 34 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 34 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 34 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 34 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 & 2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 33
\end{bmatrix}
\end{array}$$

Dengan cara yang sama pada sebelumnya diperoleh polinomial karakteristik $\boldsymbol{D_L^+}\overline{\left(\Gamma_{\langle r^3\rangle}(D_{30})\right)}$ sebagai berikut

$$p(\lambda^+) = (\lambda^+ - 28)\left(\lambda^+ - 52 - 6\sqrt{6}\right)\left(\lambda^+ - 52 + 6\sqrt{6}28\right)(\lambda^+ - 36)^3$$

$$(\lambda^+ - 33)^8(\lambda^+ - 31)^{16}$$

Dengan menetapkan $p(\lambda^+) = 0$, diperoleh nilai eigen $\lambda_1^+ = 28, \lambda_2^+ = 52 - 6\sqrt{6}, \lambda_3^+ = 52 + 6\sqrt{6}28, \lambda_4^+ = 36, \lambda_5^+ = 33,$ dan $\lambda_6^+ = 31$ dengan multiplisitas masing-masing $m(\lambda_1^+) = 1, m(\lambda_2^+) = 1, m(\lambda_3^+) = 1, m(\lambda_4^+) = 3, m(\lambda_5^+) = 8,$ dan $m(\lambda_6^+) = 16.$ Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{30})}$ adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^3\rangle}(D_{30})}\right) = \sum_{i=1}^{6} m(\lambda_i^+)|\lambda_i^+|$$

$$= (1\cdot|28|) + \left(1\cdot\left|52-6\sqrt{6}\right|\right) + \left(1\cdot\left|52+6\sqrt{6}\right|\right) + (3\cdot |36|) + (8\cdot|33|) + (16\cdot|31|)$$

$$= 1000.$$

Dari *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ di Grup Dihedral yang telah ditemukan, dapat dilihat pada tabel berikut

Tabel 3.25 Tabel pola *DSLE* pada komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$

D_{2n}	n	*DSLE*
D_{18}	9	$348 = 9\left(\frac{14\cdot 9-10}{3}\right) = \frac{14(9)^2-10(9)}{3}$
D_{24}	12	$632 = 12\left(\frac{14\cdot 12-10}{3}\right) = \frac{14(12)^2-10(12)}{3}$
D_{30}	15	$1000 = 15\left(\frac{14\cdot 15-10}{3}\right) = \frac{14(15)^2-10(15)}{3}$
$\vdots$	$\vdots$	$\vdots$
D_{2n}	n	$n\left(\frac{14n-10}{3}\right) = \frac{14n^2-10n}{3}$

Teorema 6

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ dan $\langle r^3\rangle = \{1, r^3, \dots, r^{n-3}\}$ subgrup normal dari D_{2n} untuk $n \geq 9$ dan n kelipatan 3. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{2n})$ dari grup dihedral D_{2n} adalah

$$ED_L^+\overline{\left(\Gamma_{\langle r^3\rangle}(D_{2n})\right)} = \frac{14n^2-10n}{3}$$

Bukti:

Misalkan grup dihedral $D_{2n} = \{1, r, r^2, \dots, r^{n-1}, s, sr, sr^2, \dots, sr^{n-1}\}$ untuk n kelipatan 3 dengan $n \geq 9$. Ambil $\langle r^3\rangle = \{1, r^3, \dots, r^{n-3}\}$ subgrup normal dari D_{2n}. Dengan cara yang sama pada sebelumnya dapat diperoleh matriks *distance signless Laplacian* $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ yaitu

$$\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) =$$

$$\begin{array}{c|cccccccccccccc|}
 & 1 & r & r^2 & r^3 & \dots & r^{n-2} & r^{n-1} & s & sr & sr^2 & sr^3 & \dots & sr^{n-2} & sr^{n-1} \\
r & 1 & 7n-1 & 2 & 1 & \dots & 1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
r^2 & 1 & 2 & 7n-1 & 1 & \dots & 2 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
r^3 & 2 & 1 & 1 & 7n-2 & \dots & 1 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
r^{n-2} & 1 & 1 & 2 & 1 & \dots & 7n-1 & 2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
r^{n-1} & 1 & 2 & 1 & 1 & \dots & 2 & 7n-1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\
s & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 7n-2 & 1 & 1 & 2 & \dots & 1 & 1 \\
sr & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 7n-2 & 1 & 1 & \dots & 2 & 1 \\
sr^2 & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 & 7n-2 & 1 & \dots & 1 & 2 \\
sr^3 & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 2 & 1 & 1 & 7n-2 & \dots & 1 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
sr^{n-2} & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 2 & 1 & 1 & \cdots & 7n-2 & 1 \\
sr^{n-1} & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 & 1 & 2 & 1 & \dots & 1 & 7n-2
\end{array}$$

Polinomial karakteristik $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)$ diperoleh dari $\det\left(\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \lambda^{+}\boldsymbol{I}\right)$. Dengan eliminasi baris elementer pada $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \lambda^{+}\boldsymbol{I}$ diperoleh matriks segitiga atas berikut

$$\begin{array}{c}
 \\ 1 \\ r \\ r^2 \\ \vdots \\ sr^{n-1}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & r & r^2 & \dots & sr^{n-1} \end{array} \\
\begin{bmatrix}
\left(\frac{7}{3}n-2\right)-\lambda & \dots & \dots & \dots & \dots \\
0 & -\frac{\lambda^2-\left(\frac{14}{3}n-3\right)\lambda+\frac{98}{18}n^2-7n+1}{\lambda-\left(\frac{7}{3}n-2\right)} & \dots & \dots & \dots \\
0 & 0 & -\frac{\left(\lambda-\left(\frac{7}{3}n-3\right)\right)\left(\lambda^2-\left(-\frac{2}{9}n^2+10n-31\right)\lambda-\frac{14}{18}n^2+147n-844\right)}{\lambda^2-\left(\frac{14}{3}n-3\right)\lambda+\frac{98}{18}n^2-7n+1} & \dots & \dots \\
\vdots & \vdots & \vdots & & \dots \\
0 & 0 & 0 & \dots & -\frac{\left(\lambda-\left(\frac{7}{3}n-4\right)\right)\left(\lambda-\left(\frac{8}{3}n-4\right)\right)\left(\lambda^2-\left(\frac{22}{3}n-6\right)\lambda+\frac{224}{18}n^2-\frac{64}{3}n+1\right)}{f(\lambda)}
\end{bmatrix}
\end{array}$$

Maka $\det\left(\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) - \lambda^{+}\boldsymbol{I}\right)$ merupakan perkalian unsur-unsur diagonal utama matriks segitiga atas tersebut. Sehingga diperoleh polinomial karakteristik dari $\boldsymbol{D}_{\boldsymbol{L}}^{+}\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right)$, yaitu:

$$p(\lambda^{+}) = (\lambda - (2n-2))\left(\lambda^2 - \left(\frac{22}{3}n - 6\right)\lambda + \left(\frac{224}{18}n^2 - \frac{64}{3}n + 1\right)\right)\left(\lambda - \left(\frac{8}{3}n - 4\right)\right)^3 \left(\lambda - \left(\frac{7}{3}n - 4\right)\right)^{\frac{2}{3}n-2} \left(\lambda - \left(\frac{7}{3}n - 4\right)\right)^{\frac{4}{3}n-4}$$

Dengan menetapkan $p(\lambda^{+}) = 0$ maka diperoleh $\lambda_1^{+} = 2n-2, \lambda_2^{+} = \left(\frac{11}{3}n - 3\right) - \sqrt{n^2 - \frac{2}{3}n + 1}, \lambda_3^{+} = \left(\frac{11}{3}n - 3\right) + \sqrt{n^2 - \frac{2}{3}n + 1}, \lambda_4^{+} = \frac{8}{3}n - 4, \lambda_5^{+} = \frac{7}{3}n - 4$, dan $\lambda_6^{+} = \frac{7}{3}n - 4$ dengan multiplisitas masing-masing $m(\lambda_1^{+}) = 1, m(\lambda_2^{+}) = 1, m(\lambda_3^{+}) = 1, m(\lambda_4^{+}) = 3, m(\lambda_5^{+}) = \frac{2}{3}n - 2$, dan $m(\lambda_6^{+}) = \frac{4}{3}n - 4$. Maka *DSLE* dari komplemen graf subgrup $\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}$ adalah

$$\begin{aligned}
ED_L^{+}\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) &= \sum_{i=1}^{6} m(\lambda_i)|\lambda_i| \\
&= (1\cdot|2n-2|) + \left(1\cdot\left|\left(\frac{11}{3}n - 3\right) - \sqrt{n^2 - \frac{2}{3}n + 1}\right|\right) + \\
&\quad \left(1\cdot\left|\left(\frac{11}{3}n - 3\right) + \sqrt{n^2 - \frac{2}{3}n + 1}\right|\right) + \left(3\cdot\left|\frac{8}{3}n - 4\right|\right) + \\
&\quad \left(\left(\frac{2}{3}n - 2\right)\cdot\left|\frac{7}{3}n - 4\right|\right) + \left(\left(\frac{4}{3}n - 4\right)\cdot\left|\frac{7}{3}n - 4\right|\right) \\
&= \frac{14n^2 - 10n}{3}.
\end{aligned}$$

3.6 Perhitungan Energi dalam Pandangan Islam

Anjuran tolong menolong antar sesama manusia dalam firman Allah Swt surat al-Maidah ayat 2 telah dijelaskan dalam Bab II, di mana Allah Swt memerintahkan kepada hamba-Nya untuk saling tolong menolong dalam kebajikan dan taqwa. Berdasarkan penjelasan ayat وَتَعَاوَنُوا عَلَى الْبِرِّ وَالتَّقْوَى *"Dan tolong menolonglah kamu dalam (mengerjakan) kebajikan dan takwa"* di dalam tafsir Al-Qurtubi bahwa Allah Swt menganjurkan saling tolong menolong dalam kebajikan yang disertai dengan ketakwaan. Karena dengan ketakwaan akan menghadirkan keridhaan Allah. Selain itu Al Mawardi berkata bila terdapat keridhaan manusia dalam mengerjakan kebajikan dan disertai keridhaan Allah Swt maka sempurnalah kebahagian dan nikmat akan diluaskan oleh Allah Swt. Ini menunjukkan bahwa saling menolong antar sesama manusia dalam hal kebajikan dan takwa sangat penting dilakukan.

Dengan meneladani anjuran di dalam al-Quran untuk saling tolong menolong dalam hal kebajikan tersebut, maka bagi peneliti salah satu bentuk menolong atau memudahkan urusan orang lain adalah menentukan nilai energi dari beberapa graf yang akan dipolakan, di mana pola tersebut menjadi suatu rumus yang disertai bukti. Langkah pertama dalam menentukan energi dari suatu graf adalah menggambar grafnya terlebih dahulu. Gambar graf tersebut diperoleh dari perhitungan operasi komposisi antar anggota suatu grup. Jika hasil komposisi tersebut adalah anggota subgrup, maka dua titik yang dikomposisikan dapat digambarkan graf yang terhubung. Sebaliknya, jika hasil komposisi bukan anggota subgrup, maka dua titik yang dikomposisikan tidak digambarkan graf yang terhubung. Selanjutnya dari gambar graf tersebut dapat diperoleh matriks.

Matriks pertama yang harus diperoleh adalah matriks *distance*. Dalam menentukan matriks *distance* peneliti harus mengetahui panjang lintasan terpendek dari setiap titik pada suatu graf. Matriks-matriks *distance* dari beberapa graf dapat dipola umumkan. Begitupula dengan matriks transmisi, *distance Laplacian*, dan *distance signless Laplacian*.

Berdasarkan matriks dari suatu graf tersebut dapat diperoleh suatu nilai eigen berbeda dan multiplisitas masing-masing dari nilai eigen. Setelah nilai eigen dan multiplitasnya diperoleh, maka dapat ditentukan nilai energi suatu graf dengan penjumlahan setiap nilai eigen berbeda yang dimutlakkan dengan dikalikan multiplisitas masing-masing. Oleh karena itu, nilai energi dari beberapa graf yang telah diperoleh dapat dirumuskan.

Dengan demikian, berdasarkan surat al-Maidah ayat 2 mengenai anjuran untuk saling tolong menolong dalam kebaikan, kita bisa melihat bahwa Allah Swt selalu memudahkan jalan hamba-Nya untuk saling tolong menolong sesuai dengan kemampuan masing-masing. Sebagai seorang hamba dengan kewajiban untuk terus memperbanyak pahala dari Allah Swt, maka sepatutnya melaksanakan setiap anjuran didalam al-Quran, termasuk ahli matematika dalam menentukan suatu rumus energi dari suatu graf yang dapat diterapkan dalam bidang teori graf aljabar.

BAB IV

PENUTUP

4.1 Kesimpulan

Berdasarkan pembahasan maka dapat diperoleh kesimpulan beberapa pola umum *DLE* dan *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral D_{2n}.

1. *DLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral D_{2n} adalah.

   a. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_{2n})$ untuk $n \geq 3$ dan *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah

   $$ED_L\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

   b. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^2\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah

   $$ED_L\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$

   c. *DLE* dari komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{2n})$ untuk $n \geq 6$ dengan n kelipatan tiga adalah

   $$ED_L\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) = \frac{14n^2 - 10n}{3}$$

2. *DSLE* dari komplemen graf subgrup $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$ dari grup dihedral D_{2n} adalah

a. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r\rangle}(D_{2n})$ untuk $n \geq 3$ dan *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2,s\rangle}(D_{2n})$, $\Gamma_{\langle r^2,sr\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2n})}\right) = ED_L^+\left(\overline{\Gamma_{\langle r^2,sr\rangle}(D_{2n})}\right) = 6n^2 - 4n$$

b. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^2\rangle}(D_{2n})$ untuk $n \geq 4$ dengan n bilangan genap adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^2\rangle}(D_{2n})}\right) = 5n^2 - 4n$$

c. *DSLE* dari komplemen graf subgrup $\Gamma_{\langle r^3\rangle}(D_{2n})$ untuk $n \geq 9$ dengan n kelipatan tiga adalah

$$ED_L^+\left(\overline{\Gamma_{\langle r^3\rangle}(D_{2n})}\right) = \frac{14n^2 - 10n}{3}$$

4.2 Saran

Penelitian ini hanya menentukan energi dari komplemen graf subgrup dari grup dihedral untuk subgrup normal $\langle r\rangle, \langle r^2\rangle, \langle r^2, s\rangle, \langle r^2, sr\rangle$, dan $\langle r^3\rangle$. Penelitian selanjutnya diharapkan dapat menemukan teorema terkait energi dari komplemen graf subgrup dari grup dihedral untuk subgrup normal yang lainnya atau dari grup lainnya.

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Nabila Umar, lahir di Gresik pada tanggal 15 Februari 1997, biasa dipanggil Nabila. Anak ke-lima dari lima bersaudara dari pasangan bapak Umar (Alm) dan ibu Siti Asiyah.

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No	Tanggal	Hal	Tanda Tangan
1.	16 Januari 2019	Konsultasi BAB I, II	1.
2.	15 Januari 2019	Konsultasi Keagamaan BAB I	2.
3.	28 Januari 2019	Konsultasi BAB II &III	3.
4.	4 Februari 2019	Konsultasi & Revisi BAB II	4.
5.	6 Februari 2019	Konsultasi BAB IV	5.
6.	7 Februari 2019	Konsultasi Keagamaan BAB II	6.
7.	7 Februari 2019	ACC Keagamaan	7.
8.	7 Februari 2019	ACC BAB I, II, III	8.
9.	30 April 2019	Konsultasi Keagamaan BAB IV	9.
10.	7 Mei 2019	Konsultasi BAB IV & V	10.
11.	9 Mei 2019	ACC Keagamaan	11.
12.	9 Mei 2019	ACC Keseluruhan	12.

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